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THE RATIONAL EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE AT THE POLISH INTERBANK MARKET

1. INTRODUCTION

In this paper we report the results of testing for the rational expectations hypothesis (REH) at the interbank market in Poland. When doing so we utilize a set of monthly sampled WIBORs (Warsaw Interbank Offered Rates) for maturities of 1, 3, 6, 9 and 12 months from the period January 1999-December 2007. The analysis draws on the co-integrating technique and methodology proposed in Campbell and Shiller [5], [6], and extended in Tzavalis and Wickens [27], and Cuthbertson and Nitzsche [11]. We use a three-variable vector autoregressive model (VAR) including the yield spread, the change in the short-term interest rate and the excess holding period return and provide with the set of restrictions on its parameters and statistics to test for the time-varying term premium (TVP) and to link short term interest rates changes best forecasts (theoretical spreads) with actual spreads. We find much support for the REH with the TVP in the data.

The rest of the paper proceeds as follows. Section 2 introduces the REH of the term structure with the TVP, describes how it can be tested for within the VAR framework and summarizes the recent results in that field. Section 3 discusses our empirical findings. The last section briefly concludes.

2. RATIONAL EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE WITH THE TIME-VARYING TERM PREMIUM

2.1.THEORETICAL FRAMEWORK

The REH of the term structure that allows for a time-varying term premium (REHTVP) claims that the expected one-period holding period return on a bond that has n periods to maturity equals the return on one-period bond increased by the term premium, i.e. (see [27], p. 231 and [11], p. 419)

$$E_t h_{t+1}^{(n)} = E_t \left[\ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} \right] = R_t^{(1)} + \theta_t^{(n)}, \tag{1}$$

where: $P_t^{(n)}$ is the price at time t of pure discount bond with face value of Pln 1 and n periods to maturity, $R_t^{(1)}$ is the certain (riskless) one-period interest rate, E_t is the expectations operator conditional on information available in period t, and $\theta_t^{(n)}$ is a time-varying term premium, perceived at time t, which compensates for the risk of investing in long-term bonds. In case $\theta_t^{(n)} = 0$ or to some other constant the REH stands in its pure form (PREH).

Relation (1) indicates that the expected excess one-period holding period return, $E_t h_{t+1}^{(n)} - R_t^{(1)}$, reflects changes in the (one-period) time-varying term premium, $\theta_t^{(n)}$. Since $\ln P_t^{(n)} = -nR_t^{(n)}$, where $R_t^{(n)}$ is the spot yield on the long-term bond,

$$h_{t+1}^{(n)} = nR_t^{(n)} - (n-1)R_{t+1}^{(n-1)} = n\left[R_t^{(n)} - (n-1/n)R_{t+1}^{(n-1)}\right], n \geq 1. \tag{2}$$

Substituting (2) into the expected one-period holding period return, $E_t h_{t+1}^{(n)}$, and rearranging terms leads to

$$E_t\left[R_{t+1}^{(n-1)} - R_t^{(n)}\right] = (1/n-1)\left[R_t^{(n)} - R_t^{(1)}\right] + (1/n-1)\theta_t^{(n)}, \tag{3}$$

which iterated forwards gives

$$R_t^{(n)} = (1/n)\sum_{i=0}^{n-1} E_t R_{t+i}^{(1)} + \Theta_t^{(n)}. \tag{4}$$

Equation (4) shows that the n-period interest rate is the average expected one-period interest rate over n periods increased by the rolling over term premium, $\Theta_t^{(n)} = (1/n)\sum_{i=0}^{n-1} E_t \theta_{t+i}^{(n-i)}$, which itself is the average (arithmetic mean) of the current and expected future one-period holding premia, $\theta_{t+i}^{(n-i)} (i = 0, 1, \ldots, n-1)$.

Subtracting $R_t^{(1)}$ from both sides of equation (4), after some manipulations yields

$$S_t^{(n,1)} = E_t \sum_{i=1}^{n-1}(1 - i/n)\,\Delta R_{t+i}^{(1)} + \Theta_t^{(n)}, \tag{5}$$

so that the observed yield spread should equal the optimal forecast of future changes in the short-term rates, $E_t \sum_{i=1}^{n-1}(1 - i/n)\,\Delta R_{t+i}^{(1)}$ ('perfect foresight spread'), and the rolling over term premium, conditional on information available to market participants. At time t no other information apart from that contained in the yield spread and the rolling over term premium should help predict future changes in the short-term interest rates. A weak implication of the latter is that $S_t^{(n,1)}$ Granger causes $\Delta R_{t+i}^{(1)}$. Additionally, in the case term premium $\theta_t^{(n)}$ is not time-varying, the expected excess one-period holding period return is a constant and should not depend upon its past values as well as past values of the actual spread and changes in the future short-term interest rates.

Equation (4) can be used to decompose an unanticipated change ('surprise') in the one-period holding period return, $eh_{t+1}^{(n)} = h_{t+1}^{(n)} - E_t h_{t+1}^{(n)}$. Substituting (4) into $eh_{t+1}^{(n)}$ gives (see [27], p. 232 and [11], pp. 421-422)

$$eh_{t+1}^{(n)} = -(E_{t+1} - E_t)\sum_{i=1}^{n-1} R_{t+i}^{(1)} - (E_{t+1} - E_t)\sum_{i=1}^{n-1} \theta_{t+i}^{(n-i)} = -\left[eR_{t+1}^{(1)} + e\theta_{t+1}^{(n)}\right], \qquad (6)$$

where $eR_{t+1}^{(1)} = (E_{t+1} - E_t)\sum_{i=1}^{n-1} R_{t+i}^{(1)}$ is the 'news' about future short-term interest rates, and $e\theta_{t+1}^{(n)} = (E_{t+1} - E_t)\sum_{i=1}^{n-1} \theta_{t+i}^{(n-i)}$ is the 'news' about future term premia. The first term is due to a revision to expectations about future short-term interest rates $R_{t+i}^{(1)}$, the latter is due to a revision to expectations about the future term premia, $\theta_{t+i}^{(n-i)}\,(i = 1, 2, \ldots, n-1)$.

2.2. TESTING FOR THE REHTVP WITHIN THE VAR FRAMEWORK

In testing for the REHTVP within the VAR framework we follow Tzavalis and Wickens [27] and Cuthbertson and Nitzsche [11] who extended the two-variable VAR of Campbell and Shiller [5], [6] including the yield spread, $S_t^{(n,1)} = R_t^{(n)} - R_t^{(1)}$, and the change in the short-term interest rate, $\Delta R_t^{(1)}$, into its three-variable counterpart. They added a third variable, the ex-post excess one-period holding period return, $h_t^{(n)} - R_{t-1}^{(1)}$. Then, having its parameters estimated, they proposed several metrics to test for the validity of the REHTVP and to find out the proportions in which an unanticipated change in the one-period holding period return can be attributed to the 'news' about future short-term interest rates and the 'news' about future term premia. In doing so we proceed as follows.

Firstly, we assume that both the long and the short-term interest rates are $I(1)$ variables. Equations (1) and (5) imply that in such circumstances $x = \left[S_t^{(n,1)} \;\; \Delta R_t^{(1)} \;\; h_t - R_{t-1}^{(1)}\right]'$ is a stationary vector process. Next we claim that x_t can be written as a demeaned VAR of order $p, \tilde{x}_t = \sum_{j=1}^{p} A_j \tilde{x}_{t-j} + \xi_t$, where $A_j = \begin{bmatrix} a_{11}^{(j)} & a_{12}^{(j)} & a_{13}^{(j)} \\ a_{21}^{(j)} & a_{22}^{(j)} & a_{23}^{(j)} \\ a_{31}^{(j)} & a_{32}^{(j)} & a_{33}^{(j)} \end{bmatrix}$ and $\xi_t = \left[\xi_{1t} \;\; \xi_{2t} \;\; \xi_{3t}\right]'$. Since such a system can be stacked into a companion form, $z_t = Az_{i-1} + u_t$, where:

$$A = \begin{bmatrix} A_1 & A_2 & \ldots & A_{p-1} & A_p \\ I_3 & 0 & \ldots & 0 & 0 \\ 0 & I_3 & \ldots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \ldots & I_3 & 0 \end{bmatrix}_{(3p \times 3p)}, z_t = \left[\tilde{x}_t \;\; \tilde{x}_{t-1} \;\ldots\; \tilde{x}_{t-p+1}\right]', \text{ and}$$

$u_t = \left[\xi_{1,t} \;\; \xi_{2,t} \;\; \xi_{3,t} \;\; 0 \;\ldots\; 0\right]'$, vector z_t summarizes the whole history of x_t up to time t. We employ vector z_t as our information set. We also note that $E\left(z_{t+k} \middle| x_t, x_{t-1}, \ldots\right) = E\left(z_{t+k} \middle| z_t\right) = A^k z_t$.

Now, using $(3p \times 1)$ selection vectors $e1$, $e2$ and $e3$ with unity in the first, second, and third rows, respectively, and zeros elsewhere, we are able to pick out form the VAR the actual spread, $S_t^{(n,1)} = e1'z_t$, the change in the short-term interest rate, $\Delta R_t^{(1)} = e2'z_t$, the excess one-period holding period return, $h_t^{(n)} - R_{t-1}^{(1)} = e3'z_t$, and use them to forecast the expected excess one-period holding period return, $E_t h_{t+1}^{(n)} - R_t^{(1)}$, as well as the future changes in short-term interest rates, $\Delta R_{t+i}^{(1)}$.

The prediction of the expected excess one-period holding period return from the VAR is $E_t h_{t+1}^{(n)} - R_t^{(1)} = e3'z_{t+1} = e3'Az_t$, which in the case of time-invariant term premium should equal to some constant. Since all variables in the system are expressed as deviations from their means, the above requires a set consisted of $3p$ linear restrictions to be such that $e3'A = 0$. This is tested with the use of a Wald test. The rejection of the null hypothesis stating that all parameters in the third equation of the VAR are zeros indicates that the data supports the REHTVP of the term structure. Otherwise, the PREH is more likely.

The linear prediction of the yield spread from the VAR ('theoretical spread') can be expressed as (see [6], [1])

$$S_t^{*(n,1)} = E_t \sum_{i=1}^{n-1} (1 - i/n)\, \Delta R_{t+i}^{(1)} = e2'A\left[I - (1/n)(I - A^n)(I - A)^{-1}\right](I - A)^{-1} z_t = e2'\Lambda z_t, \quad (7)$$

where $\Lambda = A\left[I - (1/n)(I - A^n)(I - A)^{-1}\right](I - A)^{-1}$. In the absence of time-varying expectations about the rolling over term premium $\Theta_t^{(n)}$ it should equal the actual spread, $S_t^{(n,1)} = e1'z_t$. If this is the case, the set of nonlinear cross-equation restrictions on the VAR parameters is imposed, $f(a) = e1' - e2'\Lambda = 0$, that can be tested for with the use of a Wald test. The relevant test statistics,

$$W = f(a)' \times \left[\frac{\partial f(a)}{\partial a'} \hat{\Sigma}_{aa} \frac{\partial f(a)}{\partial a}\right]^{-1} \times f(a), \quad (8)$$

where $\hat{\Sigma}_{aa}$ is either the standard or the Eicker-White heteroscedasticity consistent variance-covariance matrix of VAR parameters estimator, under standard conditions of error term u_t is distributed as the χ^2 variable with $3p$ degrees of freedom.

As Campbell and Shiller [6] note that formal tests of the VAR restrictions may lead to the rejection of the PREH even though deviations of $S_t^{(n,1)}$ from $S_t^{*(n,1)}$ are negligible, additional metrics are used to compare behaviour of both series. They include variance ratio $\sigma^2\left[S_t^{*(n,1)}\right]/\sigma^2\left[S_t^{(n,1)}\right]$ and correlation coefficient $corr\left[S_t^{(n,1)}, S_t^{*(n,1)}\right]$, which both should be close to unity if the REH is true and expectations about the rolling over term premium are not time-varying. Additionally, the graphs of two series should move together.

Since $corr\left[S_t^{(n,1)}, S_t^{*(n,1)}\right] \Big/ \sqrt{\sigma^2\left[S_t^{*(n,1)}\right]/\sigma^2\left[S_t^{(n,1)}\right]}$ is the unbiased OLS estimate of the slope coefficient in the regression of actual spread onto theoretical spread that should equal unity if the PREH adequately reflects the term structure, either both the correlation and the standard deviation estimates must be close to unity or one of them must be an approximate inverse of the other (see [27], p. 236).

If the standard deviations ratio is less (more) than unity and correlation is close to unity, then the slope would be less (more) than unity and the actual spread is more (less) volatile than the theoretical spread, the optimal predictor of future short rates. That is to say long-term interest rates overreact (underreact) to the current available information about future short-term interest rates. In the case neither the slope nor the correlation coefficient are close to unity, the actual spread behaves differently from the theoretical spread and the overreaction (underreaction) could be the consequence of a time-varying risk premium.

The decomposition of an unanticipated change in the one-period holding period return is based on the VAR residuals. To proceed recall equation (6) and note that

$$eR_{t+1}^{(1)} = (E_{t+1} - E_t)\sum_{i=1}^{n-1} R_{t+1}^{(1)} = (E_{t+1} - E_t)\left[(n-1)R_t^{(1)} + \sum_{i=1}^{n-1}\sum_{j=1}^{i} \Delta R_{t+j}^{(1)}\right] = \\ = (E_{t+1} - E_t)\sum_{i=1}^{n-1}\sum_{j=1}^{i} \Delta R_{t+j}^{(1)}. \tag{9}$$

This implies that the 'news' about future term premia can be calculated as (see [10], p. 396 and [11], p. 422)

$$e\theta_{t+1}^{(n)} = -eR_{t+1}^{(1)} - eh_{t+1}^{(n)} = \\ = e2'\left[(n-1)I + (n-2)A + (n-3)A^2 + \ldots + \left(n(n-1)A^{n-2}\right)\right]u_{t+1} - \xi_{3,t+1}. \tag{10}$$

The first term in equation (10) reflects the 'news' about future short-term interest rates, the second term reflects the 'surprise' in the holding period return. The A^s matrices represent the degree of persistence in the 'news' about future short term interest rates ($s = 1, 2, \ldots, n-2$).

If a revision to expectations about future term premia is very small $\left(e\theta_{t+1}^{(n)} \approx 0\right)$, it is deducted that $eh_{t+1}^{(n)} \approx -eR_{t+1}^{(1)}$, which in turn results in $\sigma^2\left[eR_{t+1}^{(1)}\right]/\sigma^2\left[eh_{t+1}^{(n)}\right] \approx 1$ and $corr\left[eR_{t+1}^{(1)}, eh_{t+1}^{(n)}\right] \approx -1$. In view of $h_{t+1}^{(n)} - R_t^{(1)} = \theta_t^{(n)} - eR_{t+1}^{(1)} - e\theta_{t+1}^{(n)}$ it is concluded that $(1 - R^2)$ of the one-period holding period return equation in the VAR indicates a proportion of the excess holding period return that is due to variation in the 'news' about future short rates.

2.3. PREVIOUS RESEARCH ON THE REHTVP USING THE VAR

The empirical research on the REHTVP in financial markets employing a three-variable VAR is rather rare. The exemptions include that of Tzavalis and Wickens [27], Cuthbertson and Bredin [10], Cuthbertson and Nietzsche [11] and Tzavalis [26]. All of them except for Cuthbertson and Bredin [10] deliver much support for the TVP. Nevertheless, it is revealed that in all cases the volatility of the ex-post excess holding period return is almost solely the result of a revision to expectations about the future short-term interest rates.

Tzavalis and Wickens [27] use annualized and continuously compounded monthly estimates of the yields to maturity for zero-coupon 3, 6, and 12 month US government bonds from the period January 1970-February 1992, taken from McCulloch and Kwon [25]. On the ground of integration and co-integration analysis they ascertain that the variables included in the VAR are stationary. The VAR metrics enable them to reject the null hypothesis stating that the term premia are not time-varying, but the volatility of ex-post excess holding period returns is found to be due mainly to the 'news' about the short-term interest rates and not to variation in term premia.

The findings of Cuthbertson and Bredin [10] relate to the Irish spot rates for maturities of 5, 10 and 15 years. The data set from January 1989 through October 1997 is sampled monthly. The general conclusion stemming from their analysis is that the term premia are constant. This is reached upon the appropriate Wald test statistics which supports $H_0 : e3' A = 0$. The other VAR metrics to a great extent support $H_0 : S_t^{(n,1)} = S_t^{*(n,1)}$ and prove a negligible role of the 'news' about future term premia in explaining any possible variation in the ex-post excess holding period returns.

Cuthbertson and Nitzsche [11] examine the UK spot rates calculated by the Bank of England from coupon bonds for several maturities from 2 to 25 years. The data are sampled monthly and cover the period February 1975-December 1998. The findings support the existence of the stationary TVP but only for the long maturity bonds ($n = 15, 20, 25$ years). The one-period term premia for these maturities are not persistent however, and have relatively small impact on the rolling over term premium relative to changes in expectations about future short-term rates. An almost all the variation in ex-post excess one-period return depends on revisions to forecasts about the future short-term interest rates.

Tzavalis [26] refers to the US zero-coupon bonds for maturities from 1 to 120 months. The data are from January 1947 to February 1991, and sampled monthly. The sample is split to reflect 4 different monetary regimes: an introduction and abolishment of interest rate targeting policy by the Fed, an introduction of financial innovations in the bond market, and the use of the spread as an indicator variable of inflation stabilization procedure by the Fed's governors, respectively. The main findings include those that the term premia seem to vary cyclically over time reflecting changes in the business cycle conditions of the US economy rather than the monetary regimes changes occurred during the sample and a lesser volatility of the rolling over term premium comparing to that of the holding over premia can explain the yield spread better forecasts the cumulative changes of future short-term rates than the change in long rates.

3. EMPIRICAL RESULTS

The empirical analysis begins with testing for nonstationarity of the individual WIBORs, and the variables entering the VAR (the actual yield spread, the change in short-term interest rate and the ex-post excess one-period holding period return). For testing purposes we employ the DF-GLS, the KPSS and the augmented Dickey-Fuller tests (see Elliot, Rothenberg, Stock [15], Kwiatkowski, Phillips, Schmidt, Shin [23], and Dickey, Fuller [14]). Their results are gathered in Table 1. Following the indications of the KPSS and ADF tests we conclude that all the variables entering the VAR, i.e. $S_t^{(n,1)}$, $\Delta R_t^{(1)}$ and $h_t^{(n)} - R_{t-1}^{(1)}$, are integrated of order zero ($I(0)$).

Table 1

Testing for unit roots and stationarity results

Variable	Test statistics					
	DF-GLS		KPSS		DF/ADF	
	$t_{DF\text{-}GLS}$	lag	KPSS	lag	t_{ADF}	lag
WIBORs						
1M	-1.790	3	0.117	12	-1.335[1]	0
3M	-1.321	1	0.120	12	-1.177[1]	1
6M	-1.674	2	0.122	12	-1.082[1]	1
9M	-0.410	1	0.169	11	**-2.780**[1]	1
12M	-0.499	1	0.166	11	**-2.621**[1]	1
Changes in WIBORs						
1M	-2.089	2	0.092	12	**-10.540**[1]	0
3M	-2.522	1	0.096	12	**-6.488**[1]	0
6M	-2.543	1	0.099	12	**-5.975**[1]	0
9M	**-4.556**[a]	1	0.109	11	**-5.445**[3]	0
12M	**-4.461**[a]	1	0.106	11	**-5.268**[3]	0
Yield spreads						
3M1M	**-4.797**[a]	1	0.101	11	**-6.669**[1]	0
6M1M	**-3.452**[a]	1	0.099	11	**-5.135**[1]	0
9M1M	-2.730	1	0.130	10	**-2.292**[1]	0
12M1M	-2.565	1	0.125	10	**-2.655**[1]	0
Ex-post excess one-period holding period returns						
3M	-1.874	3	0.094	12	**-5.757**[3]	0
6M	**-2.749**[b]	1	0.100	12	**-2.101**[1]	3
9M	**-5.139**[a]	1	0.099	11	**-2.337**[1]	2
12M	**-4.820**[a]	1	0.097	11	**-7.864**[1]	0

Source: own computations with the use of STATA 10.0 (DF-GLS and KPSS tests) and GAUSS 8.0 (ADF/DF test).

Notes: The maximal lag length (11 or 12, not reported here) is determined with the use of the Schwert rule. The optimal lag (third, fifth and seventh column) is set on the ground of either the Schwarz (DF-GLS and DF/ADF tests) or the Newey-West criterion (KPSS test), see Hobijn *et al*. [17]. Values of the test statistics causing the rejection of the null [series is I(1) in the DF-GLS and the DF/ADF tests; series is stationary around trend in case of the KPSS test] are in bold. Types of the DF/ADF auxiliary regressions: (1) no deterministic components, (2) constant present in the regression, (3) both constant and trend present in the regression. (a) [(b)] – significant at 5 [10] per cent significance level in the DF-GLS test, respectively; 1 and 5 per cent significance levels are used the KPSS and the DF/ADF tests, respectively; in case of the DF/ADF critical values are calculated with the use of response surfaces estimated by Cheung and Lai [4].

Some additional evidence for their stationarity emerges from Figure 1, Panels A-C, in which the relevant series are plotted against time. It also seems that they all exhibit decreasing volatility. The $\Delta R_t^{(1)}$ series (displayed in Panel C) at a slow pace converges to about a zero rate rendering the ability of the National Bank of Poland to lower the inflation and then to keep the short-term interest rate within declared bands.

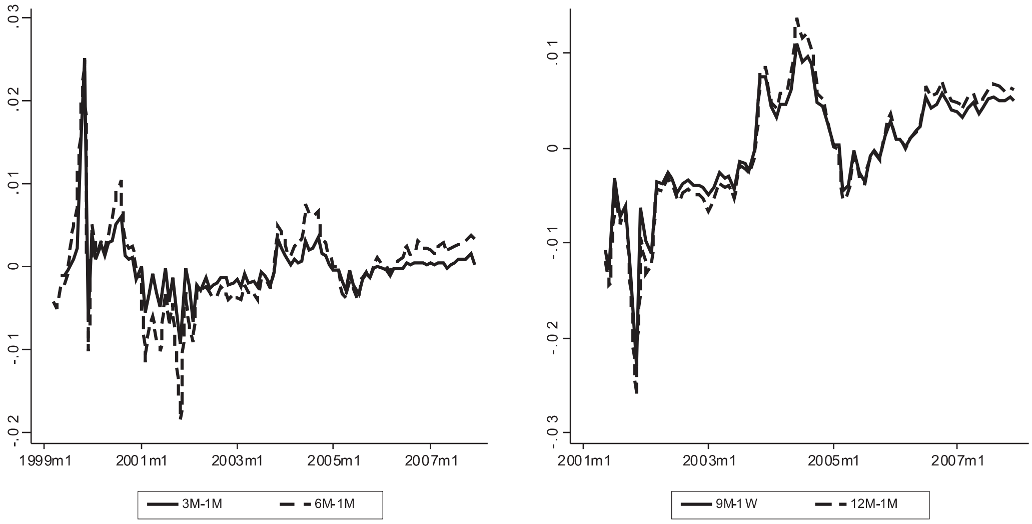
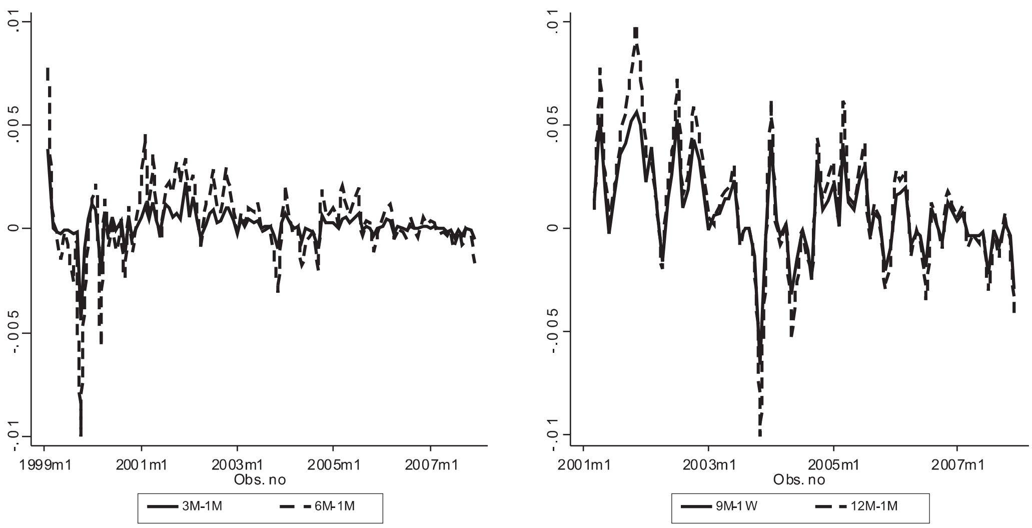
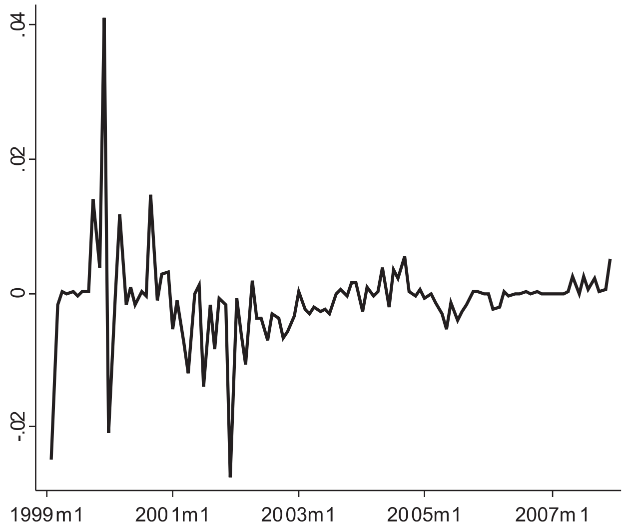


Figure 1. A. Actual yield spreads (3,1), (6,1), (9,1) and (12,1)



B. Ex-post excess one-period holding period returns (3,1), (6,1), (9,1) and (12,1)



C. Change in the short-term interest rate, $n = 1$

The results from the VAR models are stacked in Table 2. Lag p of the VAR for each maturity is set with the use of Akaike information criterion but occasionally increased to remove autocorrelation in residuals.

Table 2

Summary statistics for VAR $x_t = \left[S_t^{(n,1)}\ \Delta R_t^{(1)}\ h_t - R_{t-1}^{(1)}\right]$

$(n,1)$	VAR order	Autocorrelation[(a)]						R^2			Granger non-causality[(b)]
		LM(6)			Ljung-Box(6)						
		$S_t^{(n,1)}$	$\Delta R_t^{(1)}$	$h_t^n - R_{t-1}^{(1)}$	$S_t^{(n,1)}$	$\Delta R_t^{(1)}$	$h_t^n - R_{t-1}^{(1)}$	$S_t^{(n,1)}$	$\Delta R_t^{(1)}$	$h_t^n - R_{t-1}^{(1)}$	
(3,1)	3	9.3866 (0.1530)	5.1536 (0.5243)	4.1843 (0.6517)	5.1205 (0.5285)	3.7204 (0.7145)	1.0467 (0.9838)	0.647	0.811	0.316	139.1482 (0.0000)
(6,1)	1	6.1165 (0.4103)	8.7349 (0.1890)	5.7092 (0.4565)	4.9026 (0.5564)	8.4647 (0.2060)	3.7979 (0.7040)	0.470	0.656	0.182	29.2001 (0.0000)
(9,1)	2	10.4850 (0.1057)	15.5144 (0.0166)	5.3848 (0.4955)	4.4925 (0.6103)	10.8101 (0.0944)	4.2767 (0.6393)	0.803	0.642	0.311	36.1775 (0.0007)
(12,1)	2	7.8351 (0.2504)	13.0788 (0.0418)	5.7112 (0.4563)	3.7396 (0.7119)	9.4194 (0.1513)	5.7873 (0.4474)	0.825	0.658	0.306	45.5992 (0.0000)

Source: own computations with the use of GAUSS 8.0..

Notes: [(a)] Estimates of the LM and the Ljung-Box test statistics for autocorrelation of order 6 under the null of no autocorrelation distributed as χ^2 (6); the relevant p-values in brackets under the estimates. [(b)] Estimates of the test statistics for Granger non-causality from $S_t^{(n,1)}$ to $\Delta R_t^{(1)}$ under the null $\left[a_{21}^{(1)} = a_{21}^{(2)} = \dots = a_{21}^{(p)} = 0\right]$ distributed as χ^2 (p) variable; the relevant p-values in brackets under the estimates.

Each equation in the system has a relatively large explanatory power as reflected by their coefficient of determination estimates. Nevertheless a lot of unexplained variation in the ex-post excess one-period holding return equation is left to be attributed to revisions to the expectations about future short-term interest rates and future term premia. The estimates of individual coefficients pertaining the lagged excess returns in this equation and their corrected for heteroscedasticity standard deviations (if necessary), not reported here but available for inspection on request, are indicative for the term premium moderate persistence. The ability of the yield spread to predict future changes in the one-month WIBOR is proved for all maturities using the test statistics for Granger non-causality.

Table 3

VAR restrictions and other metrics

$(n,1)$	Excess one-period returns not time-varying	Actual spread $S_t^{(n,1)}$ and theoretical spread $S_t^{*(n,1)}$			
	$e3'A = 0$[(a)]	$S_t^{*(n,1)} = S_t^{(n,1)}$[(a)]	$\sigma^2\left[S_t^{*(n,1)}\right]/\sigma^2\left[S_t^{(n,1)}\right]$		$corr\left[S_t^{*(n,1)}, S_t^{(n,1)}\right]$[(b)]
			var. ratio[(b)]	conf. inter.	
(3,1)	W(9) = 43.8186 (0.0000)	W(9) = 49.4394 (0.0000)	1.3415 (0.2704)	1.0117 2.0137	0.9542 (0.0241)
(6,1)	W(3) = 17.0415 (0.0007)	W(3) = 9.0681 (0.0284)	2.0474 (0.3677)	1.2631 2.7089	0.9972 (0.0033)
(9,1)	W(6) = 43.6861 (0.0000)	W6 = 13.9933 (0.0297)	2.2353 (0.5767)	0.8664 3.0835	0.9910 (0.0117)
(12,1)	W(6) = 44.3434 (0.0000)	W6 = 11.1146 (0.0849)	2.5164 (0.7538)	0.7858 3.6522	0.9923 (0.0176)

Source: own computations with the use of GAUSS 8.0.

Notes: [(a)] The relevant p-values in brackets under the Wald test statistics estimates. [(b)] The relevant standard errors from the bootstrap under the variance ratio and the correlation estimates. The recursive bootstrap has been applied with 50000 replications. The bootstrap series have been used to estimate the VAR, and then to compute artificial ‘actual’ and ‘theoretical’ spreads, their correlation coefficients, variance ratios and confidence intervals.

It should be stressed however that we obtain somewhat unclear picture of the autocorrelation for the change in short-term interest rate equation for 9 and 12 month WIBORs. On the one hand the estimates of the Ljung-Box test statistics used to test for no autocorrelation of up to the 6-th order are far away from the critical value of the χ^2 variable with 6 degrees of freedom at the conventional 5 per cent significance level. On the other hand, however, the Lagrange Multiplier type test rejects the null hypothesis in both cases at the approximate 1.7 and 4.2 per cent significance levels. Since the Ljung-Box test statistics is biased downwards in autoregressive models, we rely on the LM test and conclude that the coefficients from the second VAR equation are not consistently estimated. In such circumstances for these two maturities much caution should be retained when further predictions about the theoretical spread as well as predictions based upon all VAR metrics employing $\Delta R_{t+i}^{(1)}$ are made[1].

Table 3 reports the results of testing for the validity of the REHTVP using the restrictions set on VAR parameters and the other metrics. Restriction $e3' A = 0$ is strongly rejected for all maturities. The same conclusion is reached on the ground of the Wald test statistics for $H_0 : S_t^{*(n,1)} = S_t^{(n,1)}$ for all interest rates but the 12 month WIBOR. In this case, however, the null hypothesis is rejected at 10 per cent significance level. Thus we suspect that the term premia are time-varying for all maturities.

Inspection of Figure 2 on which scatter plots of the actual spread versus the theoretical spread as well as their co-movement across time are displayed gives somewhat mixed evidence on the term premium variability. The empirical points on all scatter plots are highly concentrated around the approximate 45-degree straight line indicating that for all n correlation between $S_t^{(n,1)}$ and $S_t^{*(n,1)}$ is nearly unity. The estimates of $corr\left[S_t^{*(n,1)}, S_t^{(n,1)}\right]$ differing from unity by less than their two standard deviations enhance this preposition. For $n = 3$ and 6 this holds at the edge, however.

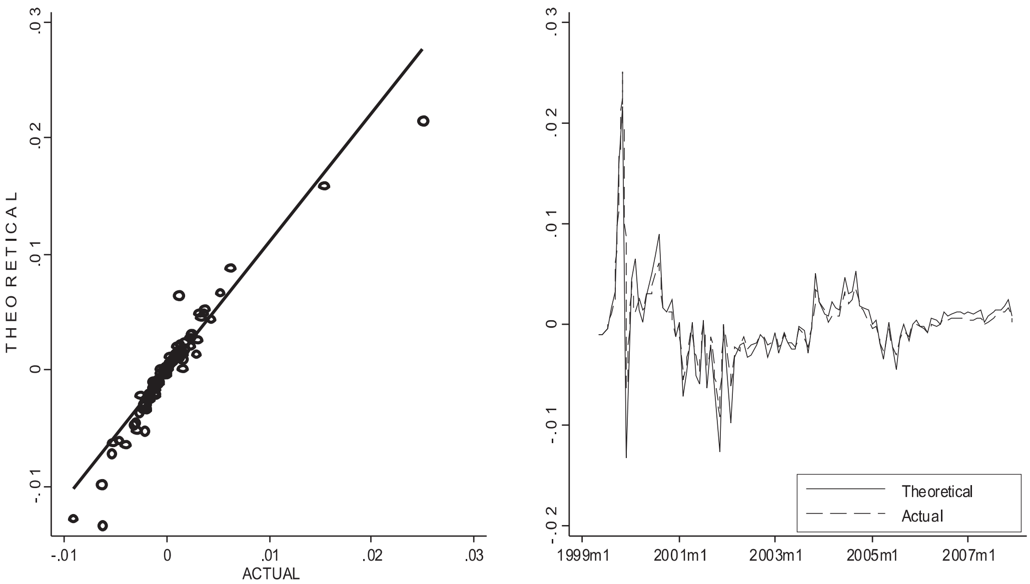
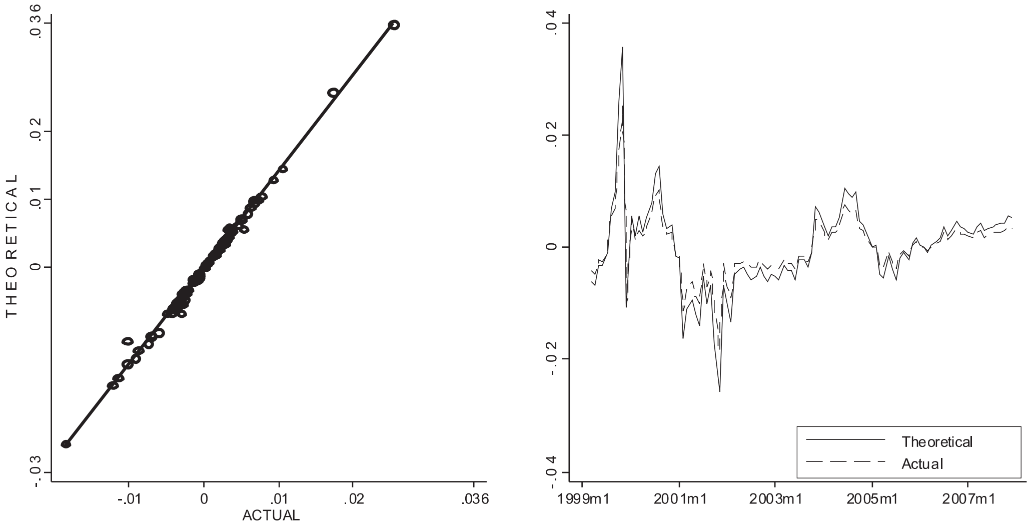
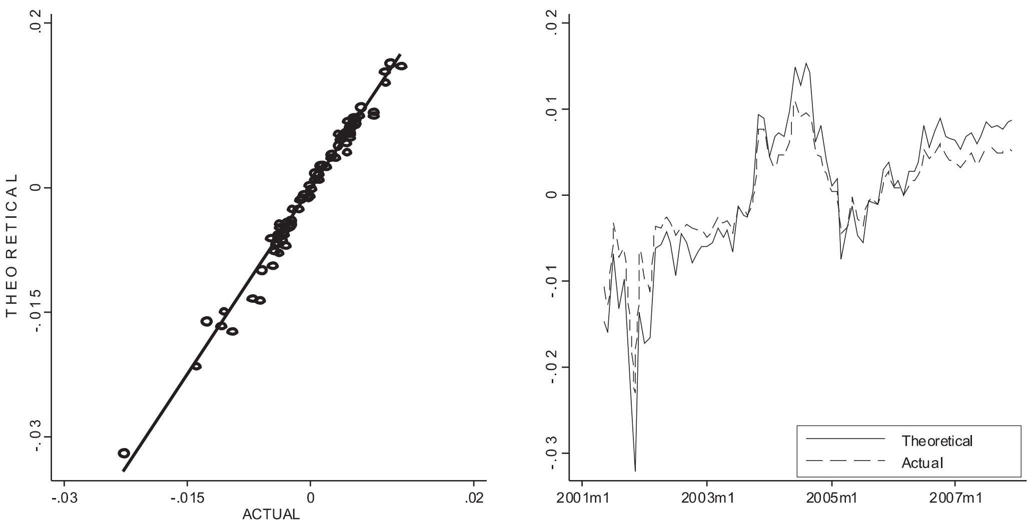


Figure 2. A. Actual and theoretical spread (3,1)

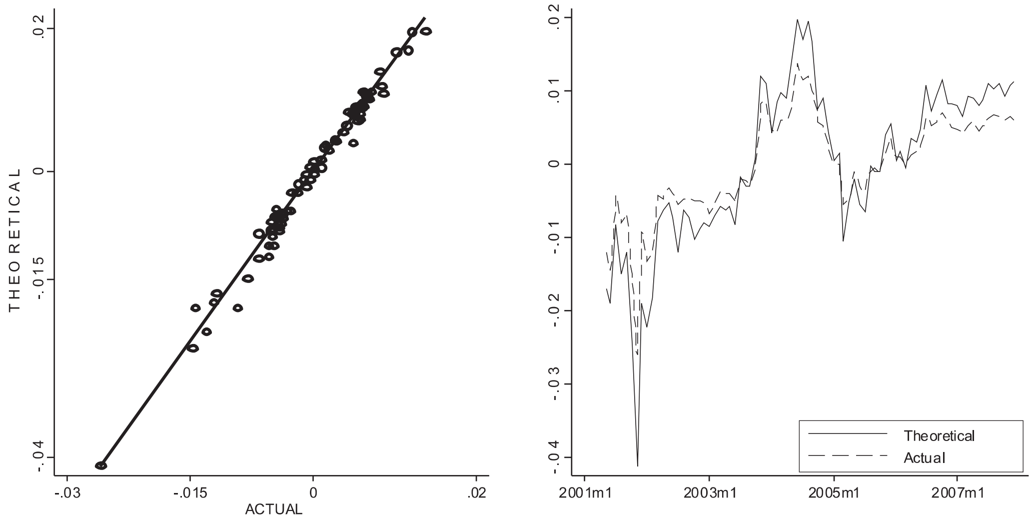
[1] For 9 and 12 month maturities we are not able to remove autocorrelation in residuals without critically overparametrizing the VAR. Setting lag p in the VAR up to 12 does not help to solve for autocorrelated residuals.



B. Actual and theoretical spread (6,1)



C. Actual and theoretical spread (9,1)



D. Actual and theoretical spread (12,1)

Although for all maturities the actual and the theoretical spread series seem to move together across time, the two shorter and the two longer WIBORs display quite different volatility. Departure from unity at most by two standard deviations for the $\sigma^2\left[S_t^{*(n,1)}\right]/\sigma^2\left[S_t^{(n,1)}\right]$ estimate is found only for the 3 month WIBOR while the 95 per cent confidence interval for the variance ratio covering unity is found for 9 and 12 month maturities. Thus we can conclude this part of the analysis stating that the time-varying term premium is likely only for the two shorter WIBORs. The two longer are rather supposed to underreact to the current available information about future changes in the one-month WIBOR. The latter crucially depends on the consistency of parameter estimates from the second VAR equation, however.

In Table 4 we compare the behaviour of unexpected one-period holding period return series $eh_{t+1}^{(n)} = h_{t+1}^{(n)} - E_t h_{t+1}^{(n)}$ with the 'news' about future changes in short-term interest rates $eR_{t+1}^{(1)}$. The estimates of correlation coefficient $corr\left[eR_{t+1}^{(1)}, eh_{t+1}^{(n)}\right]$ are close to minus unity for all maturities. The estimates of variance ratio $\sigma^2\left[eR_{t+1}^{(1)}\right]/\sigma^2\left[eh_{t+1}^{(n)}\right]$ differ from unity by less than their two standard deviations for $n = 3$, 6 and 9, and their estimated 95 per cent confidence intervals cover unity only for $n = 3$ and 6. Thus we can safely conclude for these two shorter maturities. Since the coefficient of determination R^2 estimates in the excess one-period holding period return equations for these maturities equal to 0.316 and 0.182, respectively, the proportion of the excess holding period return that is due to variation in the 'news' about future short-term interest rates counts to 69.4 and 88.8 per cent.

Table 4

Variance decomposition: news about short rates and one-period returns

$(n,1)$	$\sigma^2\left[eR^{(1)}_{t+1}\right]/\sigma^2\left[eh^{(n)}_{t+1}\right]$		$corr\left[eR^{(1)}_{t+1}, eh^{(n)}_{t+1}\right]^{(a)}$
	var. ratio[a]	conf. inter.	
(3,1)	0.5843 (0.2092)	0.3688 1.0769	-0.9458 (0.0260)
(6,1)	1.2953 (0.1969)	0.9312 1.7063	-0.9897 0.0094
(9,1)	1.8357 (0.4439)	1.0153 2.7567	-0.9882 (0.0178)
(12,1)	2.5821 (0.7560)	1.1736 4.1053	-0.9803 (0.0244)

Source: own computations with the use of GAUSS 8.0.

Notes: [a] The relevant standard errors from the bootstrap under the variance ratio and the correlation estimates. The recursive bootstrap has been applied with 50000 replications. The bootstrap series have been used to estimate the VAR, and then to compute artificial 'actual' and 'theoretical' spreads, their correlation coefficients, variance ratios and confidence intervals.

Our results complete and extend those reported by Konstantinou [22] who worked on the set of daily sampled 1 week to 6 month WIBORs from June 1995 to February 2003. Using the co-integrating technique of Johansen (see [19] and [20]) he revealed the existence of the co-integrating properties of the yield spreads coherent with the PREH, but on the boundary of the acceptance region. Although Campbell-Shiller VAR implies a vector error correction model with the yield spreads being the co-integrating vectors (see King, Kurmann [21], Appendix C), his remaining results do not contradict time-variability of the term premia. For instance, regressing the 'perfect foresight' spread onto the actual spread he proved that the latter is a biased predictor of cumulative changes in the future one-week WIBOR and the estimates of correlation coefficient between $S_t^{(n,1)}$ and $S_t^{*(n,1)}$ for 1, 3 and 6 month maturities were rather very distant from unity. The same but to a lesser extent he observed for the variance ratio estimates. Nonetheless, the both metrics are not accompanied by their standard deviations.

4. CONCLUSION

The aim of the paper has been to examine whether the REH with the TVP provides with the adequate description of the Polish interbank term structure. The analysis has been placed within a three-variable VAR including the yield spread, the change in the short-term interest rate and the ex-post excess one-period holding period return, where the latter variable enables to capture the movements of the term premium.

Having its parameters estimated we find that the equations in the system have a relatively large explanatory power. Nevertheless a lot of unexplained variation in the ex-post excess one-period holding period return equation is left to be attributed to revisions to the expectations about future short-term interest rates and future term premia. We also proved the yield spread has a strong predictive power.

For all maturities we reject (at the different but reasonable significance levels, from $\alpha = 0.0001$ to 0.1) the restrictions set on the VAR parameters that the one-period excess holding period return is not time varying ($e3'\,A = 0$ and $S_t^{*(n,1)} = S_t^{(n,1)}$), however the variance ratios of theoretical spread to the actual spread and their correlation coefficients estimated form the VARs enable us to sustain this for the two shorter WIBORs ($n = 3{,}6$). The other two ($n = 9{,}12$) we suppose to underreact to the current available information about future changes in the one-month WIBOR.

Finally, comparing the behaviour of unexpected one-period holding period return with the 'news' about future changes in short-term interest rates series estimated from the VAR we conclude that the proportion of the excess one-period holding period return that is due to variation in the 'news' about future short rates counts to 69.4-88.8 per cent, at least for the shorter WIBORs.

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REFERENCES

[1] Bataa E., Kim D.H., Osborn D.R., [2006], *Does Spread Really Predict the Short Rate? Explaining Empirical Anomalies in the Expectations Theory*, Centre for Growth and Business Cycle Research, Discussion Papers Series, No. 84.

[2] Blangiewicz M., Miłobędzki P., [2008], *Is there a nonlinear mean reversion in the term structure of interest rates at the Polish interbank market?*, [in:] Soares J.O, Pina J., Catalão-Lopes M. (eds.), New Developments in Financial Modeling, Cambridge Scholar Publishing, pp. 77-94.

[3] Blangiewicz M., Miłobędzki P., *The term structure of interest rates at the Polish interbank market. A VAR approach*, [in] Milo W., Wdowiński P. (eds.), Forecasting Financial Markets. Theory and Applications, Wydawnictwo Uniwersytetu Łódzkiego, forthcoming.

[4] Cheung Y.-W., Lai K.S., [1995], *Lag Order and Critical Values of the Augmented Dickey-Fuller Test*, „Journal Business & Economic Statistics", Vol. 13, No. 3, pp. 277-280.

[5] Campbell J.Y., Shiller R.J., [1987], *Cointegration and Tests of Present Value Models*, „Journal of Political Economy", Vol. 95, pp. 1062-1088.

[6] Campbell J.Y., Shiller R.J., [1991], *Yield Spreads and Interest Rates Movements: A Bird's Eye View*, „Review of Economic Studies", Vol. 58, pp. 495-514.

[7] Cuthbertson K., [1996], *The Expectations Hypothesis of The Term Structure: The UK Interbank Market*, „Economic Journal", Vol. 106, pp. 578-592.
[8] Cuthbertson K., Hayes S., Nitzsche D., [1996], *The Behaviour of Certificate of Deposit Rates in the UK*, Oxford Economic Papers, Vol. 48, pp. 397-414.
[9] Cuthbertson K., Hayes S., Nitzsche D., [2000], *Are German Money Market Rates Well Behaved?*, „Journal of Economic Dynamics & Control", Vol. 24, pp. 347-360.
[10] Cuthbertson K., Bredin D., [2001], *Risk Premia and Long Rates in Ireland*, Journal of Forecasting, Vol. 20, pp. 391-403.
[11] Cuthbertson K., Nitzsche D., [2003], *Long Rates, Risk Premia and Over-reaction Hypothesis*, „Economic Modelling", Vol. 20, pp. 417-435.
[12] Cuthbertson K., Nitzsche D., [2004], *Quantitative Financial Economics: Stocks, Bonds & Foreign Exchange*, Wiley.
[13] Davidson R., MacKinnon J.G., [1993], *Estimation and Inference in Econometrics*, Oxford University Press.
[14] Dickey D.A., Fuller W.A., [1979], *Distribution of the Estimators for Autoregressive Time Series with a Unit Root*, „Journal of the American Statistical Association", Vol. 74, pp. 427-431.
[15] Elliot G., Rothenberg T.J., Stock J.H., [1996], *Efficient Tests for an Autoregressive Unit Root*, „Econometrica", Vol. 64, pp. 813-836.
[16] Engsted T., Tanggaard C., [1997], *The Predictive Power of Yield Spreads for Future Interest Rates: Evidence from the Danish Term Structure*, „Scandinavian Journal of Economics", Vol. 1, pp. 145-159.
[17] Hobijn B., Franses P., Ooms M., [1998], *Generalizations of the KPSS-test for Stationarity*, „Econometric Institute", Erasmus University Rotterdam, Report 9802/A.
[18] Hurn A.S., Moody T., Muscatelli V.A., [1995], *The Term Structure of Interest Rates in the London Interbank Market*, Oxford Economic Papers, Vol. 47, pp. 418-436.
[19] Johansen S., [1988], *Statistical Analysis of Cointegrating Vectors*, „Journal of Economic Dynamics and Control", Vol. 12, pp. 231-254.
[20] Johansen S., [1991], *Estimation and hypothesis testing of cointegrating vectors in Gaussian vector autoregressive models*, „Econometrica", Vol. 59, pp. 1551-1580.
[21] King R.G., Kurmann A., [2002], *Expectations and the Term Structure of Interest Rates: Evidence and Implications*, Federal Reserve Bank of Richmond Quarterly, Vol. 88, No. 4, pp. 49-95.
[22] Konstantinou P.T., [2005], *The Expectations Hypothesis of the Term Structure. A Look at the Polish Interbank Market*, Emerging Markets Finance and Trade, Vol. 41, No. 3, pp. 70-91.
[23] Kwiatkowski D., Phillips P.C.B., Schmidt P., Shin Y., [1992], *Testing the null hypothesis of stationarity against the alternative of a unit root*, „Journal of Econometrics", Vol. 54, pp. 159-178.
[24] Luetkepohl H., [2005], *New Introduction to Multiple Time Series Analysis*, Springer.
[25] McCulloch J., Kwon H., [1993], *US Term Structure Data, 1947-1991*, Ohio State University Working Paper, No. 93-96.
[26] Tzavalis E., [2003], *The term premium and the puzzles of the expectations hypothesis of the term structure*, „Economic Modelling", Vol. 21, pp. 73-93.
[27] Tzavalis E., Wickens M., [1998], *A Re-Examination of the Rational Expectations Hypothesis of the Term Structure: Reconciling the Evidence from Long-Run and Short-Run Tests*, „International Journal of Finance and Economics", Vol. 3, pp. 229-239.

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HIPOTEZA RACJONALNYCH OCZEKIWAŃ STRUKTURY TERMINOWEJ POLSKIEGO RYNKU DEPOZYTÓW MIĘDZYBANKOWYCH

Streszczenie

W artykule przedstawiamy wyniki badania poświęconego weryfikacji hipotezy racjonalnych oczekiwań struktury terminowej stóp procentowych na rynku depozytów międzybankowych w Polsce. Badanie opieramy o trójwymiarowy model VAR, którego poszczególne równania odzwierciedlają kolejno spred stóp procentowych (ang. yield spread), zmianę stopy zwrotu dla depozytu o krótszej zapadalności (ang. change in the short rate) oraz nadwyżkową, okresową stopę zwrotu (ang. excess holding period yield). W procesie estymacji i weryfikacji modelu wykorzystujemy miesięczne szeregi czasowe stóp referencyjnych WIBOR dla depozytów o zapadalnościach 1, 3, 6, 9 oraz 12 miesięcy z okresu styczeń 1999-grudzień 2007 roku. Uzyskane rezultaty empiryczne dla wszystkich terminów zapadalności wskazują na to, że spred stóp procentowych jest przyczyną w rozumieniu Grangera dla zmiany stopy zwrotu z depozytów o krótszej zapadalności. Pozostałe wyniki są mniej jednoznaczne. Stwierdzamy, że restrykcje nałożone na współczynniki modelu VAR odpowiadające hipotezie głoszącej stałość w czasie nadwyżkowej, okresowej stopy zwrotu winny być odrzucone dla wszystkich terminów zapadalności. Odrzucone winny być także restrykcje – z wyjątkiem WIBORu 12-miesięcznego – odpowiadające hipotezie, która głosi, iż spred bieżący jest równy spredowi teoretycznemu. Niemniej takie zwyczajowe miary obliczane na podstawie oszacowanych modeli VAR (ang. VAR metrics), jak współczynnik korelacji pomiędzy spredem bieżącym i spredem teoretycznym, a także ich iloraz wariancji w znacznej mierze wspierają hipotezę oczekiwań w postaci czystej (ang. PREH – pure rational expectations hypothesis). Te pierwsze są bliskie jedności, natomiast te drugie – odchylają się o mniej niż dwa odchylenia standardowe od jedności dla depozytów o zapadalności 3-miesięcznej. Wnioski wspierające hipotezę oczekiwań w postaci czystej dla depozytów o zapadalnościach 9 i 12-miesięcznej uzyskaliśmy na podstawie eksperymentu bootstrapowego, w którym oszacowaliśmy 95-procentowy przedział ufności dla ilorazu wariancji. Oceny pozostałych miar VARowskich sugerują, że względnie duża część zmienności nieoczekiwanej stopy zwrotu jest wynikiem napływających na rynek informacji odnoszących się do przyszłych stóp zwrotu, a nie informacji odnoszących się do przyszłych przeciętnych premii płynności.

Słowa kluczowe: struktura terminowa stóp procentowych, hipoteza racjonalnych oczekiwań, zmienna w czasie premia płynności, VAR, polski rynek depozytów międzybankowych.

THE RATIONAL EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE AT THE POLISH INTERBANK MARKET

Summary

We use a three-variable VAR including the yield spread, the change in the short rate and the excess holding period yield to test for the validity of the rational expectations hypothesis (REH) at the Polish interbank market. In doing so we utilize the set of monthly sampled WIBORs (Warsaw Interbank Offered Rates) for maturities of 1, 3, 6, 9 and 12 months from the period January 1999-December 2007. Although the yield spread Granger-causes future changes in the short rate for all maturities the other testing results are somewhat ambiguous. We find the restrictions set on the VAR that the one-period excess holding period return is not time varying should be rejected for all maturities. So should be the restrictions stating the actual spread equals the theoretical spread, except for 12 month WIBOR. Nevertheless, the estimates of conventional VAR metrics such that correlation coefficients between the actual and the theoretical spread and their variance ratios to some extent support the REH in its pure form (PREH). The first are all very close to unity and the latter are less than two standard deviations from unity for 3 month maturity. The conclusions in favour of the PREH for 9 and 12 month maturities are reached upon the bootstrapping experiment in which we have estimated the 95 per cent confidence intervals for the variance ratios. The estimates of the other VAR metrics suggest that a relatively large piece of variation in the unexpected return is due to news about future short rates and not due to news about the future average term premium.

Key words: term structure of interest rates, rational expectations hypothesis, time-varying term premia, VAR, Polish interbank market.