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PAN Warszawska Drukarnia Naukowa
00-056 Warszawa, ul. Śniadeckich 8
Tel./fax:+48 22 628 76 14

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KATARZYNA BIEŃ-BARKOWSKA

EXPLAINING LIQUIDITY DYNAMICS IN THE ORDER DRIVEN FX SPOT MARKET[1]

1. INTRODUCTION

The concept of liquidity in the market microstructure literature is generally perceived as ”slippery and elusive concept” that is difficult to define (c.f. Kyle, 1985). There is a well-established consensus in the financial market literature that liquidity has at least four major dimensions: depth, tightness, resilience (c.f. Black, 1971; Kyle, 1985) and immediacy (c.f. Sarr, Lybek, 2002). In this paper we focus on the examination of the first three categories mentioned above: we investigate the market depths, the bid-ask spread and some more precise measures of the limit order book (LOB) tightness, as well as the Amihud (2002) illiquidity measure of market resilience.

The aim of this paper is to quantify and describe the intraday dynamics of different liquidity measures of the order-driven interbank EUR/PLN spot market from the perspective of time-varying fraction of informed trading. As the share of trading on private information cannot be observed directly, it has to be approximated and deduced from the quantified intensity of incoming orders. It is widely recognized in the literature that informational motives of currency dealers constitute an important driving force of FX trading. According to King et al. (2013), the amount of information heterogeneity among currency dealers may arise from different exposure to bank clients submitting unbalanced types of market orders (i.e. different amount of buy orders in comparison to sell orders), private research on market fundamentals, or even sharing the views and expectations within an informal social network. Accordingly, we intend to measure the scale of this information discrepancy and relate it to the continually changing liquidity conditions on the EUR/PLN market. The estimates of ‘rates’ of informed and uninformed trade arrivals are to be obtained from the dynamic sequential trade model proposed by Easley et al. (2008) and adjusted to the intraday setup by Bień-Barkowska (2013). As a result of this, we are able to estimate a time-varying fraction of informed trades from the continually changing differences

[1] We thank the Thomson Reuters for providing the data for our study. This research has been carried out within the project “The Microstructure of the Interbank FX Spot Market” financed by the National Science Centre in Poland upon decision No. DEC-2013/09/B/HS4/01319.

in amounts of buyer- and seller-initiated trades. This time-varying share of informed trading is to be treated at a later stage as an explanatory factor for each individual measure of liquidity provision.

Our study aims to contribute to an extremely scarce literature on liquidity determination in FX markets. To our knowledge, our findings are unique in terms of an in-depth analysis of liquidity dynamics on an intraday level. Some implications about fluctuations in the shape of the order-book can be deducted from the studies of order submissions provided by Lo, Sapp (2008) and Lo, Sapp (2010), however their studies covered Deutsche Mark-US Dollar market and the Canadian Dollar – U.S. Dollar currency pairs, hence the major currencies and not emerging ones. The novelty of our analysis lies in an documentation of a time-of-day as well as day-of-week effects (i.e. intraday and intraweek seasonality) in different measures of liquidity provision. We also evidence long memory effects of liquidity shocks in currency markets and show that the long-range dependence in liquidity can be captured by the Fractionally Integrated Autoregressive Conditional Duration models of Jasiak (1998). Additionally, we also show that the amount of liquidity supplied is closely linked to the share of informed trading in the market. Although significant impact of informed trading on the market tightness was already documented for cross sections of stocks by Brockman, Chung (1998), (1999), (2000) and Easley et al. (2008), in this study we look at liquidity from a different angle paying attention to a time series setup. Accordingly, we will be able to assess how the continually changing informational motives of trading in FX markets impact the behavior of other market participants leading to observed liquidity fluctuations.

In the market microstructure theory, adverse selection costs, the cost of dealer services and the cost of holding inventory constitute three main determinants of market tightness (c.f. Sarno, Taylor, 2002, p. 290). Although the latter two are behind the scope of this paper, the adverse selection costs can be explained in an information-oriented strand of market microstructure literature. Information models date back to the seminal study of Bagehot (1971), where in the market there are two types of traders: liquidity (uninformed) traders and informed traders. The latter can make use of private information at the expense of a market maker. Because market maker does not know with whom he trades, he widens the spread for both trading groups treating it as a premium for an adverse selection risk. Similarly, in the Glosten, Milgrom (1985) model, a market maker can additionally learn the probability of informed trading by knowing the direction (buy or sell) of orders. He cannot distinguish liquidity traders from informed traders and therefore adjusts quoted liquidity conditionally on the sign of incoming orders. The model has been further developed by Easley, O'Hara (1987), who state that not only the stream of incoming orders but also their sizes can have informative value. Thus, the existence of new information can be deduced from the sign and the size of the incoming orders. Accordingly, asymmetric information obliges market makers to update ask and sell prices and scale of market tightness is a weapon against an adverse selection problem. In many later studies bid-ask spread was also treated as a observable measure of information heterogeneity (c.f. McInish, Wood, 1992; Foster, Viswanathan, 1990; 1993).

The market depth is comprised of limit orders awaiting for an execution in the limit order book (LOB). The amount of quoted depth can be also related to the informational content of trading. De Jong and Rindi state that "(…) the choice between limit and market orders is a strategic element in any trading decision and depends on (…) the asymmetry of the personal evaluations of the risky asset between the agents who submit the orders and those who hit the existing quotes" (c.f. De Jong, Rindi, 2009, p. 134). Although there is a widespread notion that informed traders are much more likely to use market orders than limit orders, Harris (1998) points out that informed traders can also use limit orders. Moreover, liquidity traders can be discretionary, which means that they chose the time of their trading (c.f. Admati, Pfleiderer, 1988). Uninformed traders, being aware of the increased adverse selection costs during periods where informed trading can take place, may prefer to limit the risk that their stale orders will be executed at an unfavorable price. Thus, they may retreat from supplying liquidity to the market, even by canceling the previously submitted orders. Accordingly, market depth should deteriorate as a response to signs of informed trading.

2. EMPIRICAL DATA

The datasets used in this study are comprised of all incoming orders as well as trades executed during the year 2007 in the Reuters Dealing 3000 Spot Matching System with respect to the EUR/PLN currency pair. Trading of the Polish zloty takes place on offshore markets (mainly between London banks) as well as locally in Poland and the datasets used in this analysis take into account both of these trading venues. The EUR/PLN exchange rate is quoted as a quantity of zlotys per one Euro. The transaction currency is euro and the smallest order size is 1 million EUR. During the whole period under study EUR/PLN market featured appreciation trend of the Polish zloty against euro. The Reuters Dealing 3000 Spot Matching System is an electronic brokerage system that operates as an order-driven market and automatically matches incoming buy and sell orders once their prices agree. FX dealers can submit either limit or market orders; limit orders are traditionally perceived as rather passive in nature whereas market orders are liquidity-consuming and more aggressive since they are immediately realized against most competitive limit orders in the LOB. However, only the best bid and ask prices with the corresponding depths at the best ask or at the best depth are observable to other market participants on the trading screens. In our datasets, each transaction is marked with its date, exact time, rate and quantity (in millions) of EUR as well as a buy/sell indicator. Every order includes an exact date and time of submission as well as an execution/cancellation, a firm quote, the size and an indicator for the market side of the quote. The detailed structure of the datasets makes it possible to rebuild the whole order book at each moment of the market's activity. In order to limit the undesired impact of particularly thin trading periods we have excluded observations registered on weekends and on business days between the hours of 18:00 and 8:00 CET. We also omit days with exceptionally low liquidity due to national holidays. As a result of these deletions our sample covers 250 trading days of trade and order data that was aggregated into 15-minute intervals. We identify the following six liquidity measures:

– ILLIQ measure: the illiquidity measure of Amihud (2002) defined as the absolute mid price change divided by the trading volume between the times t – 1 and t, $ILLIQ_t = |\Delta P_t^{mid}| / V_t$ (where $P_t^{mid} = (P_t^{A,best} - P_t^{B,best})/2$, $P_t^{A,best}$ denotes the most competitive (lowest) ask price in the LOB, and $P_t^{B,best}$ denotes the most competitive (highest) bid price in the LOB at time t.
– Percentage bid-ask spread: the ratio of the difference between the best ask and the best bid quote prevailing in the LOB at time t and the corresponding mid price, $S_t = (P_t^A - P_t^B / P_t^{mid}) \cdot 10^4$ (in basis points).
– Market depth on the bid side of the market (and respectively, on the ask side of the market): the quantity of all limit buy (sell) orders in the LOB at time t: D_t^b (or D_t^a, respectively) (in millions of EUR).
– Quote slope for the ask side of the market (and respectively, for bid side of the market) measuring entire liquidity in the spirit of Hasbrouck, Seppi (2001). For the ask side of the market the quote slope (QS_t^A) is measured as the difference among the worst (i.e. the highest) and the best (i.e. the lowest) ask price prevailing in the LOB at time t, divided by the entire depth on the ask side of the market; $QS_t^A = (P_t^{A,worst} - P_t^{A,best}) / D_t^A$. Symmetrically, for the bid side of the market, the quote slope (QS_t^B) is defined as the difference between the best (i.e. the highest) and the worst (i.e. the lowest) bid price in the LOB at time t, divided by the entire depth on the bid side of the market; $QS_t^B = (P_t^{B,best} - P_t^{B,worst}) / D_t^B$.
– Liquidity area for the ask side of the market (and respectively, for the bid side of the market). For the ask side of the market, the liquidity area (LIQ_t^A) is defined as the area under the ask supply curve (over the mid price) that corresponds to an immediate buy of exact 5 million EUR: $LIQ_t^A = \sum_{i=1}^{5} (P_t^{A,i} - P_t^{mid})$ where $P_t^{A,i}$ indicates the zloty price for an immediate buy of i- th million of euro. Symmetrically, for the bid side of the market, the liquidity area (LIQ_t^A) is defined as

the area under the mid price (and over the bid supply curve) that corresponds to an immediate sell of exact 5 million EUR: $LIQ_t^B = \sum_{i=1}^{5}(P_t^{mid} - P_t^{B,i})$, where $P_t^{B,i}$ indicates the zloty price for an immediate sell of *i*- th million of euro.

For a better exposition of liquidity measures, in Figure 1 we present the snapshot of the LOB a couple of seconds after 8:23 CET on 9 Jan. 2007. The best (most competitive) quote offered on the ask side of the market equals 3.86 and worst (least competitive) quote equals 3.8765. On the other hand, the quote that is first to be hit on the bid side of the market is 3.857 and the least competitive bid offer is 3.848. Clearly, the bid-ask spread which amounts to 0.003 (three tenth parts of the Polish grosz; hence three thousandth parts of the Polish zloty) constitutes an extremely modest and insufficient measure of liquidity supply, similarly to the bid or ask market depths. Indeed, although the entire depth on the bid and on the ask side of the market is the same and equals 29 million EUR, the ask and bid sides of the LOB are obviously not equally tight. The discrepancy between liquidity supply on the ask and on the bid side of the market seems striking if one looks at a sequence of the most competitive ask or sell offers that play the first fiddle in the market game. The ask liquidity area (shadowed in light grey) is much larger that the bid liquidity area (shadowed in dark grey). Thus, a dealer who decides to immediately buy 5 million EUR bears much higher liquidity costs than a dealer who decides to immediately sell 5 million EUR. This is because only 1 million EUR out of 5 can be traded at the most competitive ask price. Other parts of this buy order have to be executed at less favorable prices (1 million even at 3.87, hence a quote 100 pips higher than the best ask quote). On the contrary, the liquidity provision on the bid side is considerably larger and the dominant part of a 5 million sell order can be executed at the most competitive bid price.

The motivation behind the choice of liquidity measures is the following. The Amihud (2002) measure of illiquidity is closely related to the well-known Kyle's lambda and constitutes a standard proxy for the price impact of trading. Accordingly, the ILLIQ measure captures market resiliency by reflecting a change in a quoted mid price in result of a trade. Other liquidity variables are selected to reflect the shape of a limit order book. The bid-ask spread and the bid (ask) depths are known to be the standard measures of pre-trade liquidity supply. The ask (bid) quote slopes aim to capture the entire liquidity provision on the ask (bid) side of the market. If the nominator of the ask (bid) quote slope rises (i.e. absolute difference between the best and the worst ask (bid) quote increases), so does the steepness of the ask (bid) quote slope. Similarly, the smaller the depth of ask (bid) side of the market, the steeper the quote slope. Hence, the ask (bid) quote slope tends to infinity for the infinitely illiquid market (if the depth tends to zero or the absolute difference between the best and worst price in the LOB is infinitely large). Accordingly, for the infinitely liquid market, the ask (bid) slope will be equal to zero. Although quote slopes capture the tightness of the entire LOB, they have certain drawbacks. First, in the case of only one limit order prevailing on the ask (or bid) side of the LOB, the quote slope would be equal to zero indicating an infinitely liquid market, which obviously cannot hold true. Second, quote slopes do not take into account the 'curvature' of the ask (bid) liquidity supply curves, as they neglect the quotes between the best and worst ask (bid) prices. To overcome this problem, we propose the liquidity areas as potentially more precise measures of the LOB shape. Liquidity areas measure how close the ask (bid) prices (corresponding to the pre-defined most competitive levels of the limit order book) are to the mid price. In the infinitely liquid market, the 5-million-buy or the 5-million-sell would be concluded at the best ask price or at the best sell price. Accordingly, the larger the liquidity areas, the smaller the liquidity supply and the larger are the costs of a 5-million-trade.

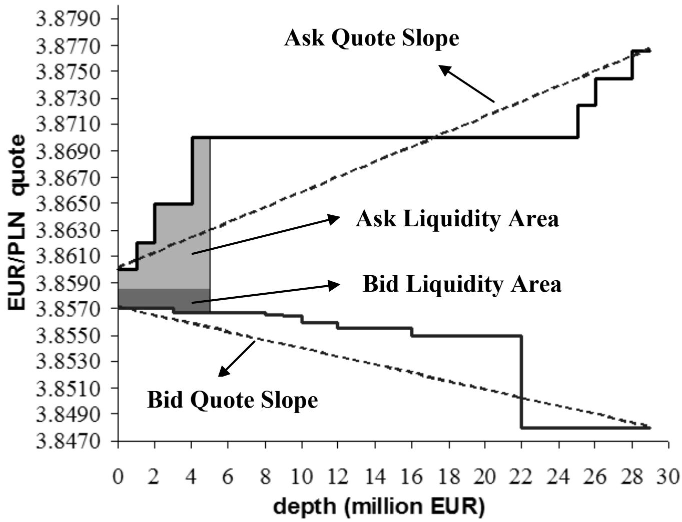

Figure 1. The snapshot of the EUR/PLN LOB on 9th January 2007 (8:23:41.34 CET)

All liquidity variables selected for the study exhibit strong intraday seasonality (diurnality). The diurnality patterns are obtained by computing the expectation of a liquidity variable conditioned on a time-of-day, separately for each day of the week, i.e. from Monday to Friday. Thus, for each day of week we derive a different shape of the intraday seasonality with a nonparametric (kernel) regression of the liquidity variable on a time-of-day indicator. The intraday seasonality factor, which was suggested by Bauwens, Veredas (2004), is given as:

$$S(\tau) = \frac{\sum_{t=1}^{T} K\left(\left(\tau - \tau_t\right)/h\right)\bar{x}_t}{\sum_{t=1}^{T} K\left(\left(\tau - \tau_t\right)/h\right)}, \tag{1}$$

where K denotes a quartic kernel function, τ is a time variable rescaled to interval [0,1] (i.e. number of seconds from 8:00 on each day was divided by the cumulative number of seconds from 8:00 to 18:00), $\bar{x}_t$ denotes a liquidity variable, i.e. $\bar{x}_t \in \{ILLIQ_t, S_t, D_t^A, D_t^B, QS_t^A, QS_t^B, LIQ_t^A, LIQ_t^B\}$, h denotes an optimal smoothing parameter selected according to the Silvermann's rule of thumb.

Diurnality patterns augmented for a day-of-week effects are depicted in Figure 2. We see that overall liquidity deteriorates in the mornings and late afternoons when trading is rather scarce. In an overnight period, when the two major headquarters of Polish zloty trading (the London market and the Polish market) are closed, the trading system is lacking liquidity. This result is consistent with many empirical studies on intraday stock trading that report an U-shaped or an inverted J-shaped curve for the intraday seasonality of the bid-ask spread (c.f. Nyholm, 2002; Nyholm, 2003; Ahn et al., 2002; Heflin et al., 2007). We document a distinct U-shaped diurnality pattern not only for the bid-ask spread, but also for the Amihud (2002) illiquidity measure as well as both ask and bid liquidity areas and both ask and bid quote slopes. Moreover, we clearly see that the interbank EUR/PLN market tends to be systematically less liquid on Mondays and Fridays in comparison to other days of the week, which relates to the uncertainty associated with a two-day-long cease in trading on weekends. On Mondays, especially in the morning, there is an increased information heterogeneity in the market because of various news releases during Saturday and Sunday. The uncertainty results in systematically wider bid-ask spread and increased quote-slopes. Similarly, deterioration in quoted liquidity on Fridays (which is especially visible for quote slopes and liquidity areas) can be attributed to increased settlement risk, because FX spot transactions are always settled two working days after they are executed. Our results are consistent with the findings of Brzeszczyński, Melvin (2006), who also document distinct intraday and intraweek seasonality patterns in trading activity for the euro FX market. Intraday seasonality patterns of the market depth are generally much more 'dispersed', but still they seem to be inversely related to these corresponding to bid-ask spread, quote slopes or liquidity areas.

In order to assess the dynamic properties of selected liquidity measures, we divided each liquidity variable by the corresponding diurnality factor $x_t = \bar{x}_t / S(\tau_t)$. This procedure allows us to disentangle between two sources of autocorrelation: intraday seasonality due to systematic and repetitive (on a daily basis) trading activities of currency dealers and the residual persistence in liquidity shocks after elimination of diurnality effects. In the sequel of the paper we use the deseasonalized liquidity variables (i.e. adjusted for both time-of-day as well as day-of-week effects), whose autocorrelation functions are depicted in Figure 3. We can see that nearly all functions exhibit a very slow hyperbolic (and non-exponential) rate of decay. Bid (ask) depths and the bid (ask) quote slopes are the most persistent and indicate long memory effects.

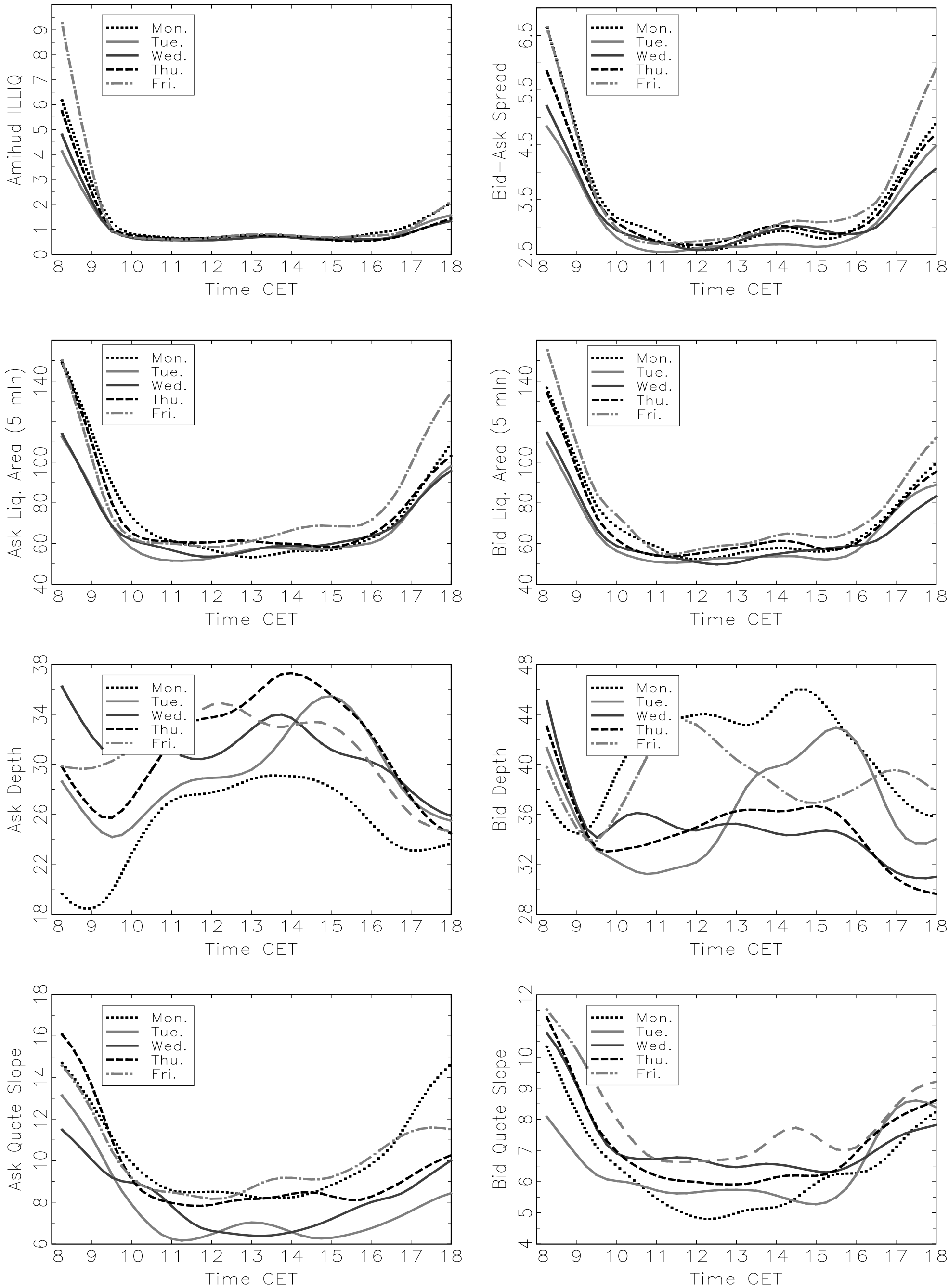

Figure 2. The day-of-week adjusted diurnality patterns for selected liquidity measures

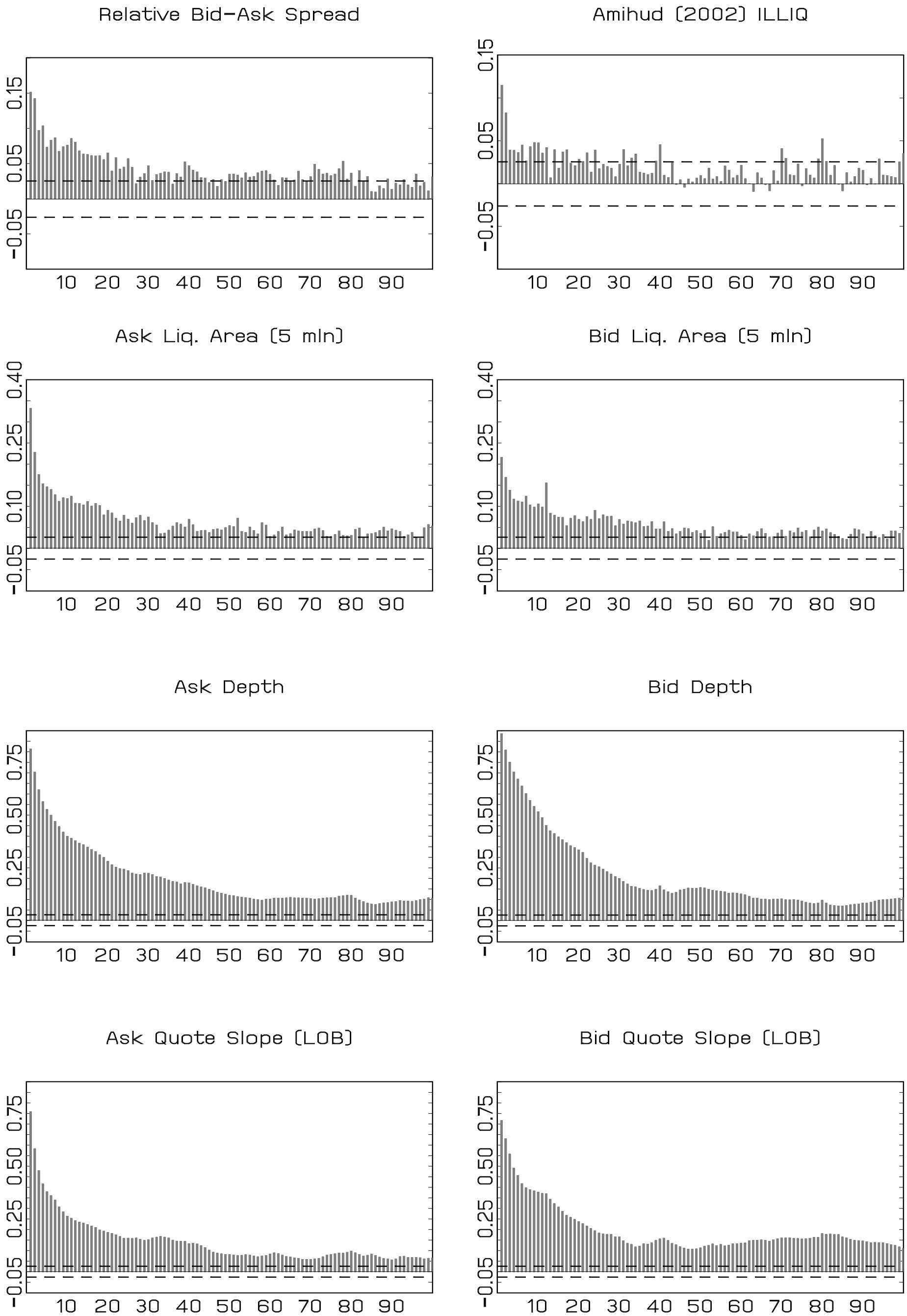

Figure 3. The autocorrelation functions for the deseasonalized liquidity measures

3. ECONOMETRIC METHODS

3.1. Fractionally Integrated ACD Models

We use Autoregressive Conditional Duration (ACD) models introduced by Engle, Russell (1998) to account for dynamic properties of variables under study. Preliminarily, ACD models were proposed to describe trading intensity and applied to autocorrelated time series of financial durations (i.e. times) between selected events (i.e. transactions or price changes). These models were also used to describe transaction volumes by Manganelli (2005) and Doman (2008), Doman (2011) or bid-ask spreads by Nolte (2008). The ACD models can explicitly capture two specific features of financial variables measured at high frequencies. First, they are designed to variables with a positive real domain. Second, they can flexibly describe processes with strong autocorrelation, often with a high degree of persistence. There is a recent upsurge in research on the ACD models, whereas vast surveys on their extensions can be found in Hautsch (2004) or Pacurar (2008). Here we use the logarithmic version of the Fractionally Integrated ACD (FIACD) model proposed by Jasiak (1998) with the Burr distribution for the error term, as suggested by Grammig, Maurer (2000). According to the ACD setup, each adjusted for time-of-day and time-of-week effect liquidity variable x_t ($x_t \in \{ILLIQ_t, S_t, D_t^A, D_t^B, QS_t^A, QS_t^B, LIQ_t^A, LIQ_t^B\}$) can be given as:

$$x_t = \Phi_t \varepsilon_t, \tag{2}$$

where $\Phi_t = E(x_t \mid F_t)$, F_t denotes an information set up to time point t and ε_t denotes the Burr-distributed error term with a property $E(\varepsilon_t) = 1$. Hence, ε_t: *i.i.d.* Burr(κ,σ^2); κ and σ^2 denote the shape parameters of the Burr distribution[2], where $0< \sigma^2 <\kappa$. We decompose the conditional expectation of x_t as:

$$\Phi_t = exp(\phi_{1,t} + \phi_{2,t}), \tag{3}$$

with the first component, i.e. $\phi_{1,t}$, designed to capture the strong persistence in liquidity with the logarithmic version of the FIACD(p,d,q) model of Jasiak (1998):

$$(1 - \beta_p(L))\phi_{1,t} = \beta_0 + \gamma_\infty(L) ln(x_{t-1}), \tag{4}$$

where β_0 is a constant, $\beta_p(L)$ denotes a scalar pth order polynomial in lag operator and $\gamma_\infty(L)$ denotes a scalar polynomial in lag operator given as:

[2] The Burr distribution has three parameters, but the assumption $E(\varepsilon_t) = 1$ makes the third (scale) parameter the function of the shape parameters κ and σ^2.

$$\gamma_{\infty}(L) = [1 - \beta_p(L) - [1 - \alpha_q(L) - \beta_p(L)](1 - L)^d]. \tag{5}$$

$\alpha_q(L)$ is a scalar qth order polynomial in lag operator and $(1 - L)^d$ (for $0<d<1$) is a fractional lag operator:

$$(1 - L)^d = \sum_{k=0}^{\infty} \varpi_k L^k \text{, where } \varpi_k = \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} = \prod_{0<j\le k} \frac{j-1-d}{j} \text{, for } k = 0,1,2,...$$

and $\Gamma(\cdot)$ is the gamma function (c.f. Nolte, 2008).

For $d = 0$, logarithmic FIACD model nests logarithmic ACD(p,q) model of Bauwens, Giot (2000) and its integrated version for $d = 1$. The second component of the conditional expectation, i.e. $\phi_{2,t}$, is designed to capture possible impact of other explanatory variables.

As explanatory variables we choose the proxy for informed trading, i.e. the measure of "probability of informed trading" PIN_t (explained in detail in the next section). In order to recover the independent impact of PIN_t on the top of other popular characteristics of market activity, we decided to enrich the model with three standard control covariates: the volume of all trades from $t - 1$ up to t (TT_t), the observed return on EUR/PLN rate during 15-minute-long interval from $t - 1$ up to t (r_t) and the proxy for volatility (given as a modulus of return $|r_t|$). In order to mitigate the multicollinearity effects, the trade volume and the proxy for volatility were deseasonalized in the same way as the liquidity measures (multiplicative intraday seasonality factor was derived with a kernel regression on a time-of-day variable separately for each day of the week). Henceforth, the component $\phi_{2,t}$ of conditional expectation of liquidity measures is given as:

$$\phi_{2,t} = \gamma_{TT} TT_t + \gamma_{vol} |r_t| + \gamma_{ret} r_t + \gamma_{PIN} PIN_t. \tag{6}$$

In the empirical analysis we will rely on the logarithmic version of the parsimonious FIACD(1,d,1) model, hence the dynamic specification of $\phi_{1,t}$ given as:

$$(1 - \beta_1 L)\phi_{1,t} = \beta_0 + [1 - \beta_1 L - (1 - \alpha_1 L - \beta_1 L)(1 - L)^d] ln(x_{t-1}). \tag{7}$$

The ACD models can be estimated with the Maximum Likelihood method. However, the "infinity" term (see $(1 - L)^d = \sum_{k=0}^{\infty} \varpi_k L^k$) has to be approximated. Therefore, we proxy infinity with 1000 and initiate first 1000 lags of $\ln(x_t)$ by the unconditional mean of $\ln(x_t)$, as in Nolte (2008). The log likelihood function of the ACD model with the Burr distribution is:

$$LogL(\Theta) = \sum_{t=1}^{T}\left[\ln\kappa - \kappa\cdot\ln\xi_t + (\kappa-1)\cdot\ln x_t - \left(\frac{1}{\sigma^2}+1\right)\cdot\ln(1+\sigma^2\cdot\xi_t^{-\kappa}\cdot x_t^{\kappa})\right], \quad (8)$$

where $\xi_t = \Phi_t \dfrac{\sigma^{2\cdot\left(1+\frac{1}{\kappa}\right)}\cdot\Gamma\left(\frac{1}{\sigma^2}+1\right)}{\Gamma\left(1+\frac{1}{\kappa}\right)\cdot\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}$ and $0<\sigma^2<\kappa$.

Application of the exponential transformation of the expectation (see equation 3) enables adding exogenous explanatory variables to the model (see equation 6). Some of these regressors might have a negative impact on the liquidity measures but this outcome will not interfere with the nonnegativity of the liquidity variable.

3.2. Probability of Informed Trading

Sequential trade models introduced by Easley et al. (1996) and developed in Easley et al. (2008) contributed to a huge upsurge in research on how the information possessed by a fraction of market participants may be unveiled to the others through the observed stream of buy and sell orders. According to the market microstructure literature, the reasons for trading can be twofold: (1) exploiting private information, and (2) satisfying liquidity needs or portfolio rebalancing. Therefore, act of trading can take place in order to exploit the information signals (informed trading) or to satisfy liquidity or inventory-related reasons (uninformed trading). Sequential trade models are used to construct a measure known as the ‘probability of informed trading’ (PIN), which reflects the forecasted fraction of all trades that are initiated by access to private information. Easley, Kiefer, O'Hara and Paperman proposed one of the first econometric parameterizations of a sequential trade model, henceforth known as the EKOP model (Easley et al. 1996).

In order to check how the predicted PIN variable influences market liquidity we apply diurnality-adjusted augmentation of the dynamic Easley et al. (2008) model suggested by Bień-Barkowska (2013). In the Easley et al. (2008) approach, buy and sell trades occur according to two independent Poisson processes with the time-varying arrival rates: $\lambda_{B,t}$ and $\lambda_{S,t}$, respectively. It is also assumed that both informed and uninformed traders may initiate trades with a time-varying rates μ_t and ε_t, respectively. Although the detailed presentation of the dynamic EKOP model can be found in Easley et al. (2008), for the sake of legibility of our analysis we sketch its major outline below.

It is assumed that at the beginning of each of the pre-defined time intervals (i.e. 15-minute spells in our setup) new information occurs with a constant probability α, or there is no news with probability $1 - \alpha$. If the information occurs, it can be either "bad" for the transaction currency (EUR) with a constant probability δ or it may be "good" with probability $1 - \delta$. Uninformed traders always conclude their trades with rates: $\lambda_B = \varepsilon_t$ (ask side) and $\lambda_S = \varepsilon_t$ (bid side), respectively. Informed traders switch into trading only after having received the information signal (with an arrival rate μ_t for both sides of the market). Accordingly, during intervals with bad information, the buy transactions are initiated by uniformed traders only and occur with an arrival rate $\lambda_B = \varepsilon_t$ but sell transactions result from both informed and uninformed traders with a rate $\lambda_s = \mu_t + \varepsilon_t$. Symmetrically, during intervals with good news, buys result from informed and uninformed traders ($\lambda_B = \mu_t + \varepsilon_t$), whereas the sells are concluded by uninformed traders only ($\lambda_s = \mu_t$).

In order to estimate the dynamic diurnality-adjusted EKOP model, the following variables have to be defined: (1) trade imbalance, given as the absolute difference between the number of buy[3] (B_t) and sell trades (S_t) that are executed between t and $t - 1$, $|B_t - S_t|$, (2) balanced trades, given as the difference between the total number of trades (TT_t) and the trade imbalance, $(TT_t) - |B_t - S_t|$. Additionally, let us by $\psi_{1,t}$ denot e the forecasted (at time t) arrival rate of uninformed trades (i.e. $\psi_{1,t} = 2\varepsilon_t$ and by $\psi_{2,t}$ the forecasted (at time t) arrival rate of informed trades (i.e. $\psi_{2,t} = \alpha\mu_t$). According to Bień-Barkowska (2013), both $\psi_{1,t}$ and $\psi_{2,t}$ are subject to a seasonality-adjusted VARMA-type dynamic specification:

$$\psi_{1,t} = \omega_1 + \phi_{11}^* \hat{\psi}_{1,t-1} + \phi_{12}^* \psi_{2,t-1} + \gamma_{11}\xi_{1,t} + \gamma_{12}\xi_{2,t}, \tag{9}$$

$$\psi_{2,t} = \omega_2 + \phi_{21}^* \psi_{1,t-1} + \phi_{22}^* \psi_{2,t-1} + \gamma_{21}\xi_{1,t} + \gamma_{22}\xi_{2,t},$$

where $\xi_{1,t} = TT_t - |B_t - S_t| - 2S(\mathbf{v},\tau) - \psi_{1,t-1}$ denotes a difference between the deseasonalized number of balanced trades (between $t - 1$ and t) and their predicted quantity at $t - 1$. Similarly, $\xi_{2,t} = |B_t - S_t| - \psi_{2,t-1} - \alpha \cdot S(\mathbf{\upsilon},\tau)$ denotes a difference between the deseasonalized number of unbalanced trades and their predicted quantity at time $t - 1$. Seasonality (diurnality) factors $S(v,\tau)$ and $S(\upsilon,\tau)$ for balanced or unbalanced trades are given as the Fourier flexible form (c.f. Andersen, Bollerslev, 1997).

[3] The main shortcoming of the EKOP model is a possible misclassification bias (c.f. Boehmer et al., 2007). It happens if the transaction datasets do not allow to directly determine which trade is a buy (has been executed with a market buy order or a marketable limit buy order) and which is a sell (has been executed by a market sell or a marketable limit sell order), and thus different classification algorithms must be applied in order to recover a trade direction indicator. In our study, we directly know which side of the market initiated a trade because we have a necessary buy/sell indicator in the dataset; hence we will not obtain biased results due to a misspecification error.

The ratio of arrival rate of informed trades to an arrival rate of all trades (informed and uninformed) results in the (deseasonalized) probability of informed trading (PIN):

$$PIN_t = \frac{\psi_{2,t}}{\psi_{2,t} + \psi_{1,t}}. \tag{10}$$

Thus, the PIN_t variable is a probability of informed trading that is forecasted for time point t on the basis of balanced trades and the trade imbalance up to this time point. In this setup news may arrive at the intra-daily frequency (at the beginning of each of 15-minute-long intervals). Having forty 15-minute intervals per day (as we use observations from 8:00 to 18:00 CET) we allow for 40 possible changes in the information set each day and for clustering in informed/uninformed trading over time.

Estimation of the seasonality-adjusted EKOP model is performed with the maximum likelihood method. The likelihood function uses the mixture of three two-dimensional Poisson distributions that refer to the arrival of 'bad news', 'no news' or 'good news' to the market (c.f. Easley et al., (2008)). The estimation results of the seasonality-augmented EKOP model for exactly the same empirical data as in this study were presented and discussed by Bień-Barkowska (2013). Because the interpretation of these parameter estimates stays beyond the scope of the current analysis, we refrain from presenting them here. However, we applied these published results to obtain the (deseasonalized) PIN_t series, as given by equation (10).

4. DISCUSSION OF EMPIRICAL RESULTS

We report the logarithmic FIACD model estimates[4] in Table 1. For all liquidity measures, the fractional differencing parameter estimates are statistically different from zero documenting the long memory effects. The smallest value corresponds to the Amihud (2002) illiquidity measure and the second smallest to the percentage bid-ask spread. Thus, these two variables are the least persistent which stays in line with the autocorrelation graphs in Figure 2. The highest degrees of persistence correspond to the quote slopes and the market depths (especially on the bid side of the market) indicating a long-range impact of individual liquidity shocks. Highest persistence of these liquidity measures that take into account the whole shape of the order book and not its first level only (i.e. most competitive quotes) may be explained by leaving many pending und uncompetitive limit orders in the LOB. The further the distance from the best quotes, where the core of the trading process takes place, the less risky

[4] All models have been pre-programmed and estimated with the application of the 'maxlik' library in the Gauss system (using the BHHH optimization algorithm). In order to ensure smooth convergence, explanatory variables were additionally divided by their standard deviations.

it is to let the behind-the-quote order wait in the LOB. Obviously, such least competitive order will be executed only in the case of huge price swings. Thus, once the limit orders are placed "sufficiently" far away from the best quote, they may be left over in the LOB for a quite long time, which results in a long-range autocorrelation of market depths and quote slope measures. In order to conserve space, we do not present the autocorrelation patterns of ACD residuals here. However, the severe autocorrelation has been reduced radically and the ACF coefficients oscillate around zero. Thus, the strong persistence in liquidity shocks have been satisfactory accommodated by the long memory ACD models.

Table 1.

Estimation results of the fractionally integrated ACD Models for selected liquidity measures. Symbols "*", "**" and "***" indicate estimates significant at 10%, 5% and 1%, respectively.

	ILLIQ Measure	**Percentage Spread**	**Ask Liquidity Area (5 mln)**	**Bid Liquidity Area (5 mln)**
β_0	-0.0717*	-0.0453	-0.2453***	-0.4075***
β_1	0.7030***	0.6370***	0.7482***	0.3313**
α_1	-0.0442***	-0.1141***	-0.0461**	-0.0712***
d	0.0598***	0.2028***	0.2297***	0.2377***
γ_{TT}	-0.0813***	-0.0520***	-0.0580***	-0.0222***
γ_{vol}	0.0983***	0.0677***	0.0772***	0.0122***
γ_{ret}	0.0002	-0.0001	0.0057***	-0.0013
γ_{PIN}	0.3217***	0.1359***	0.0175**	-0.0106 **
κ	0.8905***	2.6797***	3.0998***	3.0893***
σ^2	0.0721***	0.5258***	0.7285***	0.7143***
LogL	-8,080.9	-67,114.4	-39,987.8	-39,979.5
	Ask Depth	**Bid Depth**	**Ask Quote Slope**	**Bid Quote Slope**
β_0	0.1115***	0.0253	0.1217***	0.0701*
β_1	0.2891***	0.3637***	0.2671***	0.3176***
α_1	0.3372***	0.1917***	0.2978***	0.2202***
d	0.3674***	0.5293***	0.4709***	0.5525***
γ_{TT}	0.0090	0.0154**	-0.0192***	-0.0100

Table 1.

	Ask Depth	Bid Depth	Ask Quote Slope	Bid Quote Slope
γ_{vol}	-0.0093***	-0.0089**	0.0059	0.0137***
γ_{ret}	-0.0022***	0.0018***	0.0019***	0.0012*
γ_{PIN}	-0.0682***	0.0060	-0.0494	-0.0174
κ	5.7546***	5.6615***	5.1360***	5.1394***
σ^2	0.9814***	0.9756***	1.0854***	1.1220***
LogL	-11,278.9	-7,580.7	-23,265.8	-16,440.5

We see that trading volume is generally positively related to the LOB liquidity supply. Accordingly, we confirm that heavy trading coincidences with smaller price impact of individual trades within the next 15 minutes[5], tight bid-ask spread, larger market depths, flatter quote slopes and smaller liquidity areas. This finding clearly indicates that increased pace of market orders submissions coincide in time with increased pace of the limit order arrival. Accordingly, during heavy trading periods liquidity providers are also very active. In contrast to this, volatility has a significant negative impact on the LOB liquidity. Observed swings in the mid price enlarge the price impact of individual trades, bid-ask spread, liquidity areas and decrease the quoted depth. Previous empirical research on limit order markets have also shown that the bid-ask spread is inversely related to trading volume and positively related to volatility (cf. Brockman, Chung, (1998); (1999); (2000); and Easley et al., (2008)). Thus, in a volatile market it is more costly to place a limit order because there is an increased probability that such order will be executed with a loss if the price swings abruptly in an undesirable direction (i.e. a so called 'risk of being picked-off'). Volatility is also a common measure of uncertainty, thus its positive impact on the bid-ask spread might be closely related to increased adverse selection risk and the fear of the winner's curse. Positive EUR/PLN returns and hence the depreciation of the Polish zloty, are associated with the significant deterioration of quoted liquidity on the ask side of the market (where the limit orders to sell euro against zloty are gathered). Accordingly, market trends are continuously reflected by the shape of the LOB, even beneath the best quotes. Obviously, depreciation of the Polish zloty might be much more risky for the pending limit sell orders than it is for the pending limit buy orders. If the trend persists, than the large upward movement of the EUR/PLN rate will cause the abrupt execution of the stale and mis-priced limit sell orders. This is related to the

[5] Explanatory variables have been appropriately lagged by one period for the ILLIQ measure.

free market option risk of limit orders and a possible loss due to an unfavorable price change. On the other hand, the only risk of stale limit buy orders boils down to a risk of non-execution. This is probably why the ask side of the market reacts in a much more distinct manner to depreciation of the Polish zloty.

Apart from the impact of the control variables we can see that the PIN variable has a significantly positive impact on the Amihud (2002) illiquidity measure, the percentage bid-ask spread and ask liquidity area. Our empirical results agree with Easley et al. (2008) and confirm that the information-based motives of trading do matter for a bid-ask spread determination. We show that on top of the impact of other control variables, if the disproportion between submitted market buy and sell orders suggests that there is new information then the bid-ask spread widens, each buy or sell transaction induces larger changes in prices and the overall instantaneous liquidity of the market deteriorates. Some interesting conclusions can be formulated with respect to the measures of liquidity provision focused on the one side of the LOB only. Accordingly, having controlled for the factors reflected in transaction intensity and price variation we can see the significant impact of the PIN variable on the ask depth and the ask liquidity area. Therefore, a forecasted increase in the proportion of informed traders in the population of market participants significantly impacts the willingness to provide liquidity to the market. What is most important is that the reactions to information-motivated trading on the ask and on the bid market sides are unsymmetrical. The impact of the PIN variable on the ask depth is significantly negative, hence it deteriorates liquidity, but at the same time it is insignificant for the bid depth, or even significantly negative for the bid liquidity area. This is a very interesting result as it may suggest that the market unequally valuates investments in the emerging market currency versus the investments in Euro when confronted with incoming information. The drawback of the EKOP model is that it cannot differentiate between forecasts of informed trading evoked by good or bad information. Nevertheless, if the probability of informed trading increases (which could be initiated either by good or bad news), the quantity of limit sell orders (orders to sell EUR and to buy PLN) decreases. Accordingly, bank dealers seem to be reluctant to buy Polish zloty via limit orders. This signals that informed trading is taking place irrespective of whether it was caused by the arrival of good or bad information and thus encourages the commercial banks to secure themselves by purchasing more EUR. So, if the fraction of informed traders seems to rise, the uninformed traders are more reluctant to buy zloty and to sell Euro via limit orders than they are to sell zloty and to buy Euro. Our results point toward the conclusion that EUR seems to be perceived as a ‘safer’ currency when compared to the Polish zloty. The results show that the notion of ‘escape to the Euro’ occurs once there are premises of informed trading. It should be remembered, however, that posting limit orders is not necessarily limited to uninformed traders. Bloomfeld et al. (2005) evidence that informed traders provide even more liquidity than liquidity traders do themselves. As informed traders have superior information they limit the risk of being ‘picked-off’. The dominance of informed traders over the process of limit order submissions has been also demonstrated in the empirical work of Menkoff et al. (2010) and was devoted to studying the trading of the Russian ruble on the Moscow Interbank Currency Exchange.

5. CONCLUSION

This paper's contribution to the literature on the market microstructure of FX markets is twofold. From the econometrics perspective, we derive distinct patterns of the intraday seasonality in liquidity, whereas the diurnality patterns were additionally adjusted for the day-of-week effect. Accordingly, we show how different measures of liquidity fluctuate in systematic way over the distinct days of week. Moreover, we document long-range autocorrelation in different liquidity measures, which does not die out quickly even after adjustment for the time-of-day and day-of-week effects. Accordingly, we suggest to capture the liquidity dynamics by the long memory ACD models of Jasiak (1998). We evidence strong inertness in liquidity provision, especially beyond the best quotes, i.e. first level of the order book. We observe that the degree of persistence, reflected by the estimate of the fractional differencing parameter, rises with 'distance' from the best quotes. Accordingly, the bid-ask spread is the least persistent whereas market depths or the quote slopes that take into account the whole shape of the limit order book exhibit largest inertness. We also show that liquidity fluctuates in line with time-varying market conditions: trading intensity, volatility, previously observed returns as well as the predicted amount of 'probability of informed trading' reflecting the degree of the information heterogeneity. Interestingly, we also evidence that investment in the Polish zloty as an emerging market currency is treated as more risky in comparison to investment in the Euro, because there is a certain asymmetry in providing liquidity on the ask or bid side of the market once the probability of informed trading increases. Our results may be interesting for the academia, as they document that the currency dealers perform the constant monitoring of time-varying trading conditions and our analysis sheds some light on the process of liquidity supply. Secondly, our findings may be interesting for market participants, since we document how the publicly unobservable liquidity supply beyond the best quotes changes in parallel to the observed market characteristics. Thus, although market participants are restricted to observe the first level of the LOB only, we show what kind of 'liquidity terms' could be awaited besides this most competitive order book level.

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OPIS DYNAMIKI MIAR PŁYNNOŚCI NA KIEROWANYM ZLECENIAMI KASOWYM RYNKU WALUTOWYM

S t r e s z c z e n i e

Przedmiotem artykułu jest badanie dynamiki wybranych miar płynności systemu transakcyjnego Reuters Dealing 3000 Spot Matching, który jest głównym, kierowanym zleceniami, międzybankowym rynkiem kasowej wymiany walutowej dla pary EUR/PLN. W artykule przedstawiono schemat wewnątrzdziennej i wewnątrztygodniowej sezonowości dla różnych miar płynności rynku obrazujących kształt arkusza zleceń. Do opisu dużej persystencji płynności wykorzystano modele Autoregresyjnego Warunkowego Czasu Trwania (Autoregressive Conditional Duration, ACD) z długą pamięcią. Szczególną uwagę poświęcono oddziaływaniu napływu nowej informacji na wahania płynności. Wykazano statystycznie istotny dodatni wpływ prawdopodobieństwa zawierania transakcji na podstawie prywatnej informacji (PIN) na wielkość zmiany ceny wywołaną pojedynczą transakcją i na wielkość spreadu bid-ask, a także ujemny wpływ na podaż płynności po stronie ask rynku (zlecenia sprzedaży euro). W badaniu uwzględniono również wpływ innych zmiennych kontrolnych, takich jak wolumen transakcji, zmienność i opóźnione stopy zwrotu.

Słowa kluczowe: mikrostruktura rynku, rynek kierowany zleceniami, prawdopodobieństwo zawierania transakcji na podstawie prywatnej informacji, modele ACD

EXPLAINING LIQUIDITY DYNAMICS IN THE ORDER DRIVEN FX SPOT MARKET

A b s t r a c t

The paper investigates the dynamics of several intraday liquidity measures for the Reuters Dealing 3000 Spot Matching System that constitutes a major order driven interbank spot market for the EUR/PLN. We derive the time-of-day and the day-of-week effects for different liquidity variables representing the shape of the limit order book. In order to capture the strong persistence exhibited by liquidity, the long memory Autoregressive Conditional Duration (ACD) models are applied. Special attention is paid to the impact of information arrival on liquidity fluctuations. We document the significant positive impact of probability of informed trading (PIN) on the price impact of trading and the bid-ask spread and the negative impact of the PIN on the liquidity supply on the ask side of the market (orders to sell euro), after controlling for the effects of other covariates such as the trading volume, volatility or previously observed returns.

Keywords: market microstructure, order-driven market, probability of informed trading, ACD models

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JAKUB BORATYŃSKI

ROBUSTNESS OF CGE SIMULATION RESULTS IN THE CONTEXT OF STRUCTURAL CHANGES – THE CASE OF POLAND [1]

1. INTRODUCTION

Sensitivity analysis is a common way to address the problem of uncertainty of results in simulations based on computable general equilibrium (CGE) models. Different studies have considered the consequences of both using alternative model specifications and varying parameter values. Limiting ourselves to the latter case only, the most prominent approach in the recent literature is the systematic sensitivity analysis (SSA) – a technique in which parameter values are drawn from assumed distributions and the variances of simulation outcomes are next analyzed (Arndt, 1996; DeVuyst, Preckel, 1997; Hermeling, Mennel, 2008). SSA is usually applied to various elasticity parameters, i.e. the ones that are not derived from the CGE model's database and thus usually taken from external empirical sources (examples are Hertel et al., 2007; Domingues et al., 2008; Narayanan et al., 2010; Elliott et al., 2012). Less frequently sensitivity analysis concerns parameters obtained from calibration to benchmark equilibrium data. Using SSA in that case is more difficult, as parameter errors typically cannot be treated as independent (Dawkins, 2005). Examples of the use of SSA in this context are Dawkins (2005) and Elliott et al. (2012). Otherwise sensitivity analysis might boil down to calibrating the model to alternative benchmark equilibrium databases, e.g. data for different years (Roberts, 1994) or different estimates of data for the same year (Cardenete, Sancho, 2004).

In recent years a number of studies were published, presenting applications of various CGE models developed for Poland. For example, the ORANI-type model POLGEM was developed to study fiscal policies (Honkatukia et al., 2003). Based on another model, originally developed by the World Bank and the National Bank of Poland (Gradzewicz et al., 2006), Hagemejer et al. (2011) analyzed different strategies to reduce general government deficit, and Hagemejer, Żółkiewski (2013) estimated the impact of the EU 2020 climate and energy package on the Polish economy. Hagemejer

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et al. (2014) use a global CGE GTAP model (with Poland as one of the regions) in an analysis of liberalization of trade in services under the EU Services Directive. Zawalińska (2009) and Zawalińska et al. (2013) are examples of the use of regionally-disaggregated CGE models for Poland to study the consequences of agricultural policies. Energy and climate policies were analyzed using a model with extended treatment of energy inputs to production (Kiuila, Peszko 2006), as well as using a global economy model ROCA, with Poland as a distinguished region (World Bank 2011; see also Böhringer, Rutherford 2013). Borowski et al. (2011, 2013) assessed the impact of the preparations and organization of 2012 European Football Championships in Poland, based on a dynamic CGE model of the Polish economy. Finally, Boratyński, Borowski (2012) adopted the CGE framework to simulate the effects of a possible introduction of flat income tax. Sensitivity analysis (with respect to crucial modeling assumptions) in the cited studies is rather limited. In Hagemejer et al. (2011), Boratyński, Borowski (2012), as well as Hagemejer, Żółkiewski (2013) it amounts to performing simulations under alternative model closures. However, none of the papers has addressed the problem of uncertainty of the calibrated (share) parameters.

The present study is a follow-up to an earlier paper (Boratyński, 2011), in which systematic sensitivity analysis with respect to elasticity parameters was undertaken. These studies share the same model and simulation scenarios, but refer to distinct sources of uncertainty. The former paper analyzed consequences of uncertainty connected with unobserved behavioral (elasticity) parameters; therefore it indirectly referred to uncertainties inherent in econometric work; in terms of methodology, the cited study adopted the Gaussian Quadrature approach, as a replacement for a more computationally intensive Monte Carlo simulation. Whereas the present paper relates to the problem of uncertainty of information concerning the structure of the economy (e.g. industry/commodity composition of output/demand, technologies – including proportions of intermediate inputs, import intensities of different industries/commodities, structure of taxes etc.). These data give rise to a number of the so called “share” (or “structural” or “calibrated”) parameters of a CGE model, which – along with elasticity (behavioral) parameters – drive simulation results.[2]

The topic of uncertainties related to these calibrated share parameters is less frequently met in the CGE literature (compared to studies concerning elasticity parameters). This paper contributes to that research, firstly, by using an extensive database – a time series of annual supply and use tables for the Polish economy spanning the years 1996–2005[3] (a previous study of that type for Poland, by Roberts, 1994,

[2] In the CGE framework, the calibrated parameters are those which are derived from benchmark equilibrium data; the non-calibrated ones (e.g. elasticities) are taken from external sources, e.g. literature reporting results of econometric estimation.

[3] The database was compiled using primarily the data supplied by the Polish Central Statistical Office. As the additional sources we also used the Eurostat database and the EU KLEMS Database, March 2008 Release (see Timmer et al., 2007). An advantage over international databases such as WIOD (World Input-Output Database) is in the fact that data for *each year* are derived from official country

used aggregate, single-sector data and model). Secondly, we attempt to identify which parameter groups contribute most to variability of simulation outcomes. Thirdly, the results are reported for three different simulation exercises, comprising demand-side, supply-side and tax shocks. A practical question in the background of this paper is whether the lag between benchmark equilibrium year and the simulation period is a serious problem for the reliability of results, especially for emerging economies, such as Poland.

The paper is structured as follows. Section 2 characterizes the model and the closure used. In section 3 we examine how robust are simulation results to changes in the database used as a benchmark equilibrium for model calibration. Section 4 investigates the importance of different parameter groups for the variability of simulation results. Section 5 concludes.

2. THE MODEL AND CLOSURE

The specification of the model used in this study largely follows that of ORANI-G – a generic static single-economy, single-region computable general equilibrium model (Horridge, 2003; see also Dixon et al., 1982; for principles of CGE models, their recent developments and applications see Dixon, Jorgenson 2013). The ORANI (or MONASH) approach constitutes a long tradition in CGE modeling and has had a large number of implementations worldwide (Dixon et al., 2013). Our model represents the economy in an 18 industry/commodity breakdown. Below we characterize its key features.

- *Nested production structure*. In each industry, intermediate input composites and the primary factor bundle are combined in fixed proportions (Leontief production functions). Primary factor bundle is a CES (constant elasticity of substitution) composite of capital and labor, while intermediate inputs are CES composites of domestic and imported products.
- *Multiproduction*. Each industry produces a variety of commodities, subject to CET (constant elasticity of transformation) production frontier.
- *Household demand*. Household demand for commodities, for a single representative household, is determined in the linear expenditure system (LES) framework, which corresponds with the Klein-Rubin (or Stone-Geary) utility function.
- *Exports*. The economy faces downward sloping foreign demand schedules, so it is assumed to have some (limited) market power in international markets.
- *Sourcing of final demand*. Final demand of a given user for a given commodity is a CES composite of demand for imported and domestic commodity (as in the case of intermediate inputs in the production nest).

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supply and use tables, which is not the case of WIOD tables for Poland (and for a number of other countries); moreover in international databases original data are often subject to additional processing, in order to reach inter-country consistency.

- *Optimizing behavior*. Capital-labor substitution and product sourcing decisions are subject to the cost minimization principle. Producers adjust their product-mix to maximize revenue. Household are assumed to be utility maximizes.
- *Market structures*. We assume competitive commodity markets, and, accordingly, marginal cost pricing and zero pure profits.

However, there are some differences to the ORANI-G model. Most importantly, we assume that the composition of investment demand (e.g. the shares of demand for construction services, machinery, transport equipment etc.) are identical for all investing industries. Other differences include a more detailed, SAM-based representation of income distribution in our model, compared to ORANI-G; we do not model tariffs explicitly, but include them into the broad category of taxes on products. Finally, we also use a different notation, based on a mixed level and percentage change representation of the model equations (see e.g. Dixon, Parmenter, 1996, p. 17–21). The 18 industry/commodity breakdown was chosen as a supposedly good compromise between model detail (disaggregation) and tractability (in terms of computational burden in repeated simulations as well as presentation and analysis of the results).

In this study we adopt a long-run closure, which entails further assumptions:

- *Capital and investment*. Industry capital stocks adjust to preserve original (observed) gross rates of return, i.e. the ratios of capital rental rates to the price of new capital. Investment follows (is proportional to) capital stock in each industry. As a consequence of such a specification, capital is treated as industry-specific.
- *Labor market*. Aggregate (effective) labor supply is fixed, while labor may flow between industries, so that the wage per effective labor unit is equalized.
- *Absorption*. Aggregate consumption adjusts to facilitate a fixed (nominal) balance of trade to GDP ratio. Government and non-profit institutions' consumption follow (are proportional to) aggregate household consumption.

In the comparative static framework, simulations do not show the distribution of the analyzed effects in time (which is accomplished in either recursive-dynamic, or "fully" dynamic models with expectations). The results represent (percentage) deviations from a hypothetical baseline growth path, but without giving an explicit account of time needed by the economy to fully accommodate to the analyzed shocks. Under the long-run assumption, the length of the accommodation period is interpreted as the time necessary for the capital stock in each industry to reach its new optimum level, which is in turn related to the specificity of investment process and depreciation rates in different industries.

To avoid confusion related to interpretation, we should strongly stress that the results presented further are not the *effects for consecutive years* in the usual sense (i.e. they are not analogous to the results of a dynamic model). Rather the *data for consecutive years* are used to calibrate the model which is then solved in static long-run experiments, as described above. Calibrating the static model to data from year t entails an implicit assumption that these data – at least approximately – describe the economy's structure in year $t+s+d$, in which the effects of the shocks under consideration are expected to materialize ($t+s$ is the period in which the simulated shock is actually expected to take place, and d is the time necessary to accommodate to the shock). Given that in practice $s+d$ can often be as long as several years, this raises a question whether such obsolete structural data can reasonably approximate the future picture of the economy. This is especially an issue for emerging economies, for which it is not likely that period t structural data represent (or are close to) steady state. By calibrating the model to data from subsequent years (spanning the transition period in Poland) we test how much of a problem the structural changes are for the robustness of simulation results.

3. SIMULATION RESULTS FOR ALTERNATIVE CALIBRATION DATABASES

To perform this specific robustness analysis we repeat the same set of three simulation experiments using the same model calibrated to 10 different datasets (for the Polish economy), for subsequent years between 1996 and 2005. The results are presented as *average* responses (percentage changes) of endogenous variables to the imposed exogenous shocks, along with variation coefficients as measures of dispersion of those responses (see tables 1–3). Non-calibrated parameters, such as substitution elasticities, were held constant across simulations. The simulation scenarios where chosen arbitrarily, but they represent three distinct type of shocks – supply-side, demand-side and tax shocks. Below we briefly analyze the main mechanisms "at work" and the macroeconomic outcomes in the three experiments. Next we move to the results of sensitivity analysis itself.

Simulation 1 assumes a 20% decrease of the joint capital and labor productivity in the energy sector (electricity, gas and water supply). Such a shock might for example relate to the need of conforming with higher environmental protection standards. From the macro perspective, the shock immediately reduces the amount of *effective* primary factor inputs available in the economy – more for capital than labor, as the energy sector is capital-intensive. This makes labor relatively cheaper and induces substitution of capital for labor, which decreases capital stock even further. On the other hand, keeping the balance of trade to GDP ratio unchanged (lower activity level diminishes demand for imports, which must be followed by exports decrease) requires real appreciation of the local currency. This effect will mitigate the fall of capital stocks – by lowering the cost of investment (which is characterized by a relatively high import-intensity in Poland). Negative productivity shock and the decrease of aggregate capital stock make the real GDP fall by – on average – 0.80%. Energy prices increase by 9.47% (GDP deflator being constant, as a numeraire), while energy sector output falls by 2.04%, on average.

Simulation 2 shows the impact of decreasing the effective rates of taxes on products by 10%, while keeping government expenditure unchanged. The increased household demand stimulates output growth, requiring capital expansion (aggregate labor input is held fixed). The expansion is mitigated by the increase in production costs and terms of trade deterioration (real depreciation), making investment costs rise. The mean response of aggregate capital stock is 0.88%, and the resulting GDP increase equals 0.50%. The analyzed policy change would diminish government revenues from taxes on products by 8.62%, on average, and the total tax revenues – by 4.17%.

In *simulation 3* we assume a vertical downward shift in foreign demand schedules, such that the pre-shock quantity of goods would be exported only if the prices decreased by 5%. Such an effect can be attributed for example to an increase in import taxes paid abroad. An immediate result is the terms of trade deterioration, and real depreciation (by 6.53%, on average), necessary to preserve the original (nominal) balance of payments to GDP ratio. The increased costs dampen investment, which reduces capital stocks (by 2.80%, on average), and, as a consequence, the GDP (by 1.67%, on average). On the absorption side, it is mainly consumption and fixed capital formation that decline (by 3.21% and 2.94%, respectively), while exports volume decrease is moderate (1.5%, on average). Import volume drops by as much as 6.16% (on average), both due to constrained activity and substitution towards cheaper domestic products.

Tables 1–3 report absolute values of coefficients of variation (V) for percentage changes of endogenous variables invoked by exogenous shocks (different responses being a result of using different calibration databases). The main finding is that the results *may* be significantly sensitive to the calibration data set in use. At the same time, this need not always be the case.

In further considerations we use a coefficient of variation value of 25% as a convenient cut-off point between the ”low” and the ”high” dispersion of simulation results. In our specific sample of results a coefficient of variation greater than 25% implies that the strongest response found for a given variable is more than twice the magnitude of the weakest response.

One cannot identify variables with inherently large or small uncertainty. The scale of uncertainty crucially depends on the type of simulation experiment. The relatively highest variation of results is found in simulation 3 (the decrease in foreign demand). For 5 out of 8 reported variables, representing aggregate volumes, the coefficient of variation exceeded 25%, thus marking a considerable degree of dispersion. In the case of real tax revenues, the direction of response to the shift in foreign demand schedules is ambiguous (variation coefficients for revenues from taxes on products and total tax revenues are 182.6% and 86.6%, respectively). On the contrary, in the tax cut simulation the response of tax revenues is among the most robust results – for total tax revenues the coefficient of variation is 9.0%, while for the revenues from taxes on products – only 1.7%. In simulation 1 the sign change was found for aggregate investment reaction. Also the change in aggregate capital – although unequivocally positive – is quite sensitive to the choice of calibration database. However, in the case of 5 out of 8 aggregate volumes distinguished, the dispersion is well below 25%. Responses of aggregate prices in general appear slightly more robust than those of volumes.

Table 1.

Results for aggregates*

Variable	Simulation 1			Simulation 2			Simulation 3		
	M	*V*	*V'*	*M*	*V*	*V'*	*M*	*V*	*V'*
VOLUMES:									
GDP	-0.80	11.9	6.9	0.50	16.8	8.8	-1.67	32.3	7.4
Consumption	-0.97	5.4	5.2	0.25	45.0	23.8	-3.21	19.4	6.5
Fixed capital formation	0.20	131.3	38.5	0.92	10.2	9.5	-2.94	23.7	7.1
Exports	-0.69	15.0	7.8	0.92	7.2	6.1	-1.50	35.6	18.0
Imports	-0.46	14.5	7.4	0.49	18.5	11.1	-6.16	5.2	1.9
Capital	-0.24	41.8	21.7	0.88	10.3	9.2	-2.80	26.0	6.4
Total tax revenues	-0.51	12.7	7.2	-4.17	9.0	6.1	-0.56	86.6	20.2
Revenues from taxes on products	-0.57	30.3	12.1	-8.62	1.7	0.4	-0.33	182.6	43.0
PRICES:									
GNE	-0.06	19.6	4.9	0.10	17.9	17.4	1.54	9.6	7.1
Household consumption	0.14	12.0	10.8	-0.37	11.6	8.0	1.82	9.1	6.5
Consumption of non-profit institutions	-0.25	24.5	11.7	1.10	14.2	11.8	-0.08	190.7	184.6
Government consumption	-0.52	9.2	9.1	1.42	10.7	7.4	-1.12	17.4	17.4
Fixed capital formation	-0.27	14.6	11.2	0.38	12.1	11.9	2.86	14.3	6.5
Exports	-0.17	15.8	12.4	0.75	5.4	4.5	1.58	26.2	6.6
Imports	-0.34	13.9	8.5	0.98	4.7	4.5	6.53	4.8	1.6
Capital	-0.27	14.6	11.2	0.38	12.1	11.9	2.86	14.3	6.5
Labour	-1.22	16.9	9.7	2.30	7.1	6.7	-3.94	27.4	8.4
Real exchange rate	-0.34	13.9	8.5	0.98	4.7	4.5	6.53	4.8	1.6
Terms of trade	0.17	14.7	7.8	-0.23	7.1	6.1	-4.65	2.7	1.4

* *M* – mean response to a shock (in %), *V* – coefficient of variation (absolute value, in percentage points), *V'* – coefficient of variation after excluding trend (absolute value, in percentage points).

Table 2.

Results for commodity outputs*

Products and services	Simulation 1			Simulation 2			Simulation 3		
	M	*V*	*V'*	*M*	*V*	*V'*	*M*	*V*	*V'*
Agriculture, forestry and fishing products	-0.83	5.7	4.7	0.46	12.6	12.0	-1.24	43.4	15.3
Coal and peat	-1.34	31.6	15.1	0.13	112.7	51.9	0.63	35.6	35.2
Oil, gas and ores	-1.56	61.4	29.9	0.38	94.2	57.5	6.79	37.7	16.4
Food and tobacco	-0.42	14.5	9.4	0.61	15.4	8.8	0.10	127.5	89.3
Textiles and wearing	-0.58	16.1	12.9	0.49	16.3	11.0	-1.30	64.5	30.0
Refined petroleum products	-0.88	10.4	8.7	1.68	7.4	5.9	-2.41	21.7	13.1
Chemicals and metals, metal products	-0.97	24.7	10.8	0.79	13.4	12.9	0.17	366.9	157.5
Machinery and equipment	-0.37	21.2	12.3	0.58	11.6	11.2	0.39	203.0	80.4
Transport equipment	-0.53	16.4	15.7	1.04	13.7	13.3	-0.81	189.3	56.8
Other products	-0.59	15.5	13.6	0.67	15.9	8.2	-0.17	102.3	69.6
Electricity, gas and water supply	-2.04	14.3	4.0	0.39	33.2	13.0	-1.13	28.1	10.8
Construction services	-0.31	82.8	22.4	0.67	14.5	13.7	-2.67	24.0	8.1
Trade services	-0.64	12.0	5.2	0.56	15.7	7.7	-1.93	35.6	7.7
Transport and telecommunication	-1.08	9.4	9.4	0.63	41.9	15.3	-1.94	40.9	16.3
Financial, real-estate and business services	-0.83	8.2	7.8	0.34	13.4	11.9	-1.35	26.6	10.5
Public administration and defense	-0.95	5.7	5.1	0.26	46.0	23.6	-3.04	17.4	8.3
Education and health services	-1.07	7.1	4.7	0.02	555.1	413.3	-3.19	16.5	7.0
Other services	-0.74	8.7	8.5	0.20	71.4	10.8	-1.76	31.3	11.0

* *M* – mean response to a shock (in %), *V* – coefficient of variation (absolute value, in percentage points), *V'* – coefficient of variation after excluding trend (absolute value, in percentage points).

Table 3.

Results for commodity prices*

Products and services	Simulation 1			Simulation 2			Simulation 3		
	M	*V*	V’	M	V	V’	M	V	V’
Agriculture, forestry and fishing products	-0.14	34.9	26.6	0.48	6.9	6.7	1.88	11.3	8.4
Coal and peat	-0.55	20.6	15.8	1.43	6.6	5.2	-0.98	35.6	29.6
Oil, gas and ores	0.11	438.7	260.1	1.37	9.9	7.6	-0.91	180.3	68.8
Food and tobacco	-0.22	25.6	24.6	0.56	6.2	6.1	1.29	11.6	9.4
Textiles and wearing	-0.28	13.5	10.3	1.06	5.9	5.1	1.01	44.1	14.2
Refined petroleum products	-0.32	33.6	24.5	0.52	36.3	26.8	3.35	10.5	9.8
Chemicals and metals, metal products	-0.03	188.7	109.5	0.79	8.9	8.3	1.60	24.6	7.6
Machinery and equipment	-0.28	21.0	14.6	0.94	6.2	5.9	1.43	28.1	7.0
Transport equipment	-0.17	47.5	27.7	0.79	2.8	2.7	2.23	23.4	9.3
Other products	-0.21	31.5	25.7	0.82	7.4	6.4	1.31	18.1	6.8
Electricity, gas and water supply	9.47	3.4	3.3	0.85	9.7	9.6	0.84	19.1	9.4
Construction services	-0.32	13.5	11.4	0.77	7.1	4.6	1.32	19.0	7.6
Trade services	-0.25	29.4	10.5	0.63	5.3	5.3	1.40	7.0	6.8
Transport and telecommunication	-0.25	16.4	8.5	0.67	26.1	9.1	0.85	34.3	12.8
Financial, real-estate and business services	0.18	70.4	45.7	0.77	13.4	5.9	1.00	25.8	7.6
Public administration and defense	-0.61	17.4	9.6	1.48	9.0	7.5	-1.30	19.4	15.8
Education and health services	-0.60	11.6	11.4	1.63	7.4	6.1	-1.76	15.9	10.4
Other services	-0.12	42.3	17.0	0.84	17.2	11.0	0.61	39.7	28.3

* *M* – mean response to a shock (in %), *V* – coefficient of variation (absolute value, in percentage points), *V′* – coefficient of variation after excluding trend (absolute value, in percentage points).

As expected, the problem of sensitivity of results is more serious for sectoral than for aggregate variables. Especially the results of simulation 3 are a concern – the direction of output response to the foreign demand shock is ambiguous for 5 out of 18 sectors. In total, for 14 sectors the coefficient of variation exceeds 25%. In simulation 2 the number of sectoral output reactions with ambiguous direction is four, while in simulation 1 – two (the number of coefficients of variation exceeding 25% equals 7 and 3, respectively). Only in simulation 1 the dispersion of commodity prices' reactions is larger than that of outputs – with sign ambiguities for 3 sectors, and variation coefficient exceeding 25% for 10 sectors. In simulation 2 and 3 price responses are more robust than output responses.

A more optimistic picture of results' robustness emerges when we notice that for most variables the responses to exogenous shocks change systematically when the model is calibrated to data for subsequent years. These data carry information about the changing structure of the economy (section 4 explains more closely what aspects of this structure are taken into account), reflected in the model's calibrated – share – parameters. Therefore, systematic changes in the reported results indicate that the underlying structural change is also in some way (partly) systematic, and – to that extent – it can perhaps be subject to formalized description and prediction. Contrary to that, irregular changes in simulation results (over subsequent datasets used to calibrate share parameters) point to the more erratic part of structural changes. In order to roughly assess contributions of the systematic and the irregular components of structural change, we estimate linear "trend" for each variable's responses to the analyzed shocks[4].

In tables 1–3 *V'* represents coefficients of variation of simulation results after excluding the linear "trend" component. In most cases there is a substantial reduction of response variation after "trend" removal. Coefficients of variation for aggregate variables decrease, on average, by 39%, 22% and 55% in simulations 1, 2 and 3, respectively. The respective reductions for sectoral variables (including both output and prices) are 31%, 26%, and 49% (worth noticing, the largest variation reduction concerns the simulation with the largest volatility of endogenous variable responses – simulation 3). The above outcome shows that much part of the changes in the structure of the economy, as represented by calibration databases, can be considered in a way systematic. This implies that to improve reliability of simulation results, based on a static CGE model, one should consider updating (forecasting) calibration

[4] This can be formalized as follows: $y_i^{(t)} = \alpha_i + \beta_i \cdot t + \varepsilon_i$, where $y_i^{(t)}$ is the response (percentage change) of the i^{th} variable (i iterates over a set of all variables reported in this study) in a simulation experiment, in which the share parameters were calibrated to data from year t; α and β are the parameters (estimated using OLS), and ε is the error term. The "trend" removal amounts to the following calculation $\hat{\varepsilon}_i = y_i^{(t)} - \hat{\alpha}_i - \hat{\beta}_i \cdot t$, where $\hat{\varepsilon}$ is interpreted as the (estimated) non-systematic part of simulation results variation across alternative parametrizations of the CGE model. The term "trend" is put in quotation marks here, since it does not refer to any observable quantity, but to modeling outcomes.

databases, in order to account for changes in the economy's structure (this is in line with the findings of Dixon, Rimmer, 2002, p. 4, that in dynamic CGE modeling the baseline forecasts can significantly affect policy simulation results). Such a conclusion at least holds for Poland – an emerging economy. "Trends" in simulation results may suggest that even the use of simple techniques is potentially beneficial.

4. CONTRIBUTIONS TO RESULTS VARIATION

As a next step, we assessed relative importance of different parameter groups in generating the variation of results. The following groups of calibrated (share) parameters were separated:

(A) Macro structure of final demand (shares of final demand aggregates in total final demand);
(B) Commodity structure of final demand;
(C) Import intensities of supply of commodities (import shares);
(D) Capital and labor cost shares in value added;
(E) Value added shares in gross output;
(F) Structures of intermediate inputs (produced inputs' cost shares);
(G) Trade and transport margin rates;
(H) Rates of taxes on products;
(I) Income distribution structures;
(J) Frisch coefficient;
(K) Structure of the make matrix.

The term "income distribution structures" refers to parameters of the equations showing how value added is transformed into disposable income of institutional sectors (incl. households and government). Those parameters are mostly ratios or shares – for example the households' share in total gross operating surplus in the economy. The Frisch parameter is the negative of the reciprocal of the share of discretionary expenditure in total household consumption (Dixon, Rimmer, 2002, p. 171–173). By the "structure of the make matrix" we mean shares of various products in a given industry's output. All of the parameters (shares and ratios) listed above are based on nominal values.

We adopted the following procedure for the calculations. First, a series of simulations is performed with parameters from group (A) varying (derived from databases for subsequent years between 1996 and 2005), all other parameters being constant (derived from the 2000 database). After that we are able to calculate the variance of endogenous variables' responses to a given shock, *due to variation of parameters belonging to the (A) group*. The same is next repeated for the remaining parameter groups. In this way we obtain variances of simulation results attributed to the variation of different parameter categories. The whole procedure is repeated for each of the three simulation experiments considered.

Table 4.

Ranks of parameter groups as sources of results variation

Sources		Simulation 1		Simulation 2		Simulation 3	
		macro	sectoral	macro	sectoral	macro	sectoral
(A)	Macro structure of final demand	4	2	27	13	4	3
(B)	Commodity structure of final demand	17	15	16	18	14	8
(C)	Import shares	2	1	1	1	29	18
(D)	Capital/labour cost shares	15	9	22	26	39	46
(E)	Value added shares in gross output	14	16	2	5	6	11
(F)	Intermediate input structures	30	40	2	8	5	10
(G)	Margin rates	0	1	2	2	1	2
(H)	Rates of taxes on products	8	1	22	24	1	0
(I)	Income distribution structures	2	0	4	0	0	0
(J)	Frisch coefficient	0	1	2	1	0	0
(K)	Make matrix structure	7	13	0	1	1	2

It is noteworthy that modifying a single group of share parameters leads to an imbalance in the benchmark equilibrium data (e.g. if we impose 1996 shares of final demand aggregates on 2000 input-output flows table). Therefore at each step original flow data are re-balanced such that consistency with the desired full set of share parameters is achieved.

Partial variances resulting from the procedure described above do not sum up to the ones reported in section 3, i.e. those obtained when all parameters vary jointly. One reason is that the CGE model is non-linear. Another one is that parameter changes in time might be correlated between parameter groups. Thus, what we perform is not literally a decomposition (which is the case in global sensitivity analysis – see Saltelli et al., 2008). Nevertheless, we find it useful to rank the importance of different parameters in generating the variation of the results by adding up variances related to individual parameter groups and calculating their shares in that total. This is done for each endogenous variable of interest (i.e. for all variables listed in tables

1–3). Averages of the shares described above, calculated separately for aggregate and sectoral variables, are reported in table 4.

Looking at the results, there are only two parameter groups – namely the commodity structures of final demand (B) and the shares of capital and labor compensation in the value added (D) – which play important (although not necessarily dominant) roles in all simulation experiments, as factors contributing to the results variation. There are also parameters – including margin rates (G), income distribution structures (I), and the Frisch coefficient (J) – that proved irrelevant from the same point of view. Most importantly, however, the relevance of a given parameter group as an uncertainty source crucially depends on the shock being simulated.

For example, in simulation 1 (negative supply shock in the energy sector) the dispersion of results is driven mainly by the changing structures of intermediate inputs (F). These relate to changes in both the energy-intensity of production, as well as changes in the input composition in the energy sector itself.[5] These changes are of much smaller importance for the results of the other two experiments. In simulation 2 (tax cut), there are four parameter groups with similar contributions to the dispersion of the results – the (initial) rates of indirect taxes (H), capital and labor cost shares (D), as well as macro and micro composition of final demand (A & B). The first of the mentioned factors plays practically no role in simulations 1 & 3. The main sources of uncertainty in simulation 3 relate mainly to capital and labor cost shares (D) and import intensities of supply (C). Dispersion of sectoral and aggregate results is largely driven by the same factors, although there are certain differences in the actual contributions.

We can conclude that in order to improve the reliability of simulation results for a given policy (or other) question, one could focus on a narrow set of parameters only. Sensitivity analysis of the kind presented above could help identify those parameters (and model mechanisms) that generate a significant part of uncertainty about the simulation outcomes.

5. CONCLUDING REMARKS

It is a typical situation that a (static) CGE model is calibrated to a database that represents structural information which is not quite up-to-date. We have asked, to what extent the changes in an economy's structure, as represented by the input-output data, affect CGE simulation results. The analysis involved model calibration to the Polish data for subsequent years of the period 1996–2005, and running three distinct simulation experiments under the different parameterizations (under a long run-closure).

[5] Since what we can derive from our database is changes in cost shares only (i.e. changes in input structures in nominal terms), the dispersion of outcomes might as well be related to changes in relative prices of different inputs – e.g. being a consequence of changes in the world prices of energy resources.

The robustness of results to changes of calibration database (i.e. to the underlying structural changes) has shown to be dependent on the analyzed shock. Also, the responses of different endogenous variables to the shocks are characterized by quite different scales of dispersion within a given experiment. Hardly any regularity can be identified when analyzing the ”distribution” of uncertainty among the variables. In general, sectoral results are more sensitive than aggregate results, and volumes are usually more sensitive than prices.

Although in a majority of cases (under the three analyzed experiments) the dispersion of simulation outcomes was in an acceptable range, there was also a number of cases where robust inference was not possible, including the cases of ambiguity of the direction of a variable's response (especially with respect to sectoral variables). Thus, presuming that lagged data provide a good proxy for the current or near-future economic structure might be a potentially risky practice, at least for emerging economies, which undergo substantial restructuring. A proposed approach is to perform a thorough sensitivity analysis in order to identify uncertainty sources. As our analysis suggests, these sources confine to a subset of parameters – what subset, however, being again strictly dependent on the shock in question.

A promising finding is that when calibrating the model subsequently to the databases for consecutive years, the results – responses of endogenous variables to the imposed shocks – reveal pronounced trends. This indicates that the changes in the economic structures are in a way systematic, and thus utilizing the information inherent in the time series of calibration (benchmark equilibrium) datasets is likely to bring parameter updates improving the reliability of results.

University of Lodz

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ODPORNOŚĆ WYNIKÓW SYMULACJI NA PODSTAWIE MODELU CGE W WARUNKACH ZMIAN STRUKTURALNYCH W GOSPODARCE – PRZYPADEK POLSKI

Streszczenie

Typowym sposobem odniesienia się do problemu niepewności wyników symulacji na podstawie modeli CGE (*Computable General Equilibrium*) jest analiza wrażliwości. Większość prac poświęconych temu zagadnieniu koncentruje się na kwestii wyboru wartości różnego rodzaju elastyczności. W niniejszej pracy podejmujemy analizę wrażliwości dotyczącą parametrów opisujących strukturę gospodarki, uzyskiwanych w drodze kalibracji. Do kalibracji modelu używamy zestawów danych za kolejne lata z okresu 1996-2005, a następnie analizujemy rozrzut wyników dla trzech różnych eksperymentów symulacyjnych.

Wyniki dla części – choć nie większości – zmiennych charakteryzują się znaczącą wrażliwością na wybór bazy danych wykorzystanej do kalibracji (włączając niepewność co do kierunku reakcji). Stopień rozrzutu wyników i jego źródła istotnie zależą od rodzaju analizowanego scenariusza symulacyjnego. Skala niepewności dotyczącej poszczególnych zmiennych jest również zróżnicowana. Zaleca się zatem, aby gruntowna analiza wrażliwości była standardową częścią badania symulacyjnego. Ponadto zastosowanie nawet prostych (np. opartych na analizie trendów) metod aktualizacji bazy danych mogłoby najprawdopodobniej zwiększyć wiarygodność wyników, biorąc pod uwagę, że reakcje zmiennych endogenicznych na zadawane w symulacjach impulsy podlegają systematycznym zmianom, gdy model kalibrowany jest do danych z kolejnych lat.

Słowa kluczowe: modele CGE (*Computable General Equilibrium*), analiza wrażliwości, kalibracja

ROBUSTNESS OF CGE SIMULATION RESULTS IN THE CONTEXT OF STRUCTURAL CHANGES – THE CASE OF POLAND

A b s t r a c t

It is common to address the problem of uncertainty in computable general equilibrium modeling by sensitivity analysis. The relevant studies of the effects of parameter uncertainty usually focus on various elasticity parameters. In this paper we undertake sensitivity analysis with respect to the parameters derived from calibration to a benchmark data set, describing the structure of the economy. We use a time series of benchmark databases for the years 1996-2005 for Poland to sequentially calibrate a static CGE model, and examine the dispersion of endogenous variables' responses in three distinct simulation experiments.

We find a part – though not the most – of the results to be significantly sensitive to the choice of calibration database (including ambiguities about the direction of response). The dispersion of the results and its sources clearly depend on the shock in question. Uncertainty is also quite diverse between variables. It is thus recommended that a thorough parametric sensitivity analysis be a conventional part of a simulation study. Also, the reliability of results would likely benefit even from simple, trend-based updates of the benchmark data, as the responses of endogenous variables exhibit systematic changes, observed when the model is calibrated to the data for consecutive years.

Keywords: computable general equilibrium (CGE) modeling, sensitivity analysis, calibration

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SECOND BWANAKARE

ECONOMETRIC BALANCING OF A SOCIAL ACCOUNTING MATRIX UNDER A POWER-LAW HYPOTHESIS

1. INTRODUCTION

Contrary to many other fields, macroeconomics has neglected the link between phenomena and power-law (PL)[1], characterizing non-extensive complex systems within the class of Levy's process laws. In light of recent literature, the amplitude and frequency of macroeconomic fluctuations are not considered to substantially diverge from many other extreme events, natural or human-related, once they are explained in the same time (or space) scale. Following a few recent studies related to applying non-extensive entropy to economics, this study extends the theoretical model (e.g., Bwanakare, 2013a, b; Tsallis, 2004) and proposes a new direction for applications in solving ill-posed inverse problems. In this study, a social accounting matrix (SAM) will be balanced to illustrate this new technique.

In the rest of this introduction, the rationale of applying PL is presented. According to many studies (e.g., Bottazzi, 2007; Champernowne, 1953; Gabaix, 2008), a large array of economic laws take the form of PL, in particular, macroeconomic scaling laws, distribution of income, wealth, the size of cities and firms[2], and the distribution of financial variables such as returns and trading volume. Mantegna, Stanley (1999) have studied the dynamics of a general system composed of interacting units each with a complex internal structure comprising many subunits where they grow in a multiplicative way over a period of 20 years. They found the system following a PL distribution. It is worth noting the similarity of such a system with the internal mechanism of national account tables, like SAMs. Ikeda, Souma (2008) have worked on an international comparison of labour productivity distribution for manufacturing and non-manufacturing firms. A power-law distribution in terms of firms and sector productivity was found in US and Japan data. Testing Gibrat's law of proportionate effect, Fujiwara et al. (2004) have found, among other things, that the upper-tail of the distribution of firm size can be fitted with a power-law (Pareto–Zipf law). In a recent monograph, Bwanakare (2013b) has proposed a theorem linking low-frequency time

[1] For details about a PL, see, e.g. Gabaix (2008).
[2] See Bottazzi et al. (2007) for a different standpoint on the subject.

series macroeconomic phenomena- and thus input output accounts- with PL distribution. The above citations are not exhaustive.

The central point is that a PL displays, besides its well-known scaling law, a set of interesting characterizations related to its aggregative properties, in that it is *conserved under addition, multiplication, polynomial transformation, minimum and maximum.* Basically, non-extensive (Tsallis) entropy is a thermodynamic concept which, contrary to that of Boltzmann-Gibbs-Shannon, is characterized by complex dependency between elements of non-ergodic systems and independency from initial conditions, fitting power-law a PL distribution (Tsallis, 2009). As opposed to the Gaussian[3] family model, a non-ergodic system suggests that micro-states of the system do not display identical odds of appearing. From the microeconomic prospective[4], this suggests that some economic agents' behaviour does happen more frequently than generally expected- then a heavy queue- and may rely on distant memory and complex correlations. While the Gaussian related Shannon-Kullback-Leibler (SKL) entropy approach is well suited in cases that exhibit limited perturbations, exponential-family phenomena, it remains less appropriate for a class of more complex PL driven shocks, the ubiquity of which, as already mentioned above, now seems evident in nature or social science. Testing PL multifractal properties requires high-frequency series. The higher the series frequency is, the more significant the test outputs about these properties are. The distribution with an exponential tail might correspond to an intermediate stage between a distribution with the PL asymptotics and a very large time lag limit-a Gaussian (Dragulescu, Yakovenko, 2001; Rak et al. 2007). Recently, Nielsen, Nock (2012) have casted exponential family form into PL-related Tsallis non-extensive entropy expression and shown conditions for a closed-form. However, delimiting threshold values for law transition- which is a function of frequency level- is difficult since, to our knowledge, neither a parametric nor non-parametric test yet exists.

Thus, applying Gaussian law systematically could be misleading in the case of some aggregated series and lead in many cases to instable solutions, for example, when a random error diverges enough from the Gaussian model[5] (i.e., with q parameter equaling unity). The methodology presented below fits well with more types of series when applying q-Tsallis entropy. In fact, Gaussian law can be generalized by a class of a few types of Higher-Order Entropy Estimators (Golan, Perloff, 2001; Tsallis, 2009) among which there is Tsallis non-extensive entropy, which presents the valuable additional quality of concavity- then stability- along the existence interval

[3] Then, this law includes all discrete laws converging to normal law. This observation is important for such a study dealing with low frequency time series.

[4] A SAM reflects a general macroeconomic equilibrium based upon microeconomic behaviour of economic agents through an aggregative process.

[5] For instance, data from statistical survey might display systematic errors.

characterizing most real world phenomena. Furthermore, as we will see below, the q-Tsallis parameter presents the strong advantage of monitoring complexity of any system. Among rival methods, only it can measure how far a given random phenomenon is from the Gaussian benchmark. Since the generated empirical solution constitutes a converging case of Gaussian law, outputs of the present work should remain qualitatively comparable with those that can be produced by other rival approaches, such as the RAS approach. However, at least two advantages of the proposed technique deserve to be emphasized. The first relates to the possibility of deriving the q-Tsallis parameter, thereby allowing for assessment of the complexity level of the analyzed system. The second advantage is from an epistemological standpoint. By proposing the non-extensive entropy approach for balancing a social accounting matrix- which is a one-period time series sample- we extend one of the main laws of modern physics (the generalized second law of thermodynamics) to low frequency economic time series and, thus, propose a new competitive econometric instrument for economic modeling.

This paper is organized as follows: Section II is devoted to presenting the link between Kullback-Leibler (K-L) information divergence and non-extensive Tsallis entropy. Section III presents a generalized linear non-extensive entropy econometric model. Then, for empirical applications, a Tsallis cross-entropy econometric model for SAM parameter estimation is presented with details. Section IV proposes parameter area inference for the estimated model. Section V presents the principal theoretical aspects of a SAM structure and its balancing. Section VI presents model outputs, and the last section concludes the paper with some comments and suggestions.

2. Q-GENERALIZATION OF THE K-L RELATIVE ENTROPY AND CONSTRAINING PROBLEM

To derive non-extensive entropy formulation, one first needs to set up the three simplest differential equations and their inverse functions (see Tsallis, 2009) and, next, unify these three cases (without preserving linearity). One then gets:

$$\frac{dy}{dx} = y^q \ \ (\mathrm{y(0)=1};\ q \in \Re\ ). \tag{1}$$

We observe that this expression displays power-law distribution form.
Its solution is

$$y = [1 + (1-q)x]^{\frac{1}{1-q}} \equiv e_q^x (e_1^x = e^x)$$

and its inverse function is

$$y = \frac{x^{(1-q)} - 1}{1-q} \equiv \ln_q x \; (\ln_1 x = \ln x). \tag{2}$$

The above eq. (2) represents the non-extensive (Tsallis) entropy formula[6], which can be explained in logarithmic terms $\ln_q x$ where q stands for the basis. In particular, for q approaching unity, we get the traditional Gibbs-Shannon maximum entropy (Shannon, 1948) upon which the K-L information divergence index (IDI)[7] (Kullback, Leibler, 1951; Maasoumi, 1993) is dually related. The symbol "$y \equiv f(x)$" means y is defined to be the same as $f(x)$ under certain assumptions taken in context. This can be generalized in a straightforward manner as follows (Tsallis, 2009):

$$I_q(p, p^{(0)}) \equiv -\int dx p(x) \ln_q \left[\frac{p^{(0)}(x)}{p(x)} \right] = \int dx p(x) \frac{\left[p(x) / p^{(0)}(x) \right]^{q-1} - 1}{q-1}$$

or,

$$I_q(p, p^{(0)}) \equiv \sum p_i \frac{\left[p_i / p_i^{(0)} \right]^{q-1} - 1}{q-1} \tag{3}$$

in discrete cases. Thus, index $I_q(p,p^{(0)})$ stands for the traditional K-L IDI between hypotheses p and $p^{(0)}$, provided that q converges to unity[8]. There exist two main versions of Kullback-Leibler divergence (K-Ld) in Tsallis statistics, namely the usual generalized K-Ld shown above and the generalized Bregman K-Ld (Tsallis et al. 1998). Following Venkatesan, Plastino (2011), problems have been encountered in empirical implementation while trying to reconcile these. In their recent study, the above authors have revealed interesting aspects concerning empirical research when q-generalized cross-entropy is associated with constraining information.

Following recent literature (e.g., Abe, Bagci, 2004; Venkatesan, Plastino, 2011), the generalized Kullback-Leibler defined by eq. 3 could be more consistent with

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6 Eq. (2) can be optimized under moment restriction and then represents the generalized maximum entropy principle.

7 See, e.g., Kullback (1968) for a rich definition of this index and its connection with Bayesian formalism.

8 If we dispose of two systems P and R, the level of q-Tsallis allows for definition of three different entropies. For q < 1, the Tsallis entropy becomes a super-extensive entropy where Sq(P + R) < Sq(P) + Sq(R); for q = 1, the Tsallis entropy reduces to a standard Gibbs-Shannon extensive entropy where Sq(P + R) = Sq(P) + Sq(R); for q > 1, the Tsallis entropy becomes a sub-extensive entropy where Sq(P + R) > Sq(P) + Sq(P).

expectations and the constraints form proposed by Tsallis et al. (1998), known as *q-averages* or *escort distribution*[9]:

$$\left\langle y_q \right\rangle = \sum_i \frac{p_i^{\,q}}{\sum_i p_i^{\,q}} y_i \,.$$

3. A GENERALIZED LINEAR NON-EXTENSIVE ENTROPY ECONOMETRIC MODEL

This section applies the results of, e.g., Jaynes (1994) and Golan at al. (1996), to present the model to be later implemented for updating and balancing the social account matrix of the Polish economy. While the argument in criterion function is already known (see eq. 7), we need to reparameterize the generalized linear model which has to play the role of restrictions. Note that this presentation for the present problem is limited to methodological aspects. In fact, elements inside a SAM can be meaningfully presented by columns as the ratio explaining a sector disbursement distribution in favour of the rest of economy sectors. Each coefficient varies between zero and one and the coefficient total by column sums up to unity. Definitely, support space, usually defined a priori for the purpose of reparametrization, coincides with probability space. In this case, the accuracy of estimated parameters is higher as there is non-loss of information from this a priori data (Shen, Perloff, 2001). In any event, to be consistent, let us succinctly present the general procedure of parameter reparametrization in the case of a general inverse linear model:

$$Y = X \cdot \beta + \varepsilon , \tag{4}$$

where unknown β parameter values are not necessarily constrained between 0 and 1, which suggests the necessity of reparametrization. The term ε is an unobservable disturbance term, plausibly with finite variance, owing to the nature of economic data, exhibiting observation errors from empirical measurement or from random shocks. These stochastic errors are assumed to be driven by PL, as explained in the introductory section of this document. The variable Y represents a system and X accounts for covariates generating the system through relation parameter matrix β and unobservable disturbance ε to be estimated through observable error components e. Unlike classical econometric models, no constraining hypothesis is needed. In particular, the number of parameters to be estimated may be higher than the observed data points, and the quality of collected information data low. Additionally, as already explained,

[9] However, for computational reasons, we have definitely opted in this document for applying the Curado-Tsallis (C-T) constraints [2] of the form:
$\left\langle y_q \right\rangle = \sum_i p_i^{\,q} y_i$ where the symbol $\langle\ \rangle$ means that y_q is an average of y_i weighed by pi_q.

to increase precision of such estimated parameters from poor-quality data points, the entropy objective function allows for incorporation of all constraining consistency moments, which act as new Bayesian evidence in the model (Zellner, 1991). Thus, referring to, e.g., Jaynes (1994) and Golan et al. (1996), owing to the maximum entropy principle, each new piece of constraining information will reduce the entropy level of the system depending on the degree of its consistency with the prior.

Taking each β_k ($k = 1 \ldots K$) as a discrete, random variable with compact support (Golan et al. 1996) and $2 < M < \infty$ possible outcomes, one can estimate it by B_k, that is:

$$B_k = \sum_{m=1}^{M} p_{km} v_{km} \ , \ \forall k \in K \ , \tag{5}$$

where p_{km} is the probability of outcome v_{km} and the probabilities must be non-negative and sum up to one. Similarly, by treating each element e_i of e as a finite and discrete random variable with compact support and $2 < M < \infty$ possible outcomes centred around zero, we can express e_i as:

$$e_i = \sum_{j=1..J} r_{nj} z_{nj} \ , \tag{6}$$

where r_n is the probability of outcome z_n on the support space j. We will use the commonly adopted index n, here and in the remaining mathematical formulations, to set the number of statistical observations. It is worth note that the term e can be empirically fixed as a percentage of the explained variable as an a priori Bayesian hypothesis. Posterior probabilities within the support space may display a non-Gaussian distribution class. The element v_{km} constitutes an a priori information provided by the researcher while p_{km} is an unknown probability whose value must be determined by solving a maximum entropy problem. In matrix notation, let us rewrite $\beta = V \cdot P$, with $p_{km} \geq 0$ and $\sum_{k=1}^{K} \sum_{m>2 \ldots M} p_{km} = 1$, where again, K is the number of parameters to be estimated and M the number of data points over the support space. Also, let $e = r \cdot z$, with $r_{nj} \geq 0$ and $\sum_{n=1}^{N} \sum_{j>2 \ldots J}^{J} r_{nj} = 1$ for N the number of observations and J the number of data points over the support space for the error term. Then, the Tsallis cross-entropy econometric estimator can be stated as:

$$MinH_q(p // p^0, r // r^0, w // w^0) \equiv$$

$$\equiv \alpha \sum p_{km} \frac{\left[p_{km} / p^o{}_{km} \right]^{q-1} - 1}{q-1} + \beta \sum r_{nj} \frac{\left[r_{nj} / r^o{}_{nj} \right]^{q-1} - 1}{q-1} + \delta \sum w_{ts} \frac{\left[w_{ts} / w^o{}_{ts} \right]^{q-1} - 1}{q-1} , \tag{7}$$

s. t. $$Y = X \cdot \beta + e = X \cdot \sum_{m=1}^{M} v_m \left(\frac{p_{km}{}^q}{\sum_{m=1}^{M} p_{km}{}^q} \right) + \sum_{j=1}^{J} z_j \left(\frac{r_{nj}{}^q}{\sum_{j=1}^{J} r_{nj}{}^q} \right), \tag{8}$$

$$\sum_{k=1}^{K} \sum_{m>2 \ldots M} p_{km} = 1, \tag{9}$$

$$\sum_{n=1}^{N} \sum_{j>2 \ldots J}^{J} r_{nj} = 1,$$

$$\sum_{t=1}^{T} \sum_{s>2 \ldots S}^{S} w_{ts} = 1.$$

Additionally, k macro-aggregates can be added to the set of above constraining consistency moments as follows:

$$\sum_i \sum_j \mathrm{H}^{(d)}{}_{ij} T_{ij} = \gamma^{(d)} + \sum_{s=1}^{S} g_s \left(\frac{w_{ts}{}^q}{\sum_{t=1}^{T} w_{ts}{}^q} \right), \tag{10}$$

where H is a *dxd* aggregator matrix with ones for cells that represent the macro-constraints and zeros otherwise, and γ is the expected value of the aggregate constraint. Once again, g_s stands for a discrete point support space from $s = 2..S$. Probabilities w_{ts} stand for point weights over g_s. The real q, as previously stated, stands for the Tsallis parameter. In the empirical part of this document, the Polish gross domestic product at market and at factor prices will exemplify the above “macro-aggregates”.

Above, $H_q(p//p^0,\ r//r^0,\ w//w^0)$ is nonlinear and measures the entropy in the model. Relative entropies of the three independent systems (the three posteriors p, r and w and the corresponding priors p^0, r^0 and w^0, respectively) are then summed up using the weights α, β, δ. These are real positives summing up to unity under the given restrictions. The symbol // is a “distance metric”[10] of divergence information. We need to find the minimum divergence between the priors and the posteriors while the imposed restrictions must be fulfilled. As will be the case in the application below, the first component of the criterion function may concern the parameter structure of the table, the second component errors on column (or row) totals and the last component may concern errors around any additional consistency variable, like the GDP in the

[10] However, note that K-L divergence is not a true metric since it is not symmetric and does not satisfy the triangle inequality.

case below. As has been shown by Tsallis (2009), this form of entropy displays the same basic properties as K-L IDI or relative entropy. The estimates of the parameters and residual are sensitive to the length and position of support intervals of β parameters (eq. 5 and eq. 6) in the context of the Bayesian prior. When parameters of the proposed model are expressed under the form of elasticity or ratios, then the support space should be defined inside the interval between zero and one and will correspond to that of the usual probabilities. In such a case, no reparametrization of parameters is needed. In other cases, support space may be defined between minus and plus infinity, according to intuitive evaluation by the modeller. Additionally, within the same support space, the model estimates and their variances should be affected by the support space scaling effect, i.e., the number of affected point values (Golan et al. 1996). The higher the number of these points, the better the prior information about the system. The weights α,β,δ are introduced into the above dual objective function. The first term of "precision" accounts for deviations of the estimated parameters from the prior (generally defined under a support space). The second and the third terms of "prediction ex post" account for the empirical error term as a difference between predicted and observed data values of the model. As expected, the presented entropy model is an efficient information processing rule which transforms, according to Bayes's rule, prior and sample information into posterior information (Zellner, 1991).

4. PARAMETER CONFIDENCE AREA

In this section we will propose an inference information index $s(a_j)$ as an equivalent to a standard parameter error measure in the case of classical econometrics. An equivalent of determination coefficient R^2 will be proposed, too, under the entropy symbol $S(\mathrm{Pr})$. The departure point is that the maximum level of entropy-uncertainty is reached when non-relevant information-moment constraints are enforced. This leads to a uniform distribution of probabilities over the k states of the system. As we add each piece of informative data in the form of a constraint, a departure from the uniform distribution will result, which means uncertainty shrinkage. Thus, the value of the proposed $S(\mathrm{Pr})$ below should reflect a global departure from the maximum uncertainty for the whole model. Let us follow formulations presented by Golan et al. (1996) and propose a normalized non-extensive entropy measure of $s(a_j)$ and $S(\mathrm{Pr})$. From the Tsallis entropy definition, $S_q > 0$, let us consider now all possible micro-states of the model. This number varies with the number of support space data points i (i=1..M) and the number of parameters of the model $j(j = 1..J)$. Entropy S_q vanishes (for all q) in the case of $M = 1$; and for $M \succ 1$, $q > 0$, whenever one of the p_i $(i = 1..M)$ occurrence equals unity, the remaining probabilities, of course, vanish. A global, absolute maximum of S_q (for all q) is obtained, in the case of uniform distribution, i.e., when all $p_i = \frac{1}{M}$. In such an instance, we have for both systems the maximum entropy equal to:

$$S_q\left(a_j\right)=\left(M^{1-q}-1\right)\cdot\left(1-q\right)^{-1} \tag{11}$$

and

$$S_q\left(r\right)=\left(n^{1-q}-1\right)\cdot\left(1-q\right)^{-1}. \tag{12}$$

In eq. 11, n varies with the number of support space data points and the number of observations of the model. We propose below a normalized entropy index in which the numerator stands for the calculated entropy of the system and the denominator displays the highest maximum entropy as shown above (eq. 11 and 12):

$$s\left(a_j\right)=p_{ij}\frac{\left(p_{ij}/p_{ij}{}^{0}\right)^{q-1}-1}{q-1}/\left(M^{1-q}-1\right)\!\left(1-q\right)^{-1}=p_{ij}\frac{\left(p_{ij}/p_{ij}{}^{0}\right)^{q-1}-1}{\left(1-M^{1-q}\right)} \tag{13}$$

with j varying from 1 to J (number of parameters of the system) and i belonging to M (number of support space points), with $M > 2$; with the total number micro-states, which is obtained by multiplying number of model parameters J by number of support space points M with $M > 2$. Then $s(a_j)$ reports precision on the estimated parameters. Equation 14 reflects the non-additivity Tsallis entropy property for two independent systems. The first term $S(p)$ is related to parameter probability distribution and the second $S(r)$ to error disturbance probability:

$$S\left(\hat{\mathrm{P}}\mathrm{r}\right)=\left[S\left(\hat{p}+\hat{r}\right)\right]=\left\{\left[S\left(\hat{p}\right)+S\left(\hat{r}\right)\right]+\left(1-q\right)\cdot S\left(\hat{p}\right)\cdot S\left(\hat{r}\right)\right\}, \tag{14}$$

where $S\left(P\right)=\sum\sum p_{ij}\frac{\left(p_{ij}/p^{0}{}_{ij}\right)^{q-1}-1}{\left(q-1\right)\left(M^{1-q}-1\right)}$, and $S\left(r\right)=-\sum r_{nf}\frac{\left(r_{nf}/ro_{nf}\right)^{q-1}-1}{\left(q-1\right)n\left(1-F^{1-q}\right)}$.

$S\left(\hat{\mathrm{P}}\mathrm{r}\right)$ is then the sum of normalized entropies related to parameters of the model $S\left(\hat{p}\right)$, and to disturbance term $S\left(\hat{r}\right)$. Likewise, the latter value $S\left(\hat{r}\right)$ is derived for all observations n, with F the number of data points on the support space of estimated probabilities r related to the error term. As it results from the above formulation, the values of these normalized entropy indexes $S(\hat{a}_{ij})$, $S\left(\hat{\mathrm{P}}\mathrm{r}\right)$ vary between zero and one. Its values, near unity, indicate a poor informative variable- with higher entropy- while lower values are, on the contrary, an indication of a better informative variable about the model. From *information properties* and the above formulation of the q-gener-alized cross-entropy concept (see eq. 3), the reader can observe that both indexes fulfil basic Fisher-Rao-Cramer information index properties, among them continuity, symmetry, maximum, and additivity.

5. THEORETICAL ASPECTS OF BALANCING A SAM

A SAM is a quadratic table that encompasses information about complex processes of supply and demand of a real, open economy involving, under optimizing behaviors, different economic agents and endowments for a given time period and region. Regarding SAM construction and components (see, e.g., Pyatt, Round, 1985), general equilibrium (e.g., Wing, 2004) implies that respective row and column totals are expected to balance. Conceptually, this model is based on the laws of *product and value conservation* which guarantee conditions of zero profit, market clearance, and income balance (Scrieciu, Blake, 2005). However, different stages of statistical data processing remain concomitant with observation and measurement errors, and the SAM will not balance. This means that an unknown number of economic transaction values within the matrix are inconsistent with the data generating macroeconomic system. For clarity, let us use Table 1 to explain these imbalances, noting, for instance, a difference between the activities row and column totals as follows:

$$(aT + u_1) - (aT + \varepsilon_1) = (u_1 - \varepsilon_1). \tag{15}$$

The term on the left hand side of the above expression stands for the difference between two erroneous and unequal totals of the activity account. Its origin is the plausibly different stochastic errors u_1 and ε_1 on column and row totals, respectively. In Table 1, the first alphabetical letter of symbols inside each cell stands for the first letter of the row (supply) account, and the second letter represents the first letter of the corresponding (demand) column. For instance, in the prototype SAM below, the symbol “ca” stands for the purchases by the *activity sector* of goods and services from the *commodity sector*.

Table 1.

A simplified stochastically non-balanced SAM

	Activities	Commodities	Factors	Institutions	Capital	World	Total
Activities	0	ac	0	Ai	0	aw	aT+ε_1
Commodities	Ca	0	0	Ci	cc	0	cT+ε_2
Factors	Fa	0	0	0	0	0	fT+ε_3
Institutions	Ia	ic	If	Ii	0	iw	iT+ε_4
Capital	0	0	0	Ci	0	cw	cT+ε_5
World	0	wc	0	Wi	0	0	wT+ε_6
Total	aT+u_1	cT+u_2	fT+u_3	iT+u_4	cT+u_5	wT+u_6	

Source: own presentation.

The objective is to find, out of all probability distributions, the one (the posterior) closest to Table 2 (the prior) and ensuring its balance while satisfying other imposed consistency moments and normalization conditions. Referring to Shannon entropy, one may consider post entropy structural coefficients and disturbance errors, respectively, as *signal* and *noise*. The first step consists of computing a priori coefficients by column, from real data from Table 2, by dividing each cell account by the respective column total. Next, we treat these column coefficients as analogous to probabilities, and column totals as expected column sums, weighted by these probabilities (see eq. 7). Coefficient values in initial Table 2 will serve as the starting, best prior estimates of the model. Two other types of priors to initialize the solution concern errors on column totals (eq. 8) and on gross domestic product (GDP) at factor and market prices (eq. 10). GDP variables are added to the model with the purpose of restricting the model to meet consistency macroeconomic relationships for different accounts inside the SAM. The proposed approach combines *non-ergodic Tsallis entropy with Bayes's rule to solve a generalized random inverse problem*. We may optionally consider only some cell values as certain[11] while the rest of the random accounts are unknown. Once again, this is one of the strongest points of the entropy approach over most rival mechanical techniques of balancing national account tables. All row and column totals are not known with certainty. It is apparent that the potential number of degrees-of-freedom of parameters to estimate $n(n-1)$ remains significantly higher than n observed data points (column totals). In the particular case of a SAM, and due to empty cells, that number of unknown parameters may be much lower. Nonetheless, that will not generally prevent us from dealing with an ill-behaved inverse stochastic problem. The next important step is that of initializing the above defined error trough, a reparameterizing process. *A five point support space symmetric around zero* is defined. To scale the error support space to real data, we apply Chebychev's inequality and Three Sigma rule (Golan at al. 1996; Pukelsheim, 1994). Corresponding optimal probability weights are then computed so as to define the prior noise component (Robinson, El-Said, 2000).

6. BALANCING A SOCIAL ACCOUNTING MATRIX OF POLAND AND OUTPUTS

This section presents one of the plausible applications of the non-extensive cross-entropy approach. Readers acquainted with the Shannon entropy approach[12] and its economic applications may know its particular role in recent years for balancing social accounting matrices of many countries (e.g., Miller, Matthews, 2012; Robinson at al., 2000). In the present case, we have used this new technique to balance the

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[11] In the present case, only transaction accounts with the rest of the world (import, export, external current balance), plus government commodity consumption accounts are concerned.

[12] We recall here that Shannon-Gibbbs entropy remains a converging case of Tsallis non-extensive entropy.

Polish SAM of 2005. Technically, the problem of cross-entropy is to find a new set of SAM coefficients (posteriors) that minimize the so-called Kullback-Leibler (1951) divergence measure of the Tsallis "cross-entropy" (CE) between the prior (the initial, unbalanced SAM) and the posteriori SAM, under given restrictions. These are related to data moments, normalization condition, or any other a priori information presenting consistency with posterior probabilities in the criterion function (see eq. 7–10). For the model computations, we have used the GAMS code and the solver Minos5. Table 2 and Table 3 present the non-balanced and the post entropy-balanced SAM, respectively. The statistical data used come from the Polish Main Public Office of Statistics (http://www.stat.gov.pl/gus/), and from EUROSTAT (www.eurostat.eu). In Table 2, the number values in the total column marked in bold are related to the non-balanced sectors. As suggested in the preceding section, such imbalances and inconsistencies mainly result from the complexity of economic information gathering at country scale, where various institutions constitute different and contradictory sources of information. Furthermore, other human error during statistical table compilation remains plausible. In their recent work, trying to balance the Polish SAM for 2010, Tomaszewicz, Trębska (2013) have noticed the lack of direct data values of current and capital transfers in the case of Polish statistical data. As explained in the celebrated work of Golan et al. (1996), and based on various simulations, entropy formalism acts as a Bayesian efficient processing rule. Then, independent of the prior information level, when new data (new evidence) is consistent with the data generating process, the entropy formalism allows the estimator to quickly converge toward the minimum variance. However, in the real world, the data generating system is unknown and the assessment of a new methodology may rely on mere opinion. In fact, an official balanced SAM may still contain many conflicting errors, for instance, those related to the selected closure rule. There are other SAM balancing techniques. The RAS approach remains the most popular among them. In a recent, thorough study on the comparative performance of cross-entropy and RAS techniques, Chisari at al. (2012) concluded that cross-entropy had a more general character for the reasons listed below:

a. It does not need all the new totals of rows or columns (although prediction will be less accurate).
b. It does not need a balanced initial matrix (the sum of rows could be more/less than the sum of columns).
c. New rims could contain an error term.
d. New rims can be non-fixed parameters.
e. Many values on the final matrix could be fixed (not necessarily a parameter).
f. It allows non-linear constraints.

Referring to their simulation outputs, the authors propose *a rule of thumb consisting of preferring the RAS method if and only if no constraint or one constraint is enforced*. This seems to explain why the RAS approach continues to be successfully applied in different prediction studies. In a recent study conducted by Bwanakare

Table 2.

Initial unbalanced Polish SAM (2005)

	aAct	pCom	Labor	Capital	Pollfees	Hou	Ent	GRE	CapAc	RoW	Total
aAct	0.0	160.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	36.5	196.6
pCom	108.6	0.0	0.0	0.0	0.0	71.6	0.0	7.8	18.9	0.0	207.0
Labor	35.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	35.2
Capital	50.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.5
Pollfees	2.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.3
Hou	0.0	0.0	31.7	27.9	0.0	0.0	7.2	27.1	0.0	1.8	95.7
Ent	0.0	0.0	0.0	24.8	0.0	0.0	0.0	0.0	0.0	0.0	24.8
GRE	0.0	10.3	0.0	0.0	2.3	22.4	7.0	0.0	0.0	0.0	42.0
CapAc	0.0	0.0	0.0	0.0	0.0	6.5	11.0	0.5	0.0	0.9	18.9
RoW	0.0	37.2	0.0	0.0	0.0	0.0	1.9	0.0	0.0	0.0	39.1
Total	196.6	207.7	31.7	52.7	2.3	100.5	27.2	35.5	18.9	39.1	

Source: own compilation.

Table 3.

Balanced, post non extensive entropy Polish SAM (2005); weight equals (0.05; 0.94; 0.01)

	Aact	Pcom	Labor	Capital	Pollfees	Hou	Ent	Gre	Capac	Row	Total
Aact		160.2								36.5	196.7
Pcom	109.4					71.13		7.85	18.94		207.3
Labor	33.46										33.46
Capital	51.61										51.61
Pollfees	2.272										2.272
Hou			33.46	25.62			6.9	30.3		1.8	98.1
Ent				25.99							25.99
Gre		9.848			2.272	20.13	6.52				38.76
Capac						6.843	10.6	0.61		0.86	18.94
Row		37.2					1.94				39.13
Total	196.7	207.3	33.46	51.61	2.272	98.1	26	38.8	18.94	39.1	

Source: own compilation.

(2013b) consisting of balancing the EU input output matrix, the author- after having applied only a single constraint- found the outputs from the RAS approach slightly better compared with those from the cross-entropy technique. Thus, the conclusion from that study seems to support the one presented above by Chisari at al. (2012). However, this suggestion does not seem to be consistent with the investigations done by Robinson, El-Said (2000) on the Mozambique economy. These authors have found that the RAS and Shannon entropy approaches produce the same performance when no additional restriction is imposed. More investigations are needed to contradict or confirm the findings of the authors mentioned in this paragraph. Nevertheless, taking its stochastic characteristics into account, cross-entropy potentially has a higher performance than the RAS approach, particularly when statistical data are known with uncertainty.

The main purpose of the figures displayed below is to put emphasis on some model output characteristics through selected parameters or indices. In particular, the impact of q-Tsallis variation and weights in criterion function on computed outputs is underscored. Increasing this parameter is equivalent to a kind of "complexifying" of interrelations between economic actors or sectors inside the economy (Foley, Smith, 2008), such as reinforcing competitive conditions. Three distinct weight components (eq. 7) {(0.94;0.05;0.01)_p; (0.333; 0.334; 0.333)_nw; (0.05; 0.94; 0.01)_w1} have been assigned in the entropy criterion function and each weight inside each set corresponds, respectively, to distribution of SAM coefficients, column totals, and GDP disturbance errors. GDP accounts deserve relatively lower importance as they are only connected with a limited number of SAM accounts (production factors and tax income). Then, symbols _p, _nw, and _w1 on the right hand side of each of the above weight set underscore the dominant probability in each set. In particular, the _nw corresponds to the case equivalent weights. Figure 1 compares model goodness according to weights assigned to different components in the criterion function, for different q lying inside Gaussian attractor interval [1-5/3]. Increasing weights on the parameter probability component should enhance post-entropy SAM coefficient precision while worsening error estimation, thus at the cost of model ex-post-prediction (Golan at al., 1996). As has already been said, the model entropy encompasses statistical losses in the parameter space (precision) and in the sample space (prediction). Analytically, it can be directly shown that Lagrange multipliers stand for implicit *nonlinear* function of weights imposed in the generalised cross-entropy criterion function. Changes in weights thus alter the corresponding optimal solution value. In general, as in most constrained optimisation problems, *smaller* Lagrange multipliers for a q cross-entropy formulation should imply smaller impact of constraints on the objective, at least for q around unity, i.e., the Gaussian case. The above defined three weight types correspond, respectively, to three goodness indices "S(Pr)": $good_{-p}, good_{-nw}, good_{-w1}$, where S(Pr) is the total normalized entropy of the system (eq. 14). This index then tells us, given the unbalanced prior SAM, to what extent new evidence reflected in constraining moment conditions and the estimated model has discriminated in favour of the balanced post entropy SAM for different levels of the q-Tsallis parameter. In the present model, its highest value is around 0.99 once higher weight has been imposed on column total errors (_w1) for a *q*- parameter evolving around unity. We recall that this inference index varies between zero and one. Figure 2 analyses the precision-prediction loss trade-off between the two random sources of model sensitivity by the above selected weights and different *q*-Tsallis parameters. We compare two extreme weighting cases A= {(0.94; 0.05; 0.01)_p and B=(0.05; 0.94; 0.01)_w1}. The symbol "P*PI shrink*" is a precision index for each q-Tsallis parameter. To get the measure, we first calculate the relative differences (in absolute value) between the SAM post-entropy probability from cases A and B. Next, we calculate the arithmetical divergence mean by summing up, in absolute values, those differences divided by the number of structural probabilities being parameters within the table.

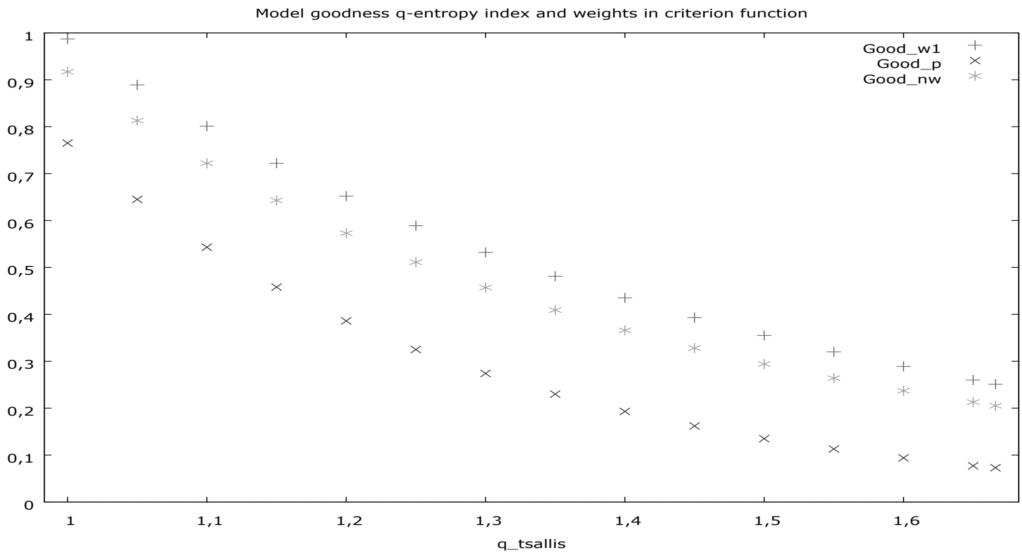

Figure 1. Model of goodness q-entropy index and weights in criterion function

The next prediction index values, "Sigma shrink," are obtained in the same way as "*PI shrink*" described above with the difference that, in this last case, attention is drawn to standard disturbance error affecting column totals. As we can observe, reducing weights on the SAM probability component in favor of the column total errors component relatively increases information divergence related to SAM coefficients between the prior and the posterior. Impact of such a weight change is to reduce standard disturbance error on column totals. This is described by Figure 2, where the best outputs are reflected by values at the beginning of the curve in the south-eastern corner. We notice, in the present case, a higher sensitivity of error component to weight change than the one from SAM coefficients. The index varies between approximately 0 and 0.9 while in the last case it varies between -0.12 and zero.

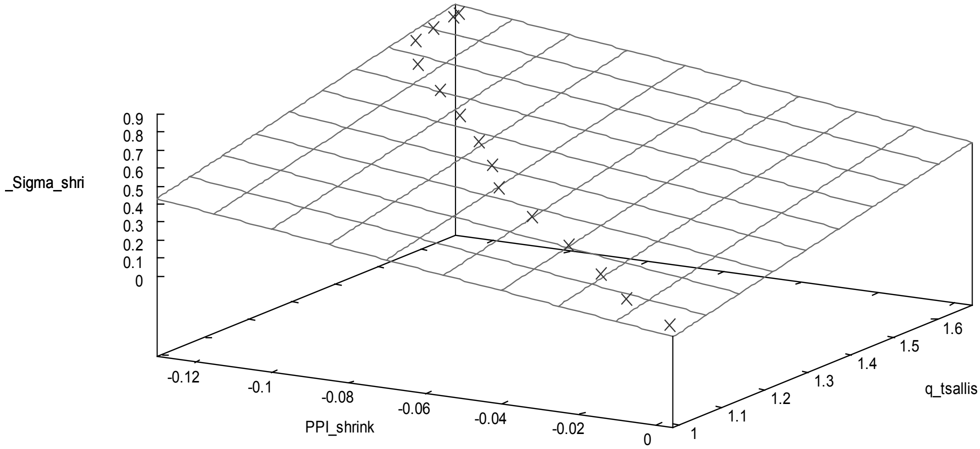

Figure 2. Precision and prediction loss tradeoff due to weight change in c.f. for different q_ parameters

7. CONCLUDING REMARKS

This paper aims at extending applications of a non-extensive entropy approach to modeling generalized inverse problems in the case of stochastically balanced systems. A Polish SAM, as a case study, has been optimally balanced. However, because the existing SAM represents only an approximation of the unknown true values of the macroeconomic transactions, it is difficult to accurately assess outputs of the estimated model. We found optimal outputs for q-Tsallis close to unity, suggesting the Gaussian structure of the SAM. Statistical inference indices proposed in this paper have been used to analyze the tradeoff between parameter precision and sample prediction for different weights in the objective function and different q-Tsallis complexity parameters. Superiority of the proposed approach should rely essentially on its generalizing attributes owing to its non-extensivity, conceptually ensuring solutions less prone to initial conditions. We suggest more investigations in other economies and other fields, particularly those in countries with different economic structures.

University of Information Technology and Management in Rzeszow

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EKONOMETRYCZNE ZBILANSOWANIE MACIERZY RACHUNKOWOŚCI SPOŁECZNEJ POD HIPOTEZĄ PRAWA POTĘGOWEGO

Streszczenie

Względna Entropia Shannon-Kullback-Leibler (SKLCE) jest szczególnie przydatna przy rozwiązaniu problemu odwrotnego systemu ergodycznego. Choć empiryczne zastosowanie podejścia Shanon-Gibbsa spotkało się ostatnim czasem ze znacznym sukcesem, cierpi jednak cały czas ze względu na charakter hipotezy ergodycznej, ograniczając wszystkie mikroelementy systemu pojawianiem się identycznego prawdopodobieństwa. Niniejszy artykuł ma na celu rozszerzenie zastosowania nieekstensywnego modelu względnej entropii (NECE) dla zbilansowania losowych macierzy wyjścia-wejścia. Model ten postuluje, że działalność ekonomiczna cechuje się długookresową pamięcią kompleksowych interakcji między podmiotami gospodarczymi lub między sektorami. Stosując własności skalowania prawa potęgowego budujemy model, który z powodzeniem zbilansuje polską macierz rachunkowości społecznej cechującą się równowagą ogólną Warlasa. Zaproponowano wnioskowanie statystyczne dla przedziału ufności indeksów informacji. Zaobserwowano, że zwiększenie wag komponentów składnika losowego dualnego kryterium funkcji prowadzi do większych wartości parametru q-Tsallisa, zaś zmniejszenie tych wag przybliża wartość parametru q-Tsallis'a do jedności. Przewagą podejścia entropii Tsallis'a nad innymi konkurującymi metodami jest możliwość uogólnienia modelu Gaussowskiego, ze względu na to, że bierze ono pod uwagę istnienie rozkładu grubego ogona. Dzięki cechom parametru q-Tsallis'a możliwą staje się również ocena kompleksowości systemu statystycznego.

Słowa kluczowe: q-uogólniana dywergencja informacji Kullback-Leibleir'a, macierz rachunkowości społecznej

ECONOMETRIC BALANCING OF A SOCIAL ACCOUNTING MATRIX UNDER A POWER-LAW HYPOTHESIS

A b s t r a c t

Shannon-Kullback-Leibler cross-entropy (SKLCE) is particularly useful when ergodic system inverse problems require a solution. Though empirical application using the Shanon-Gibbs approach has recently met with notable success, it suffers from its ergodicity, constraining all micro-states of the system to appear with identical odds. The present document aims at extending applications of a *non-extensive cross-entropy model* (NECE) for balancing an input output stochastic system. The model then postulates that economic activity is characterized by long run complex behavioural interactions between economic agents and/or economic sectors. Applying scaling property of a Power-law we present a model which successfully balances a Polish national social accounting matrix (SAM) expected to exhibit Warlasian general equilibrium features. The *Rao-Cramer-Kullback* inferential information indexes are proposed. We note that increasing relative weight on the disturbance component of the dual criterion function leads to higher values of the *q-Tsallis complexity index* while smaller disturbance weights produce q values closer to unity, the case of Gaussian distribution.

The great advantage of the approach presented over rival techniques is its allowing for the generalisation of Gaussian law enabled by its capability of including heavy tall distributions. The approach also constitutes a powerful instrument for the assessment of complexity in the analysed statistical system thanks to the q-Tsallis parameter.

Keywords: q-Generalization of K-L information divergence, social accounting matrix

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NATALIA DRZEWOSZEWSKA*

MULTILEVEL MODELLING OF BILATERAL TRADE FLOWS BETWEEN EUROPEAN UNION COUNTRIES

1. INTRODUCTION

The gravity model of international trade flows has been widely used by econometricians since Tinbergen (1962) published the first gravity equation that describes bilateral trade as directly proportional to the mass of two trading countries, namely, their national incomes, and as inversely proportional to the distance that separates them, which should approximate trade costs[1]. A popular way to approximate the trade costs, included in the theoretical gravity model proposed by Anderson, van Wincoop (2003), is the use of physical distance and a set of different dummies in the model such as, for instance, a common border, a common official language, access to the sea or sharing a trade agreement. However, the theoretical form also requires the inclusion of multilateral trade-resistance (MTR) terms, which could be approximated by the use of time dummies together with invariant country dummies (Eaton, Kortum, 2002; Helpman, 2006) or by the use of time-varying country effects in the model[2] (Baldwin, Taglioni, 2006), by the use of a simulation method with the inclusion of the elasticity of substitution[3] (Anderson, van Wincoop, 2003; Baier, Bergstrand, 2009) or by constructing the time invariant or time-varying synthetic variables, called remoteness[4] (Wei, 2000). The omission of MTR terms that are correlated with trade costs leads to the bias in the estimates (Ruiz, Villarubia, 2007, p. 18).

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[1] Transportation-, information-, communication costs, technical barriers to trade (TBTs), etc.

[2] However, the disadvantage of this method is the inability to estimate the coefficients on country-specific variables, such as national income or population, due to perfect collinearity.

[3] There is no consensus in the subject literature concerning the exact value of this parameter. Generally, the elasticity of substitution is assumed to fall in the range from 5 to 10 (Anderson, van Wincoop, 2004).

[4] Explaining the role of remoteness, Deardoff (1998) considers two pairs of countries, (*i, j*) and (*k, l*), and assumes that the distance between these countries in each pair is the same: $D_{it} = D_{kl}$. If *i* and *j* are closer to other countries, the more remote countries, *k* and *l,* will tend to trade more between each other because they do not have alternative trading partners. The definition of Deardoff's remoteness probably inspired Anderson, van Wincoop to apply MTR terms (2003).

Due to the heterogeneity that occurs by modelling international trade flows, the FE estimator is frequently applied while conducting research (Egger, 2000; Green *et al.*, 2001; Cheng, Wall, 2005; Pietrzak, Łapińska, 2014), since it improves the panel model by including fixed effects for every trading pair in the sample, which can be easily seen on the coordinate system in the plane as a set of parallel multiple regression equations[5]. The use of the FE estimator was also indicated as a better way of the approximation of MTR terms in the author's previous work devoted to the issue of alternative methods of implementing and estimating multilateral trade resistance in the panel gravity model of bilateral trade (Drzewoszewska, 2014). However, this solution ignores the average variation between trading pairs, which Egger (2000) and Cheng, Wall (2005) consider in the context of historical, political and geographical factors. Another disadvantage of the FE estimator is the fact that all the variables that are constant over time will be dropped by the estimation due to collinearity with fixed effects. On the other hand, however, the estimation of individual regressions may face sample problems and lack of generalization. Moreover, the FE estimator is inconsistent (with fixed T, $N \to \infty$) without the conditional strict exogeneity assumption and becomes inefficient when the number of clusters is high. Due to Beck, Katz (2001) submission, it would be interesting to model the differences in the basic level of trade, across trading partners, and to allow heterogeneous slopes as well. The use of mixed effects model in this study allows certain coefficients of the gravity model to vary across trading country pairs, which leads to an output where a set of regression for every trading pair is not parallel any more. According to Gelman, Hill (2007) multilevel methods generally allow consistent and efficient estimation.

The 3-level model presented in the study assumes random slope for incomes' product and the intercept in three groups: when the bilateral trade flows between old EMU-members[6] (intra-EMU trade), between old and new members or non-EMU members (inter-EMU trade) and between new and/or non-members of euro-area (outside EMU trade). The random slope at level 2 (between trading pairs) is the product of national incomes of both countries (the denominator of the basic gravity equation that reflects the combined size of the two trading countries) and their common internetization rate – the share of internauts in the whole population of both trading countries, which reflects the quality of the network infrastructure of a specific trading pair. The study assumes three research hypotheses. According to the first one, the more both trading countries are globalized, which is indicated by higher values of the globalization factors in the gravity model, the more intensive the bilateral trade

[5] Other popular estimation methods for gravity panel models which are more complex than simple pooled model include the Poisson pseudo maximum likelihood (PPML) estimator for the dependent variable at the levels proposed by Santos Silva, Tenreyro (2006) as an alternative for NLS, tobit model for panel data (Soloaga, Winters, 2001; Baldwin, DiNino, 2006; Tripathi, Leitão, 2013), HT estimator (Serlenga, Shin, 2004; Belke, Spies, 2008; Drzewoszewska, Pietrzak, Wilk, 2012) or probit – with Heckman's approach (Linders, de Groot, 2006; Martin, Pham, 2008).

[6] The old EMU-members are understood here as the first countries that created the union in 1999.

exchange between them becomes. Following the second hypothesis, the Eurozone causes the pure trade creation effect (bilateral trade flows increase if both exchange partners are members of the EMU) with no trade diversion effect[7]. The third hypothesis assumes that the bilateral flows between two European countries rise with the probability that both of them are able to communicate in English – the world's *lingua franca*. The first part of the paper presents some extensions of the gravity model's form in the empirical research. The second part describes the methodology of an empirical multilevel model and the outcome of the research conducted.

2. EMPIRICAL RESEARCH OF TRADE FLOWS WITH THE APPLICATION OF THE GRAVITY MODEL

Empirical investigation of the border puzzle effect on the inter and intra-trade was the inspiration for Anderson, van Wincoop (2003, 2004) to create their theoretical structural gravity model. Namely, they continued the research of McCallum (1995), who analysed the implication of trade patterns between Canadian provinces and U.S. states with the result that bilateral Canadian provinces' trade is 22 times more intensive than the exchange with U.S. states. After the introduction of MTR terms to the model, with the assumption that the elasticity of substitution $\sigma = 8$, Anderson, van Wincoop (2003) decomposed the border effect into the impact border barriers and multilateral resistance effects. Finally they found that Canadian provinces trade 10.7 times more than provinces with states due to the existing country border. That was the result of including MRT terms in the model, the omission of which is the crucial factor for the biased estimation of the border effect.

The gravity model of trade became a popular tool for analysing the effects of trade liberalization (McCallum, 1995) or common currency on trade (Rose, 2000). Investigating the trade or monetary union effects leads to the problem of endogeneity – due to 'natural trading partners' hypothesis[8]. However, the implication of dummies describing the RTA is still a common procedure, since it allows for analysing the trade creation and trade diversion effects of the agreement (Kandogan, 2005). The details are described in a further part of this study.

When investigating the EMU effects, there arises also the question whether the EMU is close to the optimum currency area. According to the idea of the optimum

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[7] The analysed trade diversion effects indicate the reallocation of imports from the most/less efficient source on the global market to more inefficient/efficient sources within the Eurozone. Since all EU members in the sample have reached a similar level of economic development, the expected trade diversion effects are insignificant.

[8] In the panel model the use of FE can help to overcome part of the endogeneity problem due to the omitted variable bias, although time-varying omitted variables remain a problem. Among another possible ways to estimate such a gravity model the popular method is IV estimation with instruments, namely Hausman-Taylor estimator, which uses exogenous time-varying regressors X_{it} (from periods other than the current one) as instruments.

currency area (OCA) described by Mundell (1961), openness to capital mobility and price, and wage flexibility across the region are expected. The reason is that the market forces of supply and demand automatically distribute capital and goods to where they are needed. However, in practice this does not work perfectly as there is no true wage flexibility. According to the study of Baldwin (2006), the 'euro effect' suggests that the single currency has increased trade by 5 to 15 percent in the Eurozone with comparison to the trade between non-euro countries. In order to find out whether the growth of intra-Eurozone trade is greater than international EMU trade the dummy variables can be used to describe the participation in EMU (Micco *et al.*, 2002, 2003).

Most of the research conducted on the gravity model of trade considers only the influence of the official common language, finding that sharing language translates into greater trade intensity (Glick, Rose, 2002; Santos Silva, Tenreyro, 2006; or Baldwin, Taglioni, 2006). However, international commerce is increasingly conducted in English, even if neither side of the transaction is from an English speaking country. Hence, Melitz (2008) used Ethnologue database and proposed additional variables describing all indigenous or established languages spoken in the country, taking into account also the fraction of the population speaking those languages. He found that 'open-circuit' languages (those that are official or are spoken by at least 20% of the population in both countries; measured as dummy variables) and 'direct-communication' languages (those that are spoken by at least 4% of population in both countries; measured as 'communicative probability' that two randomly chosen individuals from both countries can communicate directly in any direct-communication language) increase bilateral trade. However, the limitation of Ethnologue database is that it investigates only native speakers or ethnic-minority populations (primary speakers). The analysis of Melitz (2008) showed that 'direct-communication' is about three times more effective than indirect-communication in promoting trade, and taking them both into account, the impact of a common language becomes nearly twice as high as in the traditional gravity model. Additionally, the English language seems to have no particular advantage in foreign trade (insignificant and even a negative sign of estimates), opposite to the European languages (German, French and Spanish) as a whole.

The next step in the languages' influence on bilateral trade flows – the approach proposed by Fidrmuc, Fidrmuc (2009) – was based on the results of Eurobarometer surveys on Europeans' ability to speak various languages[9], which were carried out at the end of 2005. Here the consideration of both primary and secondary speakers is possible. Eurobarometer surveys are nationally representative what allows to estimate the share of each country's population that speaks each of 32 investigated languages[10] and finally, the probabilities that two randomly chosen individuals from two different

[9] Eurobarometer 243, *Europeans and Their Languages*, European Commission, 2005.

[10] In the final estimations the authors focused on the measurement of the effect of languages spoken by at least 10% of the population in at least three countries – what yielded English, German, French and Russian.

countries will be able to communicate ('communicative probability'). The authors created the gravity model of trade for all members and candidates countries of the EU in the time period 2001–2007. After including two additional sets of indicators on bilateral language relationships in the model estimated with OLS and 2SLS methods, they found that the command of English raises trade flows in the area of EU15, as it does between the new members and candidates countries. The results obtained for other languages were varied. In fact, the effects of the languages investigated were non-linear, displaying diminishing returns[11], which was shown by the authors with the application of the quantile regression. The results showed a hump-shaped effect on trade flows with the peak on the communicative probability in English which equals 70% for the countries with relatively higher trade intensity.

3. MULTILEVEL GRAVITY MODELS OF BILATERAL TRADE FLOWS – METHODOLOGY AND RESULTS OF THE RESEARCH

The mixed effects models described by Pinheiro, Bates (2000) are also known as random coefficients models (Longford, 1993) or multilevel models (Goldstein, 1995). A special case is the hierarchical linear model. This term was used first by Lindley and Smith (1972). The observations are made on units at different levels in a hierarchy. Statistical data are often multilevel (hierarchical, nested or clustered) in the sense that lower-level units of analysis belong to higher-level units of analysis. The panel data are multilevel as well – years are nested[12] within given countries. Multilevel models account for the dependence (clustering or correlation) found in hierarchical data. In the opposite, single-level models ignore this dependency and, therefore, may result in drawing wrong research conclusions, because of underestimated standard errors of the effects of covariates, too narrow confidence intervals, or incorrect statistical inferences (*i.e.*, Type 1 errors)[13].

Export flows from the same country are typically more alike than flows from different countries, even if the importing country is the same, because of a unique relation connecting two trading countries. Moreover, export flows from the same year could also be more alike than flows from other year, because of the global economic condition. The use of mixed models in the analysis of bilateral trade flows allows a relatively broader investigation of relationships that connect different trading pairs of countries, as they assume a more complex error structure. The variables that move relatively slowly over time play a role in determining the average levels of trade between two trading partners. Additionally, the model captures unspecified hetero-

[11] The return was particularly high for the countries with a relatively low level of proficiency in languages.

[12] It means that the random effects shared within lower-level subgroups are unique to the upper-level groups.

[13] See Rabe-Hesketh, Skrondal (2012).

geneity by allowing the intercept and certain slope coefficients of the model to have a stochastic component in their variation. In this study both three- and two-level hierarchical linear models are used to capture these effects, since years are nested within trading pairs, which are nested within the places of trade. Here the random effects at different levels are assumed to be uncorrelated. Each lower level residual is allowed-to-vary random departure from the higher-level departure. The error terms and random intercept are assumed to be normally distributed with the mean 0 and variances $\sigma^2_{v0}, \sigma^2_{u0}$ and σ^2_e, and to be mutually independent. The methodology used in the study is precisely described in the subject literature, see, for instance, Goldstein (1995), Osborne (2000), Raudenbush, Bryk (2002).

The economic integration of countries with free trade, free capital mobility and uncontrolled migration is the base for the globalization process (see Daly, 1999), which was the criterion for selecting certain EU countries to be included in the research sample (apart from Malta and Cyprus). The research time period (1999–2011) was chosen also based on the globalization theory – namely, the starting year is referred to by Friedman (1999) as 'the year of the Internet', opening a new era of easy outsourcing, offshoring and other new activities, leading to changes in the global trade structure.

The independent variables in the gravity model of trade can be easily divided for masses and the distance-variables (reflecting the trade costs)[14]. The first part should increase the trade flows between two countries, as it captures the wealth of trading partners, the second has a negative influence on trade, as it increases the trade costs. Considering the distance as the remoteness or the degree of countries' similarity, the relatively more similar countries should have larger bilateral trade flows. Thus, according to the idea of globalization, more globalized countries should trade more between each other. Therefore, the estimated gravity models of bilateral export flows include, additionally to typical gravity model's forces, a set of globalization factors which describes the distance of the country from the global markets. Among these variables the most important is access to the broadband Internet for country citizens, that reduces telecommunication costs for trading partners. The creation of the 'New Economy' in the world is observed by the increasing number of researchers in R&D, who are engaged in the conception or development of new knowledge, products, processes, methods or systems, and by the increase of high-technology exports products. The estimated models also contain two variables for this phenomenon: the calculated researchers rate for both trading partners and share of exporter's high-technology export in his total export value. The set of the data used is described in Table 1.

[14] The use of time effects in the gravity model reflects the variables that do not depend on *o* and *d*, such as the level of World liberalization and other global economic effects. According to the Isaac Newton's law of universal gravitation, we can call it the gravitational constant.

Table 1.

Variables included in the analysis of international bilateral trade flows

Variable	Definition	Measure unit	Source
EXPORT	Export flows from origin country to destination country	USD (current prices)	Comtrade/OECD
GNIproduct	The product of both countries Gross National Incomes[1]	USD (current prices)	WDI
Travel	Travel time by road between the national centroids[2]	hour	Google Maps
Internetization	Common internetization rate (the share of internauts in the population of both trading countries)	share in %	Author's calculation / WDI
Researchers	Common researchers rate in R&D (the share of researchers in the populations of both trading countries)	share in %	Author's calculation / WDI
HighTechExport	Share of high-technology export in the total export of exporter	share in %	WDI
EnergyUse	The sum of energy use in both trading countries	kt of oil equivalent	Author's calculation /WDI
ExEMU	1 if the exporter belongs to The Economic and Monetary Union but the exporter does not and 0 otherwise	dummy variable	
ImEMU	1 if the importer belongs to The Economic and Monetary Union but the exporter does not and 0 otherwise	dummy variable	
BothEMU	1 if both of the trading countries in the pair are members of The Economic and Monetary Union and 0 otherwise	dummy variable	
Border	1 if two trading countries share a common border and 0 otherwise	dummy variable[3]	
Sea	1 if at least one from two trading countries is not landlocked and 0 otherwise	dummy variable	
OfficialLanguage	1 if two trading countries share a common language and 0 otherwise	dummy variable	
Language Proficiency	1 if the language is official in both countries or spoken by more than 20% of populations and 0 otherwise	dummy variable	Author's calculation / Eurobarometer surveys
Language Communication	Probability that two trading partners will be able to communicate in the certain language	probability	Author's calculation / Eurobarometer surveys

[1] The use in the study GNI instead of GDP variable is intentional, as it measures income received by a country both domestically and from overseas.

[2] Great circle distance algorithm was used in the calculation.

[3] The formula to compute the effect of dummy-variables is following: $(e^{b_i} - 1) \times 100\%$, where b_i is the estimated coefficient.

Source: author's compilation.

The quality of infrastructure, another globalization indicator (Liberska, 2002, pp. 34-37), is included in the model through inclusion of the energy use of both trading countries which should lead to an increase in their trade, especially in the trade of commodities that is the subject of this study, and through the use of the travel time by road between trading countries (as an alternative to physical distance) by construction the synthetic variable of bilateral trade costs[15] (see equations 1–2). The bilateral trade costs $t_{t,od}$ formula is following:

$$t_{t,od} = \frac{DISTANCE_{od}}{IMPORTER'S\_OPENNESS_{t,od}}, \tag{1}$$

where:

$$IMPORTER'S\_OPENNESS_{t,od} = \frac{EXPORT_{t,od} + EXPORT_{t,do}}{TOTAL\_IMPORT_{t,d}}. \tag{2}$$

This approach allows a substantive advantage of the bilateral trade costs-variable, namely making it time-varying in this approach, what suits better to reality, since trade costs are not constant over time. The distance between countries in formula (1) is measured by travel time between the centroids of trading countries[16] and is divided by the share of bilateral trade exchange in the total import of the importing country, called here as importer's openness (2). This method reflects the theoretical significance of the importer's demand in the final amount of bilateral trade flows.

The creation of the 'global community' advances with the easiness of communication between people that can be approximated by their language proficiency. Hence, the second important issue in the extension of the variables of the model is the language effects. According to the last Eurobarometer survey – 'Europeans and their Languages', published in June 2012 – the update on result from 2005[17] – English dominates as the language that Europeans are most likely to be able to speak. The linguistic map of Europe is similar to that presented in 2005 – the five most widely spoken foreign languages remain English (38%), French (12%), German (11%), Spanish (7%) and Russian (5%). The survey registered a slight drop in the

[15] The synthetic variable of bilateral costs was proposed and described in the author's previous work.

[16] The use of the travel time between countries' centroids became possible owing to free Google Maps application, which time-data was downloaded on 14.03.2014 (with the use of a special software for calculating distances between items from the list of locations that was ordered and sponsored by JLU Giessen).

[17] These nationally representative surveys investigated the language skills: the mother tongues and up to three other languages that they speak well enough to have a conversation. Source: *Special Eurobarometer 386*.

proportions able to hold a conversation in German and French (-3 and -2 percentage points respectively). The citizens of the 'Old EU members' (EU15) are particularly more likely than those in NMS12 to speak French (14% vs. 6% respectively) and Spanish (8% vs. 2%). Moreover, they are particularly less likely to speak German (10% vs. 15%) and Russian (2% vs. 16%). The most significant conclusion of European Commission's surveys is that Europeans have very positive attitudes towards multilingualism and their passive skills are increasing. However, the results show that language skills are unevenly distributed both over the geographical area of Europe and over socio-demographic groups. The measurement of Europeans' ability to speak various languages is an important stringency of the analysis for international trade flows, hence, the approach in this paper uses the results of both Eurobarometer surveys[18], with the calculations following those in the study of Fidrmuc, Fidrmuc (2009). Namely, the factor of language is investigated in two ways. Firstly, three official languages, which are most widely spoken in Europe: English, German and French, are measured using dummies, if they are official in both countries or spoken by more than 20% of populations[19]. Secondly, the average proficiency rates ω[20] are used to estimate probabilities $P_{f,od}$ that two randomly chosen individuals from countries o and d will be able to communicate in a certain language f:

$$P_{f,od} = \omega_{f,o} \cdot \omega_{f,d}. \tag{3}$$

In the above approach there is no distinction between whether the individuals are native speakers of the language or whether one or both of them speak it as foreign language. The coefficients of all the languages-variables are expected to be positive since they facilitate communication and ease trade transactions.

In fact, the investigation of the language effects is focused on the case of English. It is expected that the effect of English proficiency will be the strongest and positive. English plays actually a role of the *lingua franca,* it is the most widely spoken foreign language in the World. Trade relations between remote countries, for example, between Portuguese and Polish entrepreneurs are more likely to be facilitated by English than by Portuguese or Polish. In the empirical analyses of bilateral trade flows of Fidrmuc, Fidrmuc (2009) the English effect appeared robust to alternative regression

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[18] The study uses the English proficiency that was calculated based on *Eurobarometer*, as an alternative to the EF English Proficiency Index, which has been criticized for its lack of representative sampling in each country – the respondents are self-selected and must possess access to the Internet.

[19] The results from the first Eurobarometer survey (*Eurobarometer 243*) are used to calculate the proficiency rates for the period of 1999–2005, the results from the second one (*Eurobarometer 386*) are used to reflect the proficiency rates for the time period 2006–2011.

[20] Proficiency rate ω is the share of the population speaking the language as native speakers or speaking it as foreign language with 'good' or 'very good' level. Those indicators were taken from *Special Eurobarometer 243*, as the next survey does not contain the information about the level of proficiency.

specifications (also to inclusion of other languages in the analysis) and also here are expected to have significant and positive impact on trade.

In order to investigate trade creation and trade diversion effects of the Economic and Monetary Union the following three binary variables were included in the estimated model: *BothEMU*, *ExEMU*, *ImEMU* (Viner, 1950). The first one takes a value of 1 if both countries *o* and *d* belong to the EMU and zero otherwise. A positive and statistically significant coefficient of *BothEMU* represents trade creation effects and indicates that intra-regional trade has been promoted more by the free trade agreement and is higher than normal trade levels. In the EMU area, trade flows between countries are expected to increase with the time due to a more intense integration (not only political, but also cultural). *ExEMU* takes a value of one if exporter *o* belongs to the EMU and destination country *d* does not and zero otherwise. A positive and statistically significant coefficient of *ExEMU* is interpreted as an export diversion effect of the EMU and indicates that regional integration leads to a switch of export activities from EMU members to non-EMU members. *ImEMU* takes a value of one if exporter *o* is a non-EMU member and destination country *d* belongs to the EMU and zero otherwise. Its positive and statistically significant coefficient indicates an import diversion effect in EMU – then EMU members have shifted their importing activities from non-member countries to member countries. The specification of trade creation and trade diversion in the logarithmic form of gravity model can be written as follows:

$$EX_{t,od} = EV_{t,od} + \sum_t \phi_1 BothEMU_{t,od} + \sum_t \phi_2 ExEMU_{t,od} + \sum_t \phi_3\, Im\, EMU_{t,od}, \qquad (4)$$

where: $EX_{t,od}$ – export flows, $EV_{t,od}$ – the rest of explanatory variables. The coefficient ϕ_1 measures the extent to which trade is higher than normal levels if both countries are EMU-members, ϕ_2 measures the extent to which members' exports are higher than normal levels from non-member countries and ϕ_3 the members' imports effects respectively.

According to Martínez-Zarzoso *et al.* (2009), one observation alone of intra-bloc trade (ϕ_1) is insufficient to confirm whether or not there is a net trade creation in the free trade area – for instance, an increase in intra-bloc exports ($\phi_1 > 0$) may be accompanied by reduction in imports from extra-bloc countries ($\phi_3 < 0$). These trade creation and diversion effects may offset each other and hence, besides the coefficient of *BothEMU* variable, there is still the need of examination the magnitudes and directions of trade among member and non-member countries (ϕ_2, ϕ_3). Assuming that $\phi_1, \phi_2 > 0$, which denotes that trade creation is accompanied by an increase in exports from intra-bloc countries to extra-bloc countries, this can be described as a pure trade creation in the EMU. However, a positive ϕ_1 accompanied by a negative ϕ_2 denotes a combination of trade creation effects and export diversion effects. Here, if $\phi_1 > \phi_2$, then, despite the trade creation effects are offset to a certain extent by export diversion effects, the trade creation still dominates. In the case of $\phi_1 < 0 < \phi_2$ a dominant export diversion effect representing a welfare loss on behalf of member countries[21]. In the case of decrease in intra-EMU export flows ($\phi_1 < 0$), along with a higher propensity to imports ($\phi_3 > 0$), occurs the extra-EMU import expansion.

According to the traditional gravity model, the trade flows are proportional to the product of national incomes and are divided by the distance between them. In this form only the distance is a variable (here bilateral trade costs variable) that is measured at the level of a trading pair of countries – the national incomes concern the countries which are at a higher level. However, putting the variable of the product of both incomes in the gravity model of trade is a common method as well (Sohn, 2005; Rahman, 2010; Gul, Yasin, 2011)[22] because it does not change the idea of the model and allows some estimation problems to be avoided, such as, for instance, the impossibility of the estimation of the countries' incomes effects if there are time-varying country effects used in the estimated model. Besides, the product of national incomes becomes a trading pair-level variable, which is especially helpful in the case of the multilevel modelling, where the pairs of countries compose the second level of the model. Most of the other variables are also established at the trading pair-level, namely the calculated internetization rate, researchers rate, energy use and variables describing communication in different languages. Only the share of the high-technology export remains at the country level.

In the 3-level model the random effects at different levels are assumed to be uncorrelated. Each lower level residual is allowed-to-vary random departure from the higher-level departure. For simplicity, the explanation of the form of estimated models is shown at the 2-level at first. With the above described set of covariates, the algebraic specification of random-coefficients 2-level model of bilateral trade is as follows[23]:

$$EX_{t,od} = \beta_{0,od} + \beta_{1,od} GNIproduct_{t,od} + \beta_{2,od} Internetization_{t,od} + \varepsilon_{t,od}, \tag{5}$$

with the fixed part of the model of:

$$\begin{aligned}\beta_{0,od} = \beta_{00} + \alpha_1 BTC_{t,od} + \alpha_2\, Research_{t,od} + \alpha_3 EnergyUse_{t,od} + \alpha_4 HighTechEx_{t,od} \\ + \alpha_p D_{p,t,od} + \alpha_s P_{s,f,od} + \gamma_1 I_o + \gamma_2 I_d + \gamma_3 I_t + u_{0,od},\end{aligned} \tag{6}$$

and the random part of the model:

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[21] Martínez-Zarzoso *et al*. (2009) identified such possible trade effects under FTA. For the details about interpreting static integration effects, see Table 1, p.53.

[22] Linnemann (1966) added to the equation even the product of two countries' populations.

[23] The estimated variables, except dummies and probabilities, are expressed in logarithms.

$$\begin{aligned} \beta_{1,od} &= \beta_{10,od} + u_{1,od}, \\ \beta_{2,od} &= \beta_{20,od} + u_{2,od}, \end{aligned} \tag{7}$$

where: $BTC_{t,od}$ – bilateral trade costs, $D_{p,t,od}$ – set of p dummy variables for the pair of countries (*ExEMU*, *ImEMU*, *BothEMU*, *Border*, *Sea*, *OfficialLanguage* and *LanguageProficiency*), $P_{s,f,od}$ – set of probability variables that two trading partners are able to communicate in the certain f language (*LanguageCommunication*), I_o, I_d – time-invariant individual (country) effects, I_t – time effect, u_{od} – trading pairs (level-2) random effects.

Equation (5) captures the variation in the time series, characterizes bilateral trade flows by the time varying variables with relatively larger variability: the national incomes' product and common internetization rate. The *od* subscript indicates that the intercept and slope coefficients are allowed to vary across the trading pairs found at level 2. The fixed-part of the model describes the given trading pair and the random-part at level-2 describes how 504 trading pairs vary around the average. The β_{00} coefficient measures the overall intercept across all trading pairs, $\beta_{0,od}$ is interpreted as the intercept of the dependent variable for the pair *od* (which is different from the flows from country d to country o; o describes origin and d – destination of the trade flow) and $\beta_{10,od}$, $\beta_{20,od}$ measure the overall slopes across all trading pairs. The fixed-part of 2-level model (6) captures the fixed effects that the rest of variables have on the variability of average levels of trade across trading country pairs (α_1, α_2, α_3, α_4, α_p, α_s, γ_1, γ_2, γ_3).

Since the bilateral export flows are nested not only within trading pairs, but also within particular areas such as northern and southern Europe, or inside and outside EMU area, the study considers the third level in the model, namely the place diversion: intra-EMU trade, the inter-EMU trade and outside-EMU trade.

Combining the first, second and third level models yields to the following model:

$$\begin{aligned} EX_{t,od,k} = {} & \beta_{00} + \beta_{10,od} GNIproduct_{t,od} + \beta_{20,od} Internetization_{t,od} + \alpha_1 BTC_{t,od} \\ & + \alpha_2 Research_{t,od} + \alpha_3 EnergyUse_{t,od} + \alpha_4 HighTechExport_{t,od} \\ & + \alpha_p D_{p,t,od} + \alpha_s P_{s,f,od} + \gamma_1 I_o + \gamma_2 I_d + \gamma_3 I_t \\ & + u_{0,od,k} + u_{1,od,k} GNIproduct_{t,od} + u_{2,od,k} Internetization_{t,od} \\ & + v_{0,k} + v_{1,k} GNIproduct_{t,od} + v_{2,k} Internetization_{t,od} + \varepsilon_{t,od,k}, \end{aligned} \tag{8}$$

where: t – level 1 (year), od – level 2 (state = trading pair) , k – level 3 (place), v_{0k} – the random effect at the place level (EMU diversion), an allowed-to-vary departure from the grand mean, $u_{0,od,k}$ – the random effect at the trading pairs level, a departure from the place effect, $\varepsilon_{t,od,k}$ – the random effect at the year level, a departure from the trading pair effect within a place.

Table 2 presents the estimation results for all the specified models. The 3-level and 2-level models with random coefficients for GNI product and common internetization rate, according to the equations (8) and (5) were estimated in turn. Then, the simple Pooled Model (level-1 model), which incorrectly assumes that individuals are independent and leads to underestimation of standard errors and incorrect inferences, was computed. The deviance values, together with results of LR test comparing both models (see Table 3)[24], showed that the random-intercept models are preferred.

The slope for each trading pair equals the fixed-effect slope for the whole sample, plus the random-effects slope for that pair. The calculated total effects (predicted random effects are in the sum) provide information on how the relationship between bilateral export flows and incomes' product and between bilateral export flows and common internetization rate vary across trading pairs. The coefficients of random slopes in the 3-level model (EMU diversion) are significant only for incomes' product. However, implementing the 2-level model gives significant estimates for common internatization rate, too. Hence, the final 3-level model contains random slopes for intercept and national incomes' product at every single level and a random slope for common internetization rate at the first and second levels. The estimates of the mixed models are computed by means of the maximum likelihood method, with the use of Stata 13 software[25].

Based on the average estimated random effects of Model 2, the equation (8) for the export from Germany to Poland would be:

$$\begin{aligned}E\hat{X}_{DEU-POL} &= (-17.12 - 1.33 - 5.94) + (0.61 + 0.02 + 0.09) GNIproduct + (0.15 + 0.13) Internetization \\ &- 0.13 BTC + 0.08 Researchers + 0.10 HighTechEx + 0.13 EnergyUse \\ &+ 0.08 ExEMU + 0.98 Border + 0.72 Sea + 1.92 EnglishCommuncation \\ &+ 3.48 FrenchCommunication + 3.76 exporter + 0.38 importer\end{aligned} \tag{9}$$

and alternatively, from Poland to Germany as follows:

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[24] The H_0 of the likelihood ratio (LR) test assumes that there is no significant difference between the two models.

[25] Stata's commands allow the estimation of the random effects with BLUP method – Best Linear Unbiased Prediction that show the amount of the variation for both the intercept and the estimated coefficients of *lnGNIproduct* and *lnInternetization*. According to Robinson (1991), 'BLUP estimates of the realized values of the random variables *u* are *linear* in the sense that they are linear functions of the data, *y*; *unbiased* in the sense that the average value of the estimate is equal to the average value of the quantity being estimated; *best* in the sense that they have minimum mean squared error within the class of linear unbiased estimators; and *predictors* to distinguish them from estimators of fixed effects'. The estimators of random effects are commonly called as 'predictors' while estimators of fixed effects are called 'estimators', however, as a matter of fact both are estimators.

Table 2.

Estimation results of 2-level and 3-level gravity models

lnEXPORT	3-level models		2-level models					1-level model
VARIABLES	Model 1	Model 2	Random-intercept Model	Model 3	Model 4	Model 5	Model 6	Pooled Model
lnGNIproduct	0.651*** (0.0302)	0.610*** (0.0441)	0.945*** (0.0229)	0.651*** (0.0297)	0.654*** (0.0302)	0.654*** (0.0302)	0.655*** (0.0302)	0.862*** (0.0355)
lnBTC	-0.138*** (0.00409)	-0.138*** (0.00409)	-0.202*** (0.00467)	-0.138*** (0.00409)	-0.138*** (0.00409)	-0.138*** (0.00410)	-0.138*** (0.00410)	-0.554*** (0.00454)
lnResearchers	0.0773* (0.0416)	0.0761* (0.0414)	0.00125 (0.0350)	0.0752* (0.0417)	0.0750* (0.0417)	0.0703* (0.0416)	0.0710* (0.0416)	0.176*** (0.0317)
lnInternetization	0.165*** (0.0292)	0.152*** (0.0291)	0.183*** (0.0189)	0.166*** (0.0289)	0.167*** (0.0290)	0.166*** (0.0291)	0.165*** (0.0291)	0.202*** (0.0272)
lnHighTechEx	0.103*** (0.0154)	0.103*** (0.0153)	0.218*** (0.0145)	0.104*** (0.0154)	0.104*** (0.0154)	0.104*** (0.0154)	0.104*** (0.0154)	0.0905*** (0.0236)
lnEnergyUse	0.147** (0.0708)	0.133* (0.0706)	-0.0219 (0.0664)	0.150** (0.0716)	0.149** (0.0720)	0.135* (0.0701)	0.131* (0.0706)	0.0187 (0.0188)
ImEMU	0.204*** (0.0241)	0.198*** (0.0241)	0.182*** (0.0245)	0.198*** (0.0239)	0.197*** (0.0240)	0.196*** (0.0240)	0.195*** (0.0240)	0.188*** (0.0336)
ExEMU	0.0888*** (0.0216)	0.0836*** (0.0217)	0.0402* (0.0222)	0.0835*** (0.0215)	0.0830*** (0.0216)	0.0821*** (0.0216)	0.0812*** (0.0215)	-0.103*** (0.0314)
BothEMU	0.185*** (0.0270)	0.177*** (0.0270)	0.194*** (0.0282)	0.186*** (0.0265)	0.185*** (0.0270)	0.184*** (0.0270)	0.184*** (0.0267)	0.208*** (0.0449)
Border	0.962*** (0.0763)	0.980*** (0.0762)	0.978*** (0.0772)	1.080*** (0.0762)	1.073*** (0.0774)	0.949*** (0.0752)	1.019*** (0.0742)	0.218*** (0.0194)
Sea	0.706*** (0.151)	0.719*** (0.151)	0.480*** (0.137)	0.613*** (0.158)	0.618*** (0.158)	0.694*** (0.153)	0.614*** (0.154)	0.195*** (0.0304)

lnEXPORT	3-level models		2-level models					1-level model
VARIABLES	Model 1	Model 2	Random-intercept Model	Model 3	Model 4	Model 5	Model 6	Pooled Model
OfficialLanguage	0.315*	0.303	0.243	0.314*	0.274	0.390**	0.345**	-0.00703
	(0.189)	(0.189)	(0.191)	(0.170)	(0.186)	(0.189)	(0.164)	(0.0419)
English	-0.00676	-0.0106	-0.0558***		-0.00540	-0.00688	-0.00646	-0.0182
	(0.0152)	(0.0151)	(0.0123)		(0.0152)	(0.0151)	(0.0151)	(0.0173)
German	0.00519	0.0120	-0.0475		0.00127	0.00398		-0.174***
	(0.0485)	(0.0481)	(0.0411)		(0.0469)	(0.0484)		(0.0375)
French	-2.261***	-2.255***	-2.395***		0.138	-2.930***		-0.305*
	(0.833)	(0.834)	(0.820)		(0.253)	(0.808)		(0.182)
EnglishCommunication	1.906***	1.922***	1.584***			1.808***	1.705***	0.527***
	(0.337)	(0.337)	(0.352)			(0.334)	(0.341)	(0.0791)
GermanCommunication	-0.336	-0.388	-0.0737			-0.254		0.0331
	(0.291)	(0.290)	(0.294)			(0.288)		(0.0874)
FrenchCommunication	3.551***	3.478***	4.015***			4.641***		0.621**
	(1.214)	(1.217)	(1.186)			(1.177)		(0.264)
Constant	-16.18***	-17.12***	-29.69***	-15.45***	-15.57***	-16.23***	-16.05***	-20.53***
	(1.770)	(2.040)	(1.329)	(1.731)	(1.770)	(1.763)	(1.765)	(1.718)
time-effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
country-effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
# groups	3 / 504	3 / 504	504	504	504	504	504	504
# observations	6,533	6,533	6533	6533	6533	6533	6533	6533
deviance	-528.9	-540.96	1185.64	-478,32	-478,74	-518,98	-502,54	5591.76
sd(GNIproduct)	0.105	0.102		0.1035	0.1029	0.102	0.102	
	(0.013)	(0.012)		(0.012)	(0.012)	(0.012)	(0.012)	

Table 2.

lnEXPORT	3-level models		2-level models					1-level model
VARIABLES	Model 1	Model 2	Random-intercept Model	Model 3	Model 4	Model 5	Model 6	Pooled Model
sd(Internetization)	0.187 (0.19)	0.219 (0.18)		0.212 (0.019)	0.212 (0.018)	0.214 (0.018)	0.217 (0.018)	
sd(cons)	6.306 (0.544)	5.878 (0.508)	0.473 (0.016)	6.072 (0.504)	6.046 (0.509)	5.958 (0.501)	5.982 (0.503)	
corr(GNI,Internetization)	0.644 (0.266)	0.295 (0.193)		0.426 (0.215)	0.429 (0.215)	0.430 (0.211)	0.402 (0.204)	
corr(GNI,cons)	-0.997 (0.005)	-0.991 (0.003)		-0.992 (0.004)	-0.993 (0.004)	-0.994 (0.004)	-0.992 (0.004)	
corr(Internetization,cons)	-0.666 (0.210)	-0.403 (0.166)		-0.510 (0.178)	-0.513 (0.177)	-0.513 (0.173)	-0.494 (0.169)	
sd(residual)	0.178 (0.002)	0.178 (0.002)	0.227 (0.002)	0.178 (0.002)	0.178 (0.002)	0.178 (0.002)	0.178 (0.002)	0.371 (0.003)
sd(GNIproduct)		0.054 (0.022)						
sd(_cons)	0.151 (0.068)	1.118 (0.621)						

Source: author's calculations using Stata software. Parentheses enclose standard errors; *** p<0.01, ** p<0.05, * p<0.1.

$$\begin{aligned} E\hat{X}_{POL-DEU} &= (-17.12-1.33-2.09)+(0.61+0.02+0.02)GNIproduct+(0.15+0.09)Internetization \\ &-0.13BTC+0.08\,Researchers+0.10HighTechEx+0.13\ln EnergyUse \\ &+0.20\,\mathrm{Im}\,EMU+0.98Border+0.72Sea+1.92EnglishCommuncation \\ &+3.48FrenchCommunication+0.56\,exporter+3.76importer. \end{aligned} \tag{10}$$

As both countries compose one trading pair, the fixed part and the 3-level random coefficients are common, the only differences between them are at level 2, where each pair has its own additional slope for the intercept, *lnGNIproduct* and *lnIntenetization*, own country fixed effects and trade division effect's value of dummy. As expected, the respective estimates for export flows from Germany (the country is Europe's export leader) to Poland are relatively larger than for the oppositely-directed flows.

Two residual intraclass correlations for the estimated 3-level nested model (10) can be calculated. First, the level-3 intraclass correlation at the place level, that is the correlation between annual export flows in the same trade-place but for different trading pairs that takes the following form:

$$\text{intra-place correlation} = \frac{\sigma_{v0}^2}{\sigma_{v0}^2+\sigma_{u0}^2+\sigma_e^2}. \tag{11}$$

The second, level-2 intraclass correlation at the pair-within-place level (between annual export flows of the same pair in the same place) is:

$$\text{intra-pairs correlation} = \frac{\sigma_{v0}^2+\sigma_{u0}^2}{\sigma_{v0}^2+\sigma_{u0}^2+\sigma_e^2}. \tag{12}$$

The error terms and random intercept are assumed to be normally distributed with mean 0 and variances $\sigma_{v0}^2, \sigma_{u0}^2$ and σ_e^2, and to be mutually independent.

The calculated residual intraclass correlations of Model 6 show that the annual export flows are only slightly correlated within the same place of trade (0.035), but they are extremely highly correlated within the same trading pair and place of trade, namely pair and place random effects compose approximately 99% of the total residual variance.

According to LR test results, there is a statistically significant difference also between the random-intercept model and all the relevant versions of random-coefficients models – the extended models (Models 1-6) provide a better fit. Model 5, which contains all the potential variables, is the most preferred among all of the 2-level models, however, it must be noted that not every effect is statistically significant, namely the effects of English and German proficiency and the probability of communications in German as well. An unexpected sign has French's coefficient, as it describes negative relation between trade flows and the common French proficiency.

However, the effects of communication in French are strongly positive and are more meaningful for the trade costs than the negative impact of the proficiency variable, which is actually created only by Belgium, France and Luxemburg, because only in the pairs of those countries the French language is official or spoken by more than 20% of the population. According to those results, the French speaking countries trade relatively less with each other than with other countries of the EU. The communication in English and French raises the trade exchange, which confirms the increasing importance of the quality of human capital in the international trade.

Table 3.

Results of the likelihood-ratio test

LR tests	Chi-square	P-value	Assumption	Preferred model
Model 5 / Model 2	21.98	0.0000	Model 5 nested in Model 2	Model 2
Model 5 / Model 1	9.92	0.0016	Model 5 nested in Model 1	Model 1
Random-intercept Model / Model 5	1704.62	0.0000	Random-intercept Model nested in Model 5	Model 5
Pooled Model / Random-intercept Model	4406.13	0.0000	Pooled Model nested in Random-intercept Model	Random-intercept Model

Source: author's calculations using Stata software.

The estimates of EMU dummies in the models indicate, that in the time period 1999–2011 there was significant trade creation in terms of imports with more pure effect in terms of exports ($\phi_3 > \phi_1 > \phi_2 > 0$). The intra-EMU trade is relatively larger, but the extra-EMU trade is growing as well and there is no trade diversion effect ($\phi_2, \phi_2 > 0$) – however, members' import effects are much larger than member's export effects, which does not seem to encourage non-EMU-members to join the EMU, since they still benefit from the export to EMU area. As a matter of fact, the positive net export is more desired, especially by developing economies, since it creates the national income.

4. CONCLUSION

This study uses two hierarchical linear models to examine the effects of both traditional and globalization-connected variables on bilateral trade flows between EU countries. All the considered variables, apart from proficiency in English and German as well as the probability of communication in German, exert a significant influence on the average level of trade. The estimation results are consistent with the theory of gravity model, where trade flows decrease with the rise of bilateral trade costs, which are a synthetic variable in the estimated models, based on travel time between country centroids and importer's openness.

Additionally, the impact of two variables: national incomes' product and common internetization rate, together with the intercept effect, vary across trading country pairs due to the heterogeneity in the sample. Both income and internetization have a positive impact on trade across trading pairs but the income also influences bilateral trade depending on the place of trade. The economic potential of the countries enhances the exchange more by the extra- and inter- than by intra-EMU trade. The distribution of random slopes of common internetization between the different places of trade is similar to the distribution of GNI slope, but is much closer to the estimated fixed effect of this variable. The hierarchical structure of the estimated models allows the formulation of the conclusion that the policy of increasing the national wealth and the quality of the network infrastructure leads to a relatively larger average increase of bilateral trade flows in the case of non-EMU-members than in the Eurozone. The intra-EMU trade is less dependent, however, overall larger since the common membership in EMU increases the trade flows. The estimation results are not completely accordant with the second hypothesis, which assumes the pure trade creation effect of the Eurozone. According to the model there is indeed the trade creation caused by EMU, however, in terms of the import. The positive signs of trade diversion-variables signify no trade diversion effects in the EU in the time period 1999–2011. The EMU members trade relatively more intensively not only with each other, but with non-EMU-members as well. Their economic conditions allow them for larger imports, which contributes to the trade creation effects of EMU.

All the coefficient estimates of the variables, that characterize the progress of globalization, provide grounds for the first research hypothesis verification, confirming the positive and significant impact of globalization on the international exchange.

Moreover, the estimates of language variables show that a common official language can increase the bilateral trade flows, however, not in the case of French speaking countries. According to the model, the ability of communicating in English and French increases the bilateral trade flows, when the impact of communication in German remains insignificant. Since the impact of French proficiency is significantly negative in the model, only the English language seems to be the true *lingua franca* within the area of the EU, which, in fact is the verification of the third hypothesis.

Further research could extend the model by including a larger research sample of countries, essentially other big trade partners of the European Union. Among other problems that remain open for consideration, the following should be mentioned: the use of hierarchical models by empirical analysis of other globalization processes, as migration or foreign direct investment flows, and the use of dynamic model, especially by the larger time period of research.

Nicolaus Copernicus University in Torun

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HIERARCHICZNE MODELE LINIOWE BILATERALNYCH PRZEPŁYWÓW HANDLOWYCH MIĘDZY PAŃSTWAMI UNII EUROPEJSKIEJ

Streszczenie

Empiryczne modele grawitacji międzynarodowych przepływów handlowych estymowane są często metodą FE, której wadą jest iż, mimo zastosowania stałych efektów, zróżnicowanych dla wszystkich jednostek w badanej próbie, zakłada jednakowe oceny parametrów zmiennych użytych w modelu. W niniejszej pracy problem heterogeniczności rozwiązany jest za pomocą modeli mieszanych, pozwalających na zróżnicowane efekty pomiędzy parami nie tylko dla stałej, ale dodatkowo dla produktu dochodów narodowych oraz wspólnego poziomu internetyzacji. Estymowane dwu oraz trzy poziomowe modele dla danych z okresu 1999–2011 wykazują istotny wpływ tradycyjnych zmiennych modelu grawitacji oraz czynników związanych z postępem globalizacji.

Słowa kluczowe: model grawitacji, model mieszany, bilateralne koszty handlu, biegłość językowa, globalizacja, internetyzacja

MULTILEVEL MODELLING OF BILATERAL TRADE FLOWS BETWEEN EUROPEAN UNION COUNTRIES

Abstract

Empirical research of international trade with the use of gravity model is often estimated with the FE estimator. Indeed, this method is appropriate in the face of heterogeneity, that is typical of pairs of countries, which influence the effect of the determinants of bilateral trade. However, the disadvantage of the FE approach is that it assumes all the slopes of the variables of interest are common across all trading pairs in the sample. The use of mixed effects model in this study allows the coefficients of national incomes' product and the common internetization rate of trading countries to vary across the pairs. In order to capture unspecified heterogeneity by allowing the intercept and slopes to have a stochastic component in their variation, the 2-level and 3-level hierarchical linear models are estimated based on the data from the period 1999–2011. The results indicate that not only typical gravity model factors, but also globalization factors as internetization rate, researchers rate, share of high-technology products' export, energy use, foreign languages proficiency and monetary union influence the bilateral trade between EU-members.

Keywords: gravity model, mixed-effects model, bilateral trade costs, language proficiency, globalization, internetization

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PIOTR DUDZIŃSKI

INSURANCE AND SELF-INSURANCE – SUBSTITUTES OR COMPLEMENTS?

1. INTRODUCTION

Individuals facing a potential loss may undertake various efforts to protect themselves against risk. One of them is market insurance, but there are possible alternatives to it. Ehrlich and Becker (1972) were first to present and systematically analyze concepts of self-insurance and self-protection. Self-insurance is defined as an effort made towards a reduction in the size of a loss, whereas self-protection leads to reduction in the probability of a loss. Ehrlich and Becker showed that market insurance and self-insurance are substitutes, but market insurance and self-protection may be complements or substitutes, depending on the initial probability of the loss. That result was confirmed by Courbage (2001) in Yaari's Dual Theory of Choice setting. Over time, concept of self-protection has attracted many researchers appearing to be more complex and interesting phenomenon than self-insurance. The reason is that self-insurance reduces large losses in the bad state more effectively than smaller loss in the good state and therefore may be considered as a type of insurance. However, it is no longer true in more general model that takes into the account many states of the world.

It is easy to see that under decreasing absolute risk aversion (DARA) self-insurance (as well as insurance) is inferior. However, Lee (2010a) showed that if the model provides for many states of the world then with DARA insurance is inferior but self-insurance may by inferior or normal, depending on productivity of self-insurance. Therefore, in more general setting self-insurance cannot be considered as special type of insurance.

The effect of an increase in risk aversion on self-insurance is another important question. Dionne, Eeckhoudt (1985) and Bryis, Schlesinger (1990) proved that more risk-averse individuals invest more in self-insurance. Lee (2010b) again generalized the model to many states and presented conditions for more risk averse individuals to invest more or less in self-protection.

It is therefore interesting whether Ehrlich and Becker's classical result about substitutability of market insurance and self-insurance does hold in more general and realistic model with multiple states of the world. It has not been analyzed yet in the literature. This paper fills that gap to some extent. We present sufficient conditions for self-insurance and market insurance to be substitutes or complements, making use of

Diamond and Stiglitz „single crossing condition” and the notion of supermodularity. We also provide economic interpretation of that result. The key concept here is an effectivity of self-insurance, which reflects its technology.

The result presented in this paper has certain limitations. We consider here only the case when level of market insurance is exogeneous. It represents the situation when insurance is obligatory, forced by law. It usually depends on the country, in Poland there are a number of cases of mandatory insurance regulated by the Compulsory Insurance Act (2003). As in all EU countries, all vehicles must have third party liability insurance. It is mandatory for a car owner to take out insurance against injury and damage. Third-party liability is mandatory to any person who owns a farm. Also the insurance of farm buildings from fire and other accidents is compulsory. There are several more examples of compulsory insurance and in those cases insurance cannot be considered as decision variable. The problem is, when insurance is mandatory then there is no demand in the usual sense. For that reason terms „substitutes” or „complements” may seem to be inappropriate in that context. Nevertheless, we define and use them as a description of reaction of the demand for self-insurance generated by increase in price of market insurance.

2. THE MODEL

Consider a risk-averse individual who has initial wealth w_0 that is subject to possible loss. The size of a loss depends on the state of the world θ is denoted by $l(\theta)$. θ is a continuous random variable such that $\theta \in [\underline{\theta}, \overline{\theta}]$, with the density function $f(\theta)$. Without loss of generality we assume that a state with higher θ represents larger loss, that is $l'(\theta) > 0$. We denote full insurance cost by π, and an individual has bought an insurance coverage $\alpha l(\theta)$ for a premium $\alpha\pi$ where factor α is determined by law and $\alpha \in [0,1]$. Moreover, he may independently invest in self-insurance that also reduces the loss. In this model, effects of insurance and self-insurance are separated in order to capture interactions between them. The amount invested in self-insurance is e (it is decision variable in our model), and it leads to reduction in loss by $d(e, \theta)$. By the definition, an increase in self-insurance reduces the loss and it is reasonable and customary to assume that reduction happens at a decreasing rate. Therefore we have $d_e(e,\theta) = \partial d(e,\theta)/\partial e > 0$ and $d_{ee}(e,\theta) < 0$. It is also assumed that the same self-insurance activity cannot lead to higher reduction of the loss in the worse state, so we have $d_\theta(e,\theta) \leq 0$. It seems like technical assumption, but typical examples of self-insurance show that it is not restrictive. Violation of that condition might lead to the conclusion that it would be profitable to incur larger loss, which makes no sense.

The final wealth in the state θ is thus

$$w = w_0 - e - \alpha\pi - (1-\alpha)[l(\theta) - d(e,\theta)]. \tag{1}$$

Let us denote $L(e,\theta) = l(\theta) - d(e,\theta)$. Hence

$$w = w_0 - e - \alpha\pi - (1-\alpha)L(e,\theta). \tag{2}$$

Due to the above assumptions, we have

$$L_\theta(e,\theta) = l'(\theta) - d_\theta(e,\theta) > 0, L_e(e,\theta) = -d_e(e,\theta) < 0. \tag{3}$$

As a consequence of our assumptions we obtain that

$$w_\theta = -(1-\alpha)L_\theta(e,\theta) \leq 0, \tag{4}$$

which reads that the worse state means smaller final wealth for the same level of investment in self-insurance, which is intuitive.

The individual's problem is to choose e to maximize expected utility of final wealth

$$Eu(w) = \int_{\underline{\theta}}^{\overline{\theta}} u\big(w(e,\theta)\big)f(\theta)d\theta,$$

where u denotes von Neumann-Morgenstern utility such that $u' > 0$, $u'' < 0$. The first-order condition – necessary for internal solution of the problem – is then

$$\frac{\partial Eu}{\partial e} = \int_{\underline{\theta}}^{\overline{\theta}} u'\big(w(e,\theta)\big)\big(-1-(1-\alpha)L_e(e,\theta)\big)f(\theta)d\theta = 0. \tag{5}$$

Obviously, for that to happen, the factor $w_e = -1-(1-\alpha)L_e(e,\theta)$ has to be positive for some values of θ and negative for other θ*s*.

Observe that the second-order condition is satisfied. Indeed, after straightforward calculations we have

$$\frac{\partial^2 Eu}{\partial e^2} = \int_{\underline{\theta}}^{\overline{\theta}} \Big[u''(w)\big(-1-(1-\alpha)L_e(e,\theta)\big)^2 + u'(w)(1-\alpha)d_{ee}\Big] f(\theta)d\theta < 0.$$

Due to our assumptions, the sign of the above expression is unambiguously negative. Hence the problem becomes concave and there exists its unique solution. Let us denote by e^* the optimal level of self-insurance, satisfying equation (5).

3. ANALYSIS OF THE MODEL

Our problem is to answer the question if insurance and self-insurance are substitutes or complements and to derive conditions sufficient to give the unambiguous answer. However, one must be careful about terms „substitutes” and „complements” in our context. In this paper we consider market insurance as mandatory, so demand for it does not make the usual sense. Especially use of the word „substitute” raises justified doubt. Nevertheless, price of the insurance is set always by insurer and an individual may adjust his self-insurance activity to an increase in price of the insurance. If demand for self-insurance increases in presence of increased prices of market insurance then we say that self-insurance is *substitute for* market insurance. If demand for self-insurance decreases then we say that self-insurance is *complement of* market insurance. It seems like classical definition, but it is not. When price of the insurance increases then „demand” for it remains the same as before. Therefore there is no substitution in traditional meaning. What we investigate here is the effect of increase in price of the market insurance on the demand for self-insurance. We use terms „substitutes” and „complements” in specific meaning, defined above. We do it for simplicity and because those terms are in present context as close as possible to their original sense.

There are three main, well-known types of absolute risk-aversion: decreasing, increasing and constant with regard to wealth of an individual, abbreviated to DARA, IARA and CARA respectively. However, DARA is considered a natural assumption and it is confirmed empirically. For example, under DARA, risky assets are normal goods, whereas with IARA it becomes inferior. DARA means that $A(w) = -\frac{u''(w)}{u'(w)}$, the Arrow-Pratt index of absolute risk-aversion is decreasing in wealth w (Pratt 1964), hence $A'(w) \leq 0$. IARA implies that $A'(w) \geq 0$.

Since DARA is intuitive and nonrestrictive and IARA case is symmetric, we will only cover the DARA case.

Proposition 1. Assume that individual exhibits decreasing absolute risk aversion (DARA).

(i) If the function $\left(-1-(1-\alpha)L_e(e^*,\theta)\right)$ crosses singly the θ – axis changing its sign from plus to minus then $\frac{\partial e^*}{\partial \pi}$ is negative and self-insurance is complementary to market insurance.

(ii) If the function $\left(-1-(1-\alpha)L_e(e^*,\theta)\right)$ crosses singly the θ – axis changing its sign from minus to plus then $\frac{\partial e^*}{\partial \pi}$ is positive and self-insurance is a substitute for market insurance.

Proof. By implicit function theorem, equation (5) may be written in general form:

$$F(\pi, e^*(\pi)) = 0. \tag{6}$$

It reflects the basic intuition that the demand for self-insurance depends somehow on the price of the market insurance. Our aim is to find out what happens with e^*, when the price of market insurance goes up. In other words, we are interested in the sign of $\frac{\partial e^*}{\partial \pi}$ and in what determines that sign.

By totally differentiating (6) we obtain

$$\frac{\partial e^*}{\partial \pi} = -\frac{\partial F/\partial \pi}{\partial F/\partial e^*}.$$

By the second-order condition, the sign of the denumerator of the above is negative, and therefore

$$\text{sign}\,\frac{\partial e^*}{\partial \pi} = \text{sign}\,\frac{\partial F}{\partial \pi}\,. \tag{7}$$

Consequently, we calculate:

$$\frac{\partial F}{\partial \pi} = \int_{\underline{\theta}}^{\overline{\theta}} u''\big(w(e^*,\theta)\big)(-\alpha)\big(-1-(1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta =$$

$$= \alpha \int_{\underline{\theta}}^{\overline{\theta}} \left(-\frac{u''\big(w(e^*,\theta)\big)}{u'\big(w(e^*,\theta)\big)}\right) u'\big(w(e^*,\theta)\big)\big(-1-(1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta.$$

We recognize the expression $-\frac{u''(w(e^*,\theta))}{u'(w(e^*,\theta))}$ as an Arrow-Pratt index of absolute risk aversion, which will be denoted by $A\big(w(e^*,\theta)\big)$ from now on. Hence we may write

$$\frac{\partial F}{\partial \pi} = \alpha \int_{\underline{\theta}}^{\overline{\theta}} A\big(w(e^*,\theta)\big)u'\big(w(e^*,\theta)\big)\big(-1-(1-\alpha)L_e(e^*,\theta)\big)f(\theta)d\theta. \tag{8}$$

In order to determine the sign of expression (8), we will make use the single crossing condition, a method introduced to economics by Diamond, Stiglitz (1974). Basically, it says that if the factor $\big(-1-(1-\alpha)L_e(e^*,\theta)\big)$ crosses singly the θ – axis and $A\big(w(e^*,\theta)\big)$ is monotonic in θ and has constant sign then it is possible to evaluate the sign of (8) unambiguously.

Firstly, observe that

$$\frac{\partial A\big(w(e,\theta)\big)}{\partial\theta} = A'\big(w(e,\theta)\big)w_\theta.$$

By (4), the sign of w_θ is negative, so the signs of $\frac{\partial A(w(e,\theta))}{\partial\theta}$ and $A'\big(w(e,\theta)\big)$ are opposite.

On the other hand, the sign of $A'\big(w(e,\theta)\big)$ is related to an individual risk perception of an individual. Our assumption is that the agent's preferences exhibit decreasing absolute risk aversion (DARA), hence $A'\big(w(e,\theta)\big) < 0$. Therefore the sign of $\frac{\partial A(w(e,\theta))}{\partial\theta}$ is positive and $A\big(w(e,\theta)\big)$ is increasing in θ.

Due to the risk-aversion, $A\big(w(e,\theta)\big) = -\frac{u''(w(e,\theta))}{u'(w(e,\theta))}$ is always positive. Now we are able to use method of Diamond and Stiglitz and the resut follows. ■

Unfortunately, the formulation of the above conditions itself generates certain problem. The function $L_e(e^*,\theta)$ is evaluated at the point e^*, which is unknown and it makes conditions (i) and (ii) virtually impossible to verify. Our next aim is to find verifiable sufficient conditions for insurance and self-insurance to be substitutes or complements.

By the definition, $L_e(e^*,\theta) = -\,d_e(e^*,\theta)$, hence $\big(-1-(1-\alpha)L_e(e^*,\theta)\big) = -1+(1-\alpha)d_e(e^*,\theta)$. Negative sign of the cross-derivative $d_{e\theta}$ means that the function $-1+(1-\alpha)d_e(e^*,\theta)$ is decreasing in θ, hence condition (i) follows. Analogous reasoning applies to (ii).

Obviously, the monotonicity in θ of $\big(-1-(1-\alpha)L_e(e^*,\theta)\big)$ guarantees the single-crossing conditions, hence we may formulate:

Proposition 2. Assume that individual exhibits decreasing absolute risk aversion (DARA).
(i) If $d_{e\theta} < 0$, then self-insurance is complementary to market insurance.
(ii) If $d_{e\theta} > 0$, then self-insurance is a substitute for market insurance.

Proposition 2 is then slightly weaker then Proposition 1. There is no apparent mathematical reason for function $-1+(1-\alpha)d_e(e^*,\theta)$ to be monotone in order to satisfy single-crossing condition. However, many examples of self-insurance suggest that it is usually the case. On the other hand, the condition $d_{e\theta} < 0$ is simple, verifiable and has clear economic interpretation.

Property $d_{e\theta} > 0$ is known as supermodularity of the function d, which in turn is equivalent to increasing differences notion (provided function d is twice continuously differentiable). It states that increases with regard to one variable are increasing in second variable. In general, the sign of cross-derivative $d_{e\theta}$ is intrinsically related to self-insurance technology. It reflects how self-insurance deals with the losses in different states.

4. ECONOMIC INTERPRETATION OF THE RESULTS

Single crossing condition in Proposition 1 (i) means precisely that there exists $\bar{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that $w_e = -1 + (1 - \alpha)d_e(e^*,\theta) \geq 0$ for $\theta < \theta^*$ and $w_e = -1 + (1 - \alpha) d_e(e^*,\theta) \leq 0$ for $\theta < \theta^*$. In states $\theta > \theta^*$ marginal increase in self-insurance costs more than its benefit; in that case we say that self-insurance is *ineffective*. Analogously, in states $\theta < \theta^*$ self-insurance is called *effective*. Hence single crossing condition (i) says that in good states (small θ) self-insurance is effective and in bad states (large θ) self-insurance is ineffective. For example, one can quench small fire by using fire extinguisher, but it does not help when the fire is severe. Also, bicycle helmet is effective in light accidents, but it does not help if the accident is severe.

It turns out that it is much harder to find real examples representing change from ineffectivity to effectivity of self-insurance as in (ii). If we consider hiring a lawyer as a form of self-insurance, then it may serve as an illustration for (ii). In minor cases investing in more expensive defense attorney is costly and results with small improvement. However, if losing the case means serious financial consequences then it is usually profitable to hire experienced, more expensive lawyer.

More generally, self-insurance aims often (not always) at small-scale problems and involves only small expenses. Therefore we may say that the technology of self-insurance usually has its limitations. If that is the case, then it fails at severe accidents. So it seems that (i) case is more frequent and realistic than (ii).

The economic interpretation of Proposition 1 is as follows. As before, single crossing condition (i) means that in good states self-insurance is effective and in bad states self-insurance is ineffective. In a way it is then opposite to insurance; increasing it in a good state does not benefit but increases the cost, and in the bad state it reduces the loss more than it costs. In other words, insurance is more effective in bad states than in good ones. Hence it is natural to think that self-insurance is then complementary to market insurance. On the other hand, condition (ii) works the other way around and makes self-insurance similar to market insurance. It may be easily perceived as a type of insurance. Therefore it is considered as a substitute for market insurance.

One may consider the problem from a different point of view. In the described situation, increasing price of the insurance (π) with constant insurance expenditures has the same effect as decreasing coverage of the insurance. It creates the situation of increasing underinsurance with its well-known adverse effects. The insurer then needs „more insurance”. In the case (ii), self-insurance has the same feature as the insurance. Therefore the insurer is willing to invest more in self-insurance. In the case (i) the situation is reversed. Self-insurance cannot be used as a replacement for lost part of insurance. In order to deal with increasing risk, the insurer decreases the expenditures on self-insurance.

5. CONCLUSION

With two states of the world, self-insurance and market insurance are substitutes. It turns out that this result does not extend to more general case with many states. In general setting the relation between self-insurance and market insurance becomes more complex. The key to understanding that relation is effectivity of self-insurance which reflects the technology of self-insurance. Under DARA, if in good (bad) states self-insurance is effective and in bad (good) states it is ineffective then self-insurance is complementary (substitute for) market insurance. The result presented here has its limitations: we have considered only the case of exogenous level of insurance, as it is in many cases forced by law. The general problem requires further research.

Uniwersytet Gdański

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UBEZPIECZENIE I SAMOUBEZPIECZENIE – DOBRA SUBSTYTUCYJNE CZY KOMPLEMENTARNE?

Streszczenie

Klasyczny wynik Ehrlicha i Beckera stwierdza, że samoubezpieczenie jest substytutem ubezpieczenia. Twierdzenie to zostało jednak uzyskane po analizie modelu, w którym występują tylko dwa stany świata. Niniejszy artykuł uogólnia ten model dopuszczając możliwość wystąpienia wielu stanów świata, prowadząc do wniosku, że teza twierdzenia Ehrlicha i Beckera przestaje obowiązywać w sposób bezwzględny. Artykuł poświęcony jest badaniu interakcji pomiędzy ceną obowiązkowego ubezpieczenia a popytem na samoubezpieczenie. Przedstawione zostały warunki dostateczne na to, aby samoubezpieczenie było substytutem (w specyficznie określonym sensie) lub dobrem komplementarnym względem ubezpieczenia. Zaprezentowana została także interpretacja ekonomiczna wyniku oraz jego założeń, gdzie podkreślono, że kluczową rolę dla zrozumienia badanego zjawiska odgrywa pojęcie efektywności samoubezpieczenia.

Słowa kluczowe: samoubezpieczenie, ubezpieczenie, dobra komplementarne i substytucyjne

INSURANCE AND SELF-INSURANCE – SUBSTITUTES OR COMPLEMENTS?

Abstract

Classical result by Ehrlich and Becker states that with two states of the world, market insurance and self-insurance are substitutes. However, it turns out that conclusion does not hold in the model with many states. This paper considers interactions between price of compulsory market insurance and demand for self-insurance. We present sufficient conditions for self-insurance to be complementary or substitute for market insurance. We provide economic interpretation of that result, highlighting the role of an efficiency of self-insurace as a key to understanding the phenomenon.

Keywords: self-insurance, insurance, substitution, complementarity

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REPORTS

MARIUSZ GÓRAJSKI, GRZEGORZ SZAFRAŃSKI, PIOTR WDOWIŃSKI

REPORT OF THE XI INTERNATIONAL CONFERENCE ON FORECASTING FINANCIAL MARKETS AND ECONOMIC DECISION-MAKING – FINDECON ’2014

The 11th edition of FindEcon Conference on *Forecasting Financial Markets and Economic Decision-Making* was organized on 15–16 May 2014 by the Chair of Econometrics, Institute of Econometrics at University of Łódź. This year the conference was held in post-industrial interiors of Hotel Focus in Łódź. The meeting gave the opportunity to present papers on well diversified topics starting from methodological aspects of financial modelling and macroeconomic forecasting to general reflections on 30 years of financial liberalisation.

Professor Władysław Milo acted the Chair of Programme Committee and Piotr Wdowiński (Assoc. Professor) was the Chair of Organization Committee. The other academics from the Chair of Econometrics, University of Łódź, were very active in conference preparations with dr Mariusz Górajski and dr Grzegorz Szafrański coordinating the works of Organization Committee and Programme Committee, respectively, acting as conference secretaries.

The conference was held under the patronage of the two prominent Polish institutions: National Bank of Poland (Narodowy Bank Polski) and Polish Financial Supervision Authority (Komisja Nadzoru Finansowego). The organizers also acknowledge the financial support from: Łódź Marshall Office, Faculty of Economics and Sociology at University of Łódź, CERFiN and Timberlake Consultants. The media partnership of this event was provided by Obserwator Finansowy (economics web portal) and TVP Łódź (television). Herewith we thank our sponsors and partners for ongoing support of our scientific events. They helped us to organise the event to the goodwill of all academic society.

The participants were representing ten domestic and nine foreign institutions, both of academic and financial background. The participants came from 15 different universities and 4 institutions including three central banks (National Bank of Poland, European Central Bank, Bank of Finland) and one commercial bank. They had the opportunity to take part in one invited lecture, four invited sessions and six contributed sessions. The participants presented the following topics, which were discussed after their presentations:

Invited lecture:

The inaugural lecture was given by prof. Timo Teräsvirta of Aarhus University and CREATES (Denmark). He discussed with all necessary econometric details his (yet unpublished) paper on *Specification and Testing of Multiplicative Time-Varying GARCH Models with Applications* (co-authored by Cristina Amado).

Invited sessions:

- The invited speech on *Reflections on 30 Years of Financial Liberalisation* by prof. Shanti P. Chakravarty from Bangor University (UK) was another piece of brilliant scientific reflection on the social origins of great financial crises experienced in 2007–2008. It was a thorough analysis based on the report of the most distinguished British economists in reply to the question of the British Queen asking why nobody had overseen the crises from its symptoms.
- Prof. Virmantas Račkauskas from Vilnius University (Lithuania) in his paper *Hilbert space valued GARCH with univariate volatility* (co-authored by Milda Prankevičiute) introduced new theoretical aspects of modelling volatility.
- Prof. Matti Virén from Turku University and Bank of Finland in his paper entitled *What drives loan losses in Europe?* has discussed practical aspects of macroeconomic determinants of lending activity across Europe.
- Dr Jacek Kotłowski from the Economic Institute (Deputy Director) of National Bank of Poland described from a practical perspective the forecasting process as it is introduced in NBP.

The following papers were presented during six contributed sessions (in order of appearance):

- Eliza Buszkowska (Adam Mickiewicz University in Poznań), *Forecasting the volatility of volatility with ARMA and GARCH models.*
- Witold Orzeszko (Nicolaus Copernicus University in Toruń), *An application of the NRL test to detect nonlinearity in financial time series.*
- James Sørlie (Caixa Cinzenta SA Portugal), *Grey-box Methods in Financial Markets.*
- Paweł Miłobędzki (University of Gdańsk), *The components of bid-ask spreads at the Warsaw Stock Exchange.*
- Barbara Będowska-Sójka (Poznań University of Economics), *Liquidity Needs or News Releases – What Causes Jumps on the Warsaw Stock Exchange.*
- Juliusz Jabłecki, Ryszard Kokoszczyński, Paweł Sakowski, Robert Ślepaczuk, Piotr Wójcik (University of Warsaw), *Volatility derivatives in portfolio optimization*.

- Magdalena Grothe (European Central Bank), *Market pricing of credit rating signals*.
- Vija Micune (University of Latvia), *Banking Sector in Dynamic Stochastic General Equilibrium Models.*
- Mariusz Górajski, Dominika Bogusz, Magdalena Ulrichs (University of Łódź), *Risk-sensitive optimal monetary policy rules in the Polish economy*.
- Antoni Leon Dawidowicz (Jagiellonian University), Katarzyna Brzozowska-Rup (University of Kielce), *An Online Expectation-Maximization Algorithm for Volatility Modelling*.
- Daniel Kosiorowski, Zygmunt Zawadzki (Cracow University of Economics), *Notes on optimality of predictive distribution pseudo-estimators in the CHARME models under the robust risk measures and their consequences for automatic trading strategies.*
- Sebastian Sitarz (University of Silesia), *Using the Tolerance Approach in the Market Model.*
- Agata Kliber (Poznań University of Economics), *Leverage Effect in Sovereign Credit Default Swap Spreads – Emerging Markets versus the Developed Ones.*
- Harri Ponkka (University of Helsinki), *Predicting the Direction of US Stock Markets using Industry Returns*.
- Dariusz Urban (University of Łódź), *Analysis of Investment Attractiveness of Companies Listed on Warsaw Stock Exchange for Sovereign Wealth Fund*.
- Magdalena Grothe, Jacob Ejsing, Oliver Grothe (European Central Bank), *Liquidity and credit risk premia in government bond yields.*
- Krzysztof Czerkas (CERFiN), *The foreign currency mortgage loans in the Polish banking sector and its possible macroeconomic and political consequences*.
- Grzegorz Szafrański (University of Łódź), Aleksandra Hałka (Narodowy Bank Polski), *What common factors are driving inflation in CEE countries?*

There was also the opportunity for young scientists (both graduate and PhD students) to present their work in progress during the poster session sponsored by Timberlake Consultants Ltd. The winning poster on *modelling credit risk for companies at Warsaw Stock Exchange* was presented by Artur Gądek (University of Łódź) – undergraduate student of *Business Analytics* (Faculty of Economics and Sociology, Institute of Econometrics).

Altogether different empirical research on both financial and macroeconomic topics were presented. Methodological and practical aspects of these papers were discussed. The participants were given many interesting remarks on their work in progress which will be useful in scientific work. Modelling volatility, liquidity, credit risk, inflation, monetary and macroprudential policies were among the most popular topics. They were intensively debated during regular presentations and during less formal discussions (‘off the floor’). Some of them will find their place being published in the post-conference FindEcon monograph which we plan to publish next year (see the previous volumes at http://findecon.online.uni.lodz.pl/) after reviewing the manuscripts.

Two invited lectures and the *Book of Abstracts* of other presented papers can be found on the FindEcon website (http://findecon.uni.lodz.pl/). It is also the best starting point to follow if you are interested in the next edition of FindEcon Conference we are planning in May 2016.

Mariusz Górajski, Grzegorz Szafrański, Piotr Wdowiński – University of Łódź