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CAN LOGNORMAL, WEIBULL OR GAMMA DISTRIBUTIONS IMPROVE THE EWS-GARCH VALUE-AT-RISK FORECASTS?

1. INTRODUCTION

International regulations established by the Basel Committee on Banking Supervision impose the obligation to manage the market risks, which are regarded as one of the three main risks in banking. Essential part of the risk management is its measurement. It has to be based on a Value-at-Risk in order to satisfy the basic requirement for an internal model.

According to the results obtained by researchers, it is not possible to determine the best method of measuring Value-at-Risk that would allow to achieve the best forecasts of Value-at-Risk in every situation. Therefore, the analysis of the quality of a Value-at-Risk forecasts generated on the basis of different models is a topic widely discussed in the literature (among others, in Engle, 2001; 2004; Tagilafichi, 2003; Alexander, Lazar, 2006; Angelidis et al., 2007; Engle, Manganelli, 2001; McAleer et al., 2009; Marcucci, 2005; Ozun et al., 2010; Dimitrakopoulos et al., 2010; Brownlees et al., 2011; Degiannakis et al., 2012 and Abad et al., 2014).

Moreover, McAleer et al. (2009) and Degiannakis et al. (2012) showed that different models may be better during tranquil or turbulent periods. In both cases, simple GARCH model was good for Value-at-Risk forecasting during a pre-crisis 2007–2009 period, but its quality significantly decreased during and after the crisis. McAleer et al. (2009) showed that RiskMetrics™ was the best model during the crisis but EGARCH-t model was better after the crisis. Whereas in the study of Degiannakis et al. (2012) APARCH with a skewed Student's t distribution was the best model during the crisis. These results show that less conservative models are best in tranquil periods, while during the crisis models that consider the distributions of returns with fatter tails are better. Degiannakis et al. (2012) stated that these claims are valid for both developed and developing countries.

Despite the conclusions drawn from the aforementioned articles, the use of regime switching models in Value-at-Risk forecasting has a rather niche character; it has been considered, among others, by Hamilton, Susmel (1994), Cai (1994), Gray (1996), Alexander, Lazar (2006) and McAleer, Chan (2002). A characteristic trait of the pro-

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posed models is that losses come from the same distribution but with different parameters, in all states. This feature contradicts the findings stated in McAleer et al. (2009) and Degiannakis et al. (2012), where models with different distributions were found to be the best in different states.

In order to fill this gap, an EWS-GARCH models were presented in Chlebus (2016b). In these models, the Value-at-Risk forecasts are calculated in two steps. First, a state of the portfolio is forecasted (a state of tranquillity or a state of turbulence – the approach is analogous to Early Warning System (EWS) models for crisis prediction) and then, depending on the forecasted state, a different model is used to forecast the Value-at-Risk. The EWS-GARCH models give the opportunity to use models to forecast Value-at-Risk in the state of tranquillity assuming a distribution of returns with relatively thinner tails, and in the state of turbulence, models with much more conservative assumptions.

In the aforementioned study, a GARCH(1,1), or a GARCH(1,1) with the amendment to empirical distribution of random error, were considered as a Value-at-Risk forecasting model in the state of tranquillity; whereas exponential, empirical or Pareto distributions were considered in the state of turbulence. The obtained results were promising and showed that the EWS-GARCH models concept may provide Value-at-Risk forecasts of very good quality. However, a lot of aspects remain in which the EWS-GARCH models may be improved.

The aim of the study is to examine whether incorporation of lognormal, Weibull or Gamma distributions in the Value-at-Risk forecasting model (in the state of turbulence), instead of distributions used previously, may increase a quality of the Value-at-Risk forecasts. The use of these distributions in Value-at-Risk forecasting is a practice met in an operational risk measurement (see Panjer, 2006). They may be considered as distributions in the state of turbulence, as all of them may have tail shape (when specific values of parameters assumed).

The lognormal, the Weibull or the Gamma distribution were compared to each other and with benchmark models: the GARCH(1,1), the GARCH(1,1) with the amendment to empirical distribution of random error, an EGARCH(1,1), a GARCH-t (1,1) (model was parametrised assuming unit variance and the number of the degrees of freedom greater than 2), and the EWS-GARCH(1,1) models with the exponential or the empirical distributions; in order to assess the quality of the Value-at-Risk forecasts obtained from the EWS-GARCH models. The evaluation of the quality of the Value-at-Risk forecasts was based on the Value-at-Risk forecasts adequacy (an excess ratio, a Kupiec test, a Christoffersen test, an asymptotic test of unconditional coverage and a backtesting criteria defined by the Basel Committee – both for Value-at-Risk and Stressed Value-at-Risk) and the analysis of loss functions (a Lopez quadratic loss function, an Abad & Benito absolute loss function, a 3rd Caporin loss function and an excessive cost function).

The paper is organized as follows: in the first section an EWS-GARCH models framework is discussed, in the second section a testing framework is presented, and in the third section an empirical verification of the Value-at-Risk forecasts obtained from the EWS-GARCH models with the lognormal, the Weibull or the Gamma distribution is analysed.

2. EWS-GARCH MODELS

At the beginning a brief definition of Value-at-Risk ($VaR_\alpha(t)$) should be presented. The Value-at-Risk may be defined as a value that a loss would not excess with a certain probability α within a specified period of time in normal market situation. Value-at-Risk can be defined as follows (Engle, Manganelli, 1999):

$$P(r_t < VaR_\alpha(t)|\Omega_{t-1}) = \alpha, \tag{1}$$

where r_t is a return at time t, $VaR_\alpha(t)$ is Value-at-Risk at time t and Ω_{t-1} is a set of information available at time t–1.

A Value-at-Risk forecasting procedure based on the EWS-GARCH models consists of two steps. In the first step, the state of time series for the next day is forecasted, then in the second step a Value-at-Risk for the next day is forecasted. The Value-at-Risk forecast is provided from an appropriate model regarding the state forecasted in the first step.

In the EWS-GARCH models it is proposed that the prediction of the state should be carried out by a model for binary dependent variable: logit, probit or cloglog models. Each of these models can be defined in a similar manner differing only in regard of a random error distribution. The logit model assumes a logistic distribution, the probit model a normal distribution, and the cloglog – a Gompertz distribution of random errors. These models can be defined as follows (Allison, 2005):

$$y_t^* = \boldsymbol{\beta X}_t + \varepsilon_t, \tag{2}$$

$$y_t = \begin{cases} 1 & y_t^* > 0, \\ 0 & y_t^* \leq 0, \end{cases} \tag{3}$$

where y^* is a latent dependent variable, $\boldsymbol{\beta}$ is a vector of parameters describing the relationship between independent variables and unobserved dependent variable, $\boldsymbol{X_t}$ is a vector of observations of independent variables that have an impact on an unobservable dependent variable, ε_t is a random error coming from the relevant distribution, and y_t is observable result of the modelled phenomenon. All aforementioned models are estimated using maximum likelihood estimators.

In the process of forecasting the state of turbulence, the y_t is equal to 1 for a certain percentage of the lowest observed returns (5% or 10%). Independent variables in the model describe a current situation on stock, exchange rates and short-term interest rates markets (prices and returns, 15-day moving averages of prices and returns and

15-days moving variances of prices and returns of Warsaw Stock Exchange Indices – WIG & WIG20, of most important to polish market exchange rates – EUR/PLN, USD/PLN and CHF/PLN, and of short-term interest rates – overnight and 3-month WIBOR). Moreover, a selection of an optimal cut-off point for the event forecast is considered (set up to 5% and 10% for the 5% and 10% definitions of y_t relevantly) to achieve the best possible forecasts quality. The choice of models for binary variable, the definition of the observable dependent variable, the choice of independent variables and the optimal cut-off threshold have been established in accordance with the results obtained in the study of Chlebus (2016a). Additionally, a set of independent variables will be limited only to variables statistically significant at the 5% significance level selected by a stepwise selection method.

The model to predict a state gives the opportunity to distinguish two states (the state of tranquillity and the state of turbulence) in a time series, which can vary considerably in their nature (with respect to expected returns, volatility etc.). In each state different models to forecast Value-at-Risk should be used in order to take into account different specificities of these two states. In the EWS-GARCH models a tail distribution is used only when the state of turbulence is forecasted, otherwise the entire distribution is used. During the study it is assumed that the dependent variable in the Value-at-Risk models is a continuous one-day rate of return, which may be expressed as $r_t = (\ln(p_t) - \ln(p_{t-1})) * 100$.

In the state of tranquillity, the considered Value-at-Risk forecasting models were: the GARCH(1,1) and the GARCH(1,1) with amendment to empirical distribution of random error. The GARCH(1,1) model can be written as:

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t \xi_t, \tag{4}$$

where r_t is a return on assets analysed at time t, μ_t is a conditional mean (assumed in the study to be constant – no independent variables included), ε_t is a random error in time t and ε_t can be expressed as the product of the conditional standard deviation σ_t and standardized random error ξ_t at time t, which satisfies the assumption $\xi_t \sim$ i.i.d.(0, 1). The equation of conditional variance in the GARCH(1,1) can be written as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{5}$$

where ω is a constant which satisfies the assumption $\omega > 0$, α_1 and β_1 are parameters that satisfy the assumptions $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. The GARCH(1,1) model is estimated using the maximum likelihood method.

For the GARCH(1,1) Value-at-Risk for the long position is estimated based on the following formula (Abad, Benito, 2013):

$$\mathrm{VaR}_\alpha(t) = \widehat{\mu_t} + k_\alpha * \sqrt{\hat{\sigma}_t^2}, \tag{6}$$

where $VaR_\alpha(t)$ is a forecast of Value-at-Risk on α tolerance level at time t, $\widehat{\mu_t}$ is a forecast of conditional mean at time t, k_α is a value of quantile α from assumed random error distribution and $\hat{\sigma}_t^2$ is an forecast of conditional variance at time t.

The Basel Committee requirements state that the Value-at-Risk should be estimated with the 99% confidence level (the α is assumed to be equal to 1%). The Value-at-Risk forecast from the GARCH(1,1) with the amendment to empirical distribution of random error (Engle, Manganelli, 2001) is obtained in a similar manner as in the GARCH(1,1); the difference lies in the use of a quantile from the empirical distribution of residuals instead of a quantile from the normal distribution.

In the state of turbulence, the lognormal, the Weibull or the Gamma distributions are considered. The lognormal distribution is uniquely determined by two parameters. A cumulative distribution function (cdf) can be written as:

$$F_{LN}(x) = \Phi\left(\frac{ln\,x-\mu}{\sigma}\right), \tag{7}$$

where $\Phi(x)$ is the standard normal distribution cdf, μ is a location parameter and σ is a shape parameter.

The second possible distribution is the Weibull distribution, which is a generalization of the exponential distribution. It is extended by a scaling parameter τ. In case where the parameter τ is equal to 1, the Weibull distribution reduces to the exponential distribution. The cdf can be written as:

$$F_{WEI}(x) = 1 - e^{-(x/\theta)^\tau}, \tag{8}$$

where θ is a scale parameter and τ is a shape parameter.

Last considered distribution is the Gamma distribution. The cdf can be written as:

$$F_{GAM}(x) = \frac{\gamma(\alpha;x/\theta)}{\Gamma(\alpha)}, \tag{9}$$

where $\gamma(\alpha;\ x/\theta)$ is the incomplete Gamma function, $\Gamma(\alpha)$ is the Gamma function, θ is a scale parameter and α is a shape parameter. In case when the α is a natural number, the Gamma distribution can be interpreted as the sum of exponentially distributed random variables. The formulation of the exponential distribution may be found in Chlebus (2016b). All aforementioned distributions are fitted using maximum likelihood estimators.

For the tail distributions Value-at-Risk is forecasted simply as a value of the quantile of the distribution. A problem in this case is the determination which quantile of the distribution provides the confidence level equal to 99%. Two quantiles are considered: the 99^{th} percentile of the tail returns distribution (conservative assumption) and for the 10% definition of the state of turbulence the 90^{th} percentile of the tail distribution and accordingly, for the 5% definition of the state of turbulence the 80^{th} percentile of the tail distribution (liberal assumption).

The two-stage nature of the EWS-GARCH models forecasts two elements: the state of turbulence, and the Value-at-Risk. Forecasts of the state and the Value-at-Risk at time $t+1$ are based on data available at time t. A data set to forecast the states is prepared using the recursive window approach. Data set for Value-at-Risk forecasting is prepared using the rolling window approach (the window width was set to 1004 observations, which corresponds to about 4 years of one day returns).

3. TESTING FRAMEWORK

Performing a thorough analysis of the quality of EWS-GARCH models requires the development of multi-aspect testing process. Tests of the adequacy of the Value-at-Risk forecasts and the loss functions analysis were carried out in order to confirm the quality of Value-at-Risk forecasts and comparisons of the models in terms of their quality.

As a part of the Value-at-Risk forecasts adequacy assessment, analyses of the following were performed: the excess ratio comparison, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage, and the backtesting criterion specified by the Basel Committee (see BCBS; 2006). The excess ratio and the backtesting criterion was analysed for the Value-at-Risk and the Stressed Value-at-Risk (a measure defined by the Basel Committee in the BCBS (2011)).

The excess ratio may be calculated as:

$$ER = \frac{\sum_{t=1}^{N} 1_{r_t < VaR_t}}{N}, \tag{10}$$

where N is a number of the Value-at-Risk forecasts and $1_{r_t < VaR_t}$ is a number of cases when a realized rate of return is smaller than a forecasted Value-at-Risk.

Using the excess ratio each of the Value-at-Risk models can be assigned to one of the Basel backtesting criterion zones – green, yellow or red. The Basel Committee requires comparing the quality of the models based on the Value-at-Risk forecasts results, however it is also worth to consider the quality of the models with regards to the Stressed Value-at-Risk. For this purpose, the worst excess ratio (from the set of 250 consecutive days with the highest excess ratio) from the out-of-sample was calculated. The result shows how the model works in the worst possible conditions observed. Analogously to the Value-at-Risk forecasts, in this case the excess ratio can be attributed as well to one of the backtesting zones defined by the Basel Committee.

The analysis of the backtesting zones has a one-tailed character. An important issue missing from this analysis is the negative assessment of the model forecasts due to excessive conservatism. In the backtesting, a model that does not identify any exceedances of the Value-at-Risk is assessed as very good (the green zone), although the expected and observed number of exceedances differ significantly. In order to assess the quality of forecasts from the perspective of both underestimation and overestimation of Value-at-Risk forecasts, among other, coverage tests are used.

The most popular test of this type is the Kupiec test (also called the unconditional coverage test) (see Kupiec, 1995). The idea of the test is based on a comparison of expected and observed numbers of Value-at-Risk exceedances. The test statistic comes from the asymptotic distribution of χ^2 with 1 degree of freedom and can be written as:

$$LR_{UC} = 2[ln(\hat{\alpha}^X(1-\hat{\alpha})^{N-X}) - ln(\alpha^X(1-\alpha)^{N-X})] \sim \chi^2_a(1), \tag{11}$$

where α is an expected excess ratio (according to the Basel Committee requirements it should be 1%), $\hat{\alpha}$ is an observed excess ratio, X is an observed number of Value-at-Risk exceedances and N is a number of Value-at-Risk forecasts. In the null hypothesis it is assumed that the expected and observed excess ratio is equal to each other. In contrast to the backtesting criterion, the Kupiec test identify models that both underestimate and overestimate Value-at-Risk, however there is no straightforward method to assess whether the analyzed model tends to overestimate or underestimate Value-at-Risk forecasts. Such an analysis is possible based on a backtesting criterion statistics, also called an asymptotic test of unconditional coverage (see Abad et al., 2014). The backtesting criterion statistics come from the asymptotic standard normal distribution. This test is two-tailed. Strongly negative values of the test statistics indicate overestimation of the Value-at-Risk forecasts, while strongly positive, underestimation of these forecasts. The test statistic can be calculated according to the following formula:

$$z_{BT} = \frac{(N\hat{\alpha} - N\alpha)}{\sqrt{N\alpha(1-\alpha)}} \sim N(0,1), \tag{12}$$

where α is an expected excess ratio, $\hat{\alpha}$ is an observed excess ratio and N is a number of Value-at-Risk forecasts.

The Christoffersen test (the conditional coverage test) proposed by Christoffersen (1998) is an extension of the Kupiec test. This test extends the Kupiec test by inclusion of an independency of Value-at-Risk exceedances testing. The test statistic comes from the asymptotic χ^2 distribution with 2 degrees of freedom and can be formulated as:

$$LR_{CC} = LR_{UC} + LR_{IND} \sim \chi^2_a(2), \tag{13}$$

where LR_{UC} is the Kupiec test statistics and LR_{IND} is an independency of exceedances statistics. The LR_{IND} is equal to

$$2[\ln((1-\pi_{01})^{N_{00}}\pi_{01}{}^{N_{01}}(1-\pi_{11})^{N_{10}}\pi_{11}{}^{N_{11}}) - \ln((1-\hat{\alpha})^{N_{00}+N_{10}}\hat{\alpha}^{N_{01}+N_{11}})],$$

where $\hat{\alpha}$ is an observed excess ratio, N_{ij} is a number of observation for which a state j (exceedance or not exceedance) is observed under condition that a state i (exceedance or not exceedance) was observed in the previous period, π_{01} is a probability of observing Value-at-Risk exceedances conditional on not observing them in the previous period and π_{11} is a probability of observing Value-at-Risk exceedances conditional on observing them in the previous period.

The tests presented above allow to evaluate Value-at-Risk models based on the adequacy of its forecasts. Additionally, an analysis of the cost (loss) compares on the one hand the losses resulting from exceeding the Value-at-Risk, and on the other hand, accuracy and cost efficiency of the used models. The cost (loss) functions analysis are not formal tests, during the analysis the score is calculated which allows to compare the Value-at-Risk models with each other.

The first cost (loss) function considered is the quadratic Lopez function (see Lopez, 1999), which may be defined as:

$$CL_t = \begin{cases} 1 + (r_t - VaR_\alpha(t))^2 & for \ \ r_t < VaR_\alpha(t), \\ 0 & for \ \ r_t \geq VaR_\alpha(t), \end{cases} \tag{14}$$

where r_t is a realised rate of return at the moment t and VaR_t is a Value-at-Risk forecast for the same moment t. The score is calculated as $\sum_{t=1}^{N} CL_t$ (where N is a number of Value-at-Risk forecasts). The Lopez function considers two aspects of Value-at-Risk forecasts: a number and a severity of exceedances. Each exceedance increase a score by at least 1, where the excess over 1 is calculated with respect to its severity and is calculated as $(r_t - VaR_t)^2$. The main disadvantage of the Lopez quadratic function is that it does not give an easy interpretation. The solution may be a function proposed by Abad, Benito (2013), which can be written as:

$$CA_t = \begin{cases} |r_t - VaR_\alpha(t)| & for \ \ r_t < VaR_\alpha(t), \\ 0 & for \ \ r_t \geq VaR_\alpha(t). \end{cases} \tag{15}$$

In this case a score is calculated as an average of severity of exceedances with respect to a number of Value-at-Risk forecasts considered, which can be calculated as $\sum_{t=1}^{N} CA_t/N$. This loss function differs from the previous one in two basic dimensions. Firstly, an average is minimized instead of the sum, therefore the number of exceedances is not taken into account. This may cause models with a larger number of exceedances to be preferred. Secondly, absolute deviation is analyzed, which makes the interpretation easier.

Both aforementioned functions consider non-zero values only in the case of exceedance. From a perspective of use of Value-at-Risk models in a financial institutions, it is reasonable to consider also cost (loss) functions that take into account the costs associated with both exceedances and lack of exceedances (opportunity costs). First considered function of this type is a function presented by Caporin (2008). In his study, he proposed three different cost functions, which assume that a cost of deviations of a forecasted Value-at-Risk from a realized rate of return is equally important regardless of whether the exceedance was observed or not. In the study the following cost function is considered:

$$CC_t = \begin{cases} |r_t - VaR_\alpha(t)| & for \ \ r_t < VaR_\alpha(t), \\ |r_t - VaR_\alpha(t)| & for \ \ r_t \geq VaR_\alpha(t). \end{cases} \tag{16}$$

Caporin proposes that in order to compare the Value-at-Risk forecasts, a sum of all CC_t should be used, however in the study the average of these values is considered. Both analyzes lead to similar conclusions, but the average can be interpreted as the average absolute error of the Value-at-Risk forecasts.

Additionally, an absolute excessive cost functions proposed in Chlebus (2016b) were analysed. The absolute excessive cost function, like the Caporin loss function, includes costs either in the case of the Value-at-Risk exceedance or lack of exceedance. The difference is that the analysis is focused rather on the excessive cost of the use of the model than precision of the forecast. Therefore, the process of assigning point values is divided into three cases and focuses precisely on the costs made by the model:

$$CAE_t = \begin{cases} |VaR_\alpha(t)| & for\ r_t \geq VaR_\alpha(t)\ and\ r_t \geq 0, \\ |VaR_\alpha(t) - r_t| & for\ r_t \geq VaR_\alpha(t)\ and\ r_t < 0, \\ |r_t| & for\ \quad r_t < VaR_\alpha(t). \end{cases} \tag{17}$$

Value-at-Risk models should be compared in terms of mean value of excessive cost function for the analysed number of forecasts $\overline{CAE} = \frac{\sum_{t=1}^{N} CAE_t}{N}$. The $\overline{CAE}$ may be interpreted as a measure of excessive model conservatism. The higher the $\overline{CAE}$ is, the more conservative the model is, which means that the model predicts on average more conservative Value-at-Risk than needed to cover losses arising from changes in a value of analysed assets.

The variety of Value-at-Risk forecast quality methods gives an opportunity to assess models form many different perspectives and thoroughly compare them. The empirical assessment of the quality of Value-at-Risk forecasts based on EWS-GARCH models with lognormal, Weibull and Gamma distribution are presented in the next section.

4. EMPIRICAL RESULTS

4.1. DATA

The quality of Value-at-Risk forecasts obtained from the EWS-GARCH models was analysed for 79 time series of returns of companies listed on the Warsaw Stock Exchange (a detailed list available upon request). Assets were selected randomly. Only one condition was imposed on the drawing process, that the shares have been listed on the Warsaw Stock Exchange since at least January 2006. It is a technical requirement intended to ensure the best possible quality of data used for modelling and similarity of sample for each company.

The empirical study was performed for the series of returns from the 1st January 2006 to 31st January 2012. The period from the beginning of 2006 to the end of 2009 constituted the original estimation sample; the forecast sample started from the beginning of 2010 and ended in 2012, thereby giving 525 predictions of the Value-at-Risk for each asset.

All considered models used to forecast the Value-at-Risk have been developed in such a way as to meet the requirements set by the Basel Committee for internal models of the market risk measurement. The measure of market risk is based on the one-day Value-at-Risk predictions satisfying the 99% confidence level. For the quality of Value-at-Risk forecasts only one-day predictions are required and sufficient. The assessment was carried out for 525 observations, which is more than expected in the Basel regulations of the minimum equal to 250 observations.

4.2. RESULTS

In the study, analogously to the practice used in the literature, the EWS-GARCH models are evaluated and compared on the basis of the Value-at-Risk forecasts quality, so the quality of states forecasts is not discussed in detail. Nevertheless, it is worth noting that the models for state of turbulence estimated in accordance with the procedure discussed earlier provide a good quality forecasts, as confirmed by the results obtained by Chlebus (2016a).

The discussion of the results for the EWS-GARCH model was divided into two parts. In the first part, results for the EWS-GARCH model with the GARCH(1,1) were presented, and in the second part were the results for the GARCH(1,1) with the amendment to empirical distribution of random error as a model in the state of tranquillity. In order to maintain transparency of the results, a crossover comparison between models of different EWS-GARCH groups (with different state of tranquillity models) was omitted. Additionally, results in this paper for an EWS-GARCH model with particular state of tranquillity and particular state of turbulence VaR forecasting models are presented only for one (with the lowest excess ratio) state of turbulence model. It means that even though in every case Probit, Logit and Cloglog with and without stepwise selection process were considered only best results are presented. All calculations and estimations were performed in SAS 9.4.

4.2.1. VALUE-AT-RISK FORECASTS QUALITY – THE EWS-GARCH(1,1) MODELS

The evaluation of the Value-at-Risk forecasts quality for the EWS-GARCH models began with the EWS-GARCH(1,1) models. Results for the EWS-GARCH(1,1) are presented in two tables. In table 1, results of the Value-at-Risk exceedances and the cost functions are presented, in table 2 results of the coverage tests are presented (same division was made for EWS-GARCH(1,1) with the amendment to empirical error distribution). In the tables only results for models that have lower excess ratio than the GARCH(1,1) are presented.

The GARCH-t(1,1) model is the model with the lowest excess ratio: it has the excess ratio equal to 0.24%, much below expected 1%. After this model, a group of models with the excess ratio smaller (between 0.84% and 0.96%) than 1% may be identified. Those models are: the GARCH(1,1) with the amendment to empirical distribution of random error, and the EWS-GARCH(1,1) models with conservative definition of Value-at-Risk quantile in the state of turbulence. Among the aforementioned EWS-GARCH(1,1) models more conservative are: models assuming the exponential or the empirical distribution, than models assuming the lognormal, the Weibull or the Gamma distributions; and models with the 10% definition of the state of turbulence then models with the 5% definition.

The EWS-GARCH(1,1) models with the liberal definition of Value-at-Risk quantile are generally less conservative and have the excess ratio higher or equal to 1%; the only exception is the EWS-GARCH(1,1) with the exponential distribution, which is rather conservative (the excess ratio equal to 0.89%).

Among the EWS-GARCH models with the lognormal, the Weibull or the Gamma distribution, the most conservative are models with the Weibull distribution; the only exception is the model with a conservative approach defining quantile to forecast Value-at-Risk and the 5% definition of the state of turbulence.

It can also be seen that the Lopez and the Abad and Benito loss functions generally decrease with lowering excess ratio. The EWS-GARCH(1,1) models with the lognormal, the Weibull or the Gamma distributions have higher values of these functions in comparison to models with the exponential or the empirical distributions.

Improvement in the excess ratio and the costs associated with the occurrence of exceeding (expressed by the Lopez and the Abad and Benito cost functions), is associated with an increase in the costs of the model used (expressed by the values of the Caporin and the excess costs functions). The increase in the cost of use of models is growing steadily along with the decrease of the excess ratio. Exceptions are models in which the Value-at-Risk was calculated as the 99^{th} percentile of the exponential, or the Gamma distributions at the 5% definition of the state of turbulence, in which case the increase of the cost of model is significant. It is also worth mentioning that EWS-GARCH models with the lognormal, the Weibull or the Gamma distributions cost less in comparison to models with the exponential or the empirical distributions used to forecast Value-at-Risk in the state of turbulence.

Regarding the Basel Committee backtesting procedure, it can be seen that all models characterized by the lower excess ratio than 1% were assigned to the green zone more than in 90% of cases. Most often the GARCH-t(1,1) (in 98.7% cases) and the GARCH(1,1) with the amendment to empirical distribution of random error (94.9%) were assigned to the green zone. The EWS-GARCH(1,1) models with conservative definition of Value-at-Risk quantile in the state of turbulence and the lognormal, the Weibull or the Gamma distribution were assigned to the green zone in 92.4% cases (the only exception is model with the Gamma distribution and the 10% definition of the state of turbulence). Slightly different results may be found when analysing assignation to at least the yellow zone. In this case, not only the GARCH-t(1,1) has the highest rate (equal to 98.7%), but the EWS-GARCH(1,1) models with the 10% definition of the state of turbulence and the exponential distribution and the EWS-GARCH(1,1) models with the 5% definition of the state of turbulence for any distribution, including the lognormal, the Weibull or the Gamma distribution have it as well. This result is interesting, because models with the lognormal, the Weibull or the Gamma distributions (which are less conservative) are of the same quality (regarding being at least in the yellow zone) as the GARCH-t(1,1) and better than the GARCH(1,1) with the amendment to empirical distribution of random error.

Analysing results for the Stressed Value-at-Risk, again the GARCH-t(1,1) model is the most often assigned to the green and at least the yellow zone (97.5% and 98.7% respectively). Rest of the models drop its quality in terms of the green zone assignment, but keep its quality in terms of being assigned to at least the yellow zone. Again, models with the 5% definition of the state of turbulence, including models with the lognormal, the Weibull or the Gamma distribution are of good quality and are assigned to at least the yellow zone in 93.7% cases.

Analysing results of the coverage tests it can be seen that the smallest rejection rate in the Kupiec test have the EWS-GARCH(1,1) models with the 5% definition of the state of turbulence, the conservative definition of Value-at-Risk quantile and with one out of the lognormal, the Weibull or the Gamma distributions. According to the results of the Christoffersen test, they are not the best but still of good quality (the best is the GARCH(1,1) with the amendment to empirical error distribution).

Very interesting conclusion may be drawn from the asymptotic unconditional coverage test, as this test is two-tailed, and because of that both the overestimation and the underestimation of the Value-at-Risk forecasts may be considered as a reason of rejection of the null hypothesis. According to the obtained test results, it may be stated that for the models with the 5% definition of the state of turbulence, the conservative definition of Value-at-Risk quantile and one out of the lognormal, the Weibull or the Gamma distributions rejections of the null hypothesis due to either the overestimation or the underestimation are on similar level and close to expected (5% for each tail). The Value-at-Risk forecasts from the EWS-GARCH models with the 10% definition of the state of turbulence, the conservative definition of Value-at-Risk quantile and one out of the lognormal, the Weibull or the Gamma distributions are rejected slightly more often, mainly because of the overestimation of forecasts. The models with the liberal definition of Value-at-Risk quantile in the state of turbulence are too liberal and lead to rejection rate due to the underestimation of Value-at-Risk much more often than expected. It should be also stated, that the GARCH-t(1,1) model is far too much conservative and rejected by all the formal tests in most of the cases.

The results obtained for the EWS-GARCH(1,1) with the lognormal, the Weibull or the Gamma distributions in the state of turbulence show that this models provides the Value-at-Risk forecasts of good quality. Taking all the results into account, it seems that the most appropriate are models with the 5% definition of the state of turbulence and the conservative definition of Value-at-Risk quantile in the state of turbulence. They maintain a good balance between conservatism (relatively low excess ratio, low values of the Lopez function and the Abad and Benito function, and relatively high qualification rate to the green zone, and at least the yellow zone in the backtesting procedure) and adequacy (the coverage tests) of the Value-at-Risk forecast. Additionally, regarding the Caporin and the excess cost functions using aforementioned models is relatively not expensive (an exception is the model with the Gamma distribution assumed). All three models (either with the lognormal, the Weibull or the Gamma distributions) exhibit similar quality of the Value-at-Risk forecasts, however among them the most appropriate seems to be the model with the lognormal distribution: it is relatively conservative, with relatively small cost of use.

In the end it is also worth mentioning that the GARCH-t(1,1) model is far too conservative, and in contrast the GARCH(1,1) with the amendment to empirical distribution of random error is very good and in many aspects the best from the analysed models. The GARCH(1,1) model seems to be too liberal, even if used only in the state of tranquillity (it leads to slightly too excessive number of Value-at-Risk exceedances). According to that, it is worth analysing of what quality the Value-at-Risk forecasts provided by the EWS-GARCH(1,1) with the amendment to empirical distribution models would be, as the GARCH(1,1) with the amendment to empirical distribution of random error model is slightly more conservative than the GARCH(1,1) model.

4.2.2. VALUE-AT-RISK FORECASTS QUALITY – THE EWS-GARCH(1,1) WITH THE AMENDMENT TO EMPIRICAL DISTRIBUTION OF RANDOM ERROR MODELS

The results with respect to the exceedances and the cost functions for the EWS-GARCH(1,1) with the amendment to empirical error distribution models are shown in table 3. Results of the coverage tests are presented in table 4.

For the EWS-GARCH(1,1) with the amendment to empirical error distribution only the results of models that improve (reduce) the excess ratio will be discussed. The GARCH(1,1) with the amendment to empirical error distribution is a conservative model itself – the excess ratio on average is smaller than the expected 1%. According to that, choosing EWS-GARCH(1,1) with the amendment to empirical error distribution models that provide the excess ratio closer to 1% than the GARCH(1,1) with the amendment to empirical error distribution, would lead to the choice of models with smaller conservatism than the GARCH(1,1) with the amendment to empirical error distribution in the state of turbulence, which is not a purpose of the EWS-GARCH models development and, therefore, will not be discussed.

As noted above, the GARCH(1,1) with the amendment to empirical error distribution is on average conservative. The average excess ratio is equal to 0.88%. Therefore, reducing excess ratio requires a relatively conservative approach to be used in the state of turbulences. It is possible for all models, assuming the Value-at-Risk is equal to the 99^{th} percentile of a distribution in the state of turbulence. Additionally, reduction

Table 1.

The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1) models

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	YELLOW	RED	EN	ER	GREEN	YELLOW	RED
-	GARCH-t	-	1.25	0.24%	6.3%	2.46	12.5%	11.6%	98.7%	98.7%	1.3%	1.03	0.4%	97.5%	98.7%	1.3%
PROBIT	GARCH	EX9_10	4.39	0.84%	8.4%	4.51	11.1%	10.3%	93.7%	98.7%	1.3%	3.56	1.4%	72.2%	93.7%	6.3%
PROBIT	GARCH	EM9_10	4.58	0.87%	8.8%	4.71	8.7%	7.8%	92.4%	97.5%	2.5%	3.71	1.5%	68.4%	92.4%	7.6%
-	GARCH EMP	-	4.61	0.88%	9.2%	4.67	7.2%	6.4%	94.9%	97.5%	2.5%	3.73	1.5%	68.4%	96.2%	3.8%
PROBIT	GARCH	EX0_10	4.67	0.89%	9.0%	4.80	7.9%	7.1%	92.4%	96.2%	3.8%	3.80	1.5%	65.8%	91.1%	8.9%
CLOGLOG	GARCH	EX9_5	4.76	0.91%	9.0%	4.82	17.1%	16.3%	93.7%	98.7%	1.3%	3.81	1.5%	69.6%	93.7%	6.3%
PROBIT	GARCH	WE9_10	4.82	0.92%	9.5%	4.95	7.7%	6.9%	92.4%	96.2%	3.8%	3.92	1.6%	62.0%	89.9%	10.1%
PROBIT	GARCH	LN9_10	4.86	0.93%	9.4%	4.93	7.6%	6.8%	92.4%	96.2%	3.8%	3.99	1.6%	62.0%	89.9%	10.1%
CLOGLOG	GARCH	EM9_5	4.87	0.93%	9.3%	4.94	8.6%	7.8%	92.4%	98.7%	1.3%	3.92	1.6%	64.6%	93.7%	6.3%
PROBIT	GARCH	GM9_10	4.95	0.94%	9.6%	5.02	7.5%	6.7%	89.9%	96.2%	3.8%	4.05	1.6%	60.8%	88.6%	11.4%
CLOGLOG	GARCH	LN9_5	5.01	0.95%	9.6%	5.02	7.7%	6.9%	92.4%	98.7%	1.3%	4.03	1.6%	62.0%	93.7%	6.3%
CLOGLOG	GARCH	GM9_5	5.04	0.96%	9.6%	5.04	13.1%	12.3%	92.4%	98.7%	1.3%	4.05	1.6%	62.0%	93.7%	6.3%
CLOGLOG	GARCH	WE9_5	5.06	0.96%	9.6%	5.07	7.8%	7.0%	92.4%	98.7%	1.3%	4.06	1.6%	62.0%	93.7%	6.3%
CLOGLOG	GARCH	EX8_5	5.23	1.00%	10.1%	5.23	7.3%	6.5%	91.1%	96.2%	3.8%	4.23	1.7%	58.2%	89.9%	10.1%
CLOGLOG	GARCH	WE0_10	5.91	1.13%	12.1%	5.92	6.9%	6.1%	82.3%	94.9%	5.1%	4.92	2.0%	54.4%	78.5%	21.5%
CLOGLOG	GARCH	WE8_5	5.94	1.13%	12.1%	5.94	6.9%	6.1%	86.1%	92.4%	7.6%	4.85	1.9%	48.1%	83.5%	16.5%
CLOGLOG	GARCH	EM0_10	6.01	1.15%	12.6%	6.02	6.8%	6.0%	82.3%	92.4%	7.6%	4.99	2.0%	50.6%	79.7%	20.3%

CLOGLOG	GARCH	GM8_5	6.14	1.17%	12.7%	6.15	6.8%	6.0%	79.7%	92.4%	7.6%	5.04	2.0%	48.1%	81.0%	19.0%
CLOGLOG	GARCH	LN8_5	6.20	1.18%	13.0%	6.21	6.8%	6.0%	79.7%	92.4%	7.6%	5.09	2.0%	45.6%	81.0%	19.0%
CLOGLOG	GARCH	GM0_10	6.27	1.19%	13.0%	6.27	6.8%	6.0%	79.7%	91.1%	8.9%	5.22	2.1%	44.3%	75.9%	24.1%
CLOGLOG	GARCH	EM8_5	6.29	1.20%	13.2%	6.30	6.7%	5.9%	81.0%	91.1%	8.9%	5.15	2.1%	43.0%	75.9%	24.1%
CLOGLOG	GARCH	LN0_10	6.35	1.21%	13.3%	6.36	6.7%	5.9%	79.7%	91.1%	8.9%	5.30	2.1%	43.0%	75.9%	24.1%
-	GARCH	-	6.42	1.22%	12.5%	6.42	6.6%	5.8%	78.5%	93.7%	6.3%	5.18	2.1%	39.2%	78.5%	21.5%
-	EGARCH	-	6.53	1.24%	12.5%	6.54	6.7%	5.9%	78.5%	92.4%	7.6%	5.19	2.1%	40.5%	81.0%	19.0%

In the table, white fields refer to the EWS-GARCH models with lognormal, Weibull or Gamma distributions, while grey fields to benchmark models.
The following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM – the Value-at-Risk forecasting model in a state of turbulence, EN - the average number of exceedances, ER – the average excess ratio, ABAD – the average value of the Abad & Benito cost function, LOPEZ – the average value of the Lopez cost function, CAPORIN – the average value of the Caporin cost function, EXCOST – the average value of the excessive cost function, GREEN – the average frequency of a model being in the green zone, YELLOW – the average frequency of a model being at least in the yellow zone, RED – the average frequency of a model being in the red zone. Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX – exponential distribution, EM – empirical distribution, LN – lognormal distribution, WE – Weibull distribution, GM – Gamma distribution; Q equal to 9 represents the 99^{th} percentile, 0 represents the 90^{th} percentile, and 8 represents 80^{th} percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

Table 2.

The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models – coverage tests results

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^D_{UC}	Z^G_{UC}
CLOGLOG	GARCH	WE9_5	5.06%	13.92%	8.86%	11.39%	3.80%	7.59%
CLOGLOG	GARCH	GM9_5	6.33%	12.66%	7.59%	12.66%	5.06%	7.59%
CLOGLOG	GARCH	LN9_5	6.33%	12.66%	7.59%	12.66%	5.06%	7.59%
CLOGLOG	GARCH	WE0_10	6.33%	16.46%	11.39%	18.99%	1.27%	17.72%
-	GARCH EMP	-	7.59%	8.86%	5.06%	10.13%	5.06%	5.06%
CLOGLOG	GARCH	EX8_5	7.59%	11.39%	6.33%	12.66%	3.80%	8.86%
CLOGLOG	GARCH	EX9_5	7.59%	12.66%	8.86%	12.66%	6.33%	6.33%
CLOGLOG	GARCH	EM9_5	7.59%	12.66%	8.86%	13.92%	6.33%	7.59%
-	GARCH	-	8.86%	8.86%	7.59%	24.05%	2.53%	21.52%
CLOGLOG	GARCH	EM0_10	8.86%	15.19%	11.39%	18.99%	1.27%	17.72%
CLOGLOG	GARCH	WE8_5	10.13%	12.66%	8.86%	16.46%	2.53%	13.92%
PROBIT	GARCH	GM9_10	10.13%	10.13%	8.86%	16.46%	6.33%	10.13%
-	EGARCH	-	10.13%	5.06%	8.86%	24.05%	2.53%	21.52%
PROBIT	GARCH	WE9_10	10.13%	8.86%	10.13%	13.92%	6.33%	7.59%
CLOGLOG	GARCH	LN8_5	10.13%	12.66%	10.13%	22.78%	2.53%	20.25%
CLOGLOG	GARCH	GM0_10	10.13%	16.46%	13.92%	21.52%	1.27%	20.25%
CLOGLOG	GARCH	LN0_10	10.13%	16.46%	13.92%	21.52%	1.27%	20.25%
PROBIT	GARCH	LN9_10	11.39%	8.86%	8.86%	15.19%	7.59%	7.59%
PROBIT	GARCH	EX9_10	11.39%	8.86%	11.39%	16.46%	10.13%	6.33%
CLOGLOG	GARCH	EM8_5	11.39%	11.39%	11.39%	21.52%	2.53%	18.99%
PROBIT	GARCH	EX0_10	12.66%	8.86%	10.13%	16.46%	8.86%	7.59%
PROBIT	GARCH	EM9_10	12.66%	8.86%	11.39%	17.72%	10.13%	7.59%
-	GARCH-t	-	77.22%	2.53%	51.90%	77.22%	75.95%	1.27%

In the table, white fields refer to the EWS-GARCH models with lognormal, Weibull or Gamma distributions, while grey fields to benchmark models.

The following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM – the Value-at-Risk forecasting model in a state of turbulence, LR_{UC} – the ratio of cases in which the null hypothesis was rejected in the Kupiec test, LR_{IND} – the ratio of cases in which the null hypothesis was rejected in the LR_{IND} part of the Christoffersen test, LR_{CC} – the ratio of cases in which the null hypothesis was rejected in the Christoffersen test, Z_{UC} – the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage, Z^D_{UC} – the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of alternative hypothesis that the actual excess ratio is significantly lower than expected, Z^G_{UC} – the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of an alternative hypothesis that the actual excess ratio is significantly higher than expected. All tests were performed for the 5% significance level, except the asymptotic test of unconditional coverage, where level of significance was set up to 10% (5% for each tail).

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX – exponential distribution, EM – empirical distribution, LN – lognormal distribution, WE – Weibull distribution, GM – Gamma distribution; Q equal to 9 represents the 99^{th} percentile, 0 represents the 90^{th} percentile, and 8 represents 80^{th} percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: own calculations.

of excess ratio is possible also by the models which assume the liberal approach to forecast Value-at-Risk using the exponential or the Gamma distribution in the state of turbulence. It is worth mentioning that in most cases the best state of turbulence forecasting model was the probit model, the cloglog model was better only once.

It can be seen, as well, that models with the lognormal, the Weibull or the Gamma distributions are less conservative than models with the exponential or the empirical distributions. Among models with the lognormal, the Weibull or the Gamma distributions, models with the 10% definition of the state of turbulence are slightly more conservative than models with the 5% definition, but the differences are not significant.

Use of any of the EWS-GARCH models presented in table 3 reduces the costs associated with the Value-at-Risk exceedances (both based on the Lopez and the Abad and Benito cost functions). For the models with the lognormal, the Weibull or the Gamma distributions slightly better results with respect to Abad and Benito cost function have models with the 5% definition of the state of turbulence.

The EWS-GARCH models with the conservative definition of Value-at-Risk quantile in the state of turbulence are qualified in 100% of cases to the green zone in the backtesting procedure, which is more frequent than in the case of the GARCH(1,1) with the amendment to empirical error distribution, and the much more conservative GARCH-t(1,1) model.

Regarding the Stressed Value-at-Risk values, the GARCH-t(1,1) was the most often assigned to the green zone. However the EWS-GARCH models with the conservative definition of Value-at-Risk quantile in the state of turbulence the exponential or the empirical distribution for both definitions of the state of turbulence, or with the lognormal, the Weibull or the Gamma distributions and the 5% definition of the state of turbulence, were assigned to at least the yellow zone in 100% cases, which is again even more than for the GARCH-t(1,1).

The improvement of all the discussed measures, as in previous cases, is associated with an increase of excess costs of using the model. Again, the excess cost grows steadily with the reduction of excess ratio (except models in which the excessive cost is inappropriately high – it happened in the models assuming that Value-at-Risk forecasts are calculated as the 99^{th} percentile of the exponential or the Gamma distribution with the 5% definition of the turbulent state). Among the EWS-GARCH models the excess costs of using the model are relatively small for models with the lognormal or the Weibull distributions.

In the results of the coverage tests it can be seen that for the EWS-GARCH models with the lognormal, the Gamma or the Weibull distributions the Kupiec test is rejected more often than for the GARCH(1,1) with the amendment to empirical error distribution, but according to the asymptotic unconditional coverage test, this happened only due to the fact that for these models the excess ratios are lower than expected. Moreover, according to the same tests it may be noted that for the EWS-GARCH models analysed the excess ratio is never higher than expected.

Table 3.

The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models with the amendment to empirical distribution of random error

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)											STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)		
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	YELLOW	RED	EN	ER	GREEN	YELLOW	RED
-	GARCH-t	-	1.25	0.24%	6.3%	2.46	12.5%	11.6%	98.7%	98.7%	1.3%	1.03	0.4%	97.5%	98.7%	1.3%
PROBIT	GARCH EMP	EX9_10	3.06	0.58%	6.4%	3.27	11.6%	10.7%	100.0%	100.0%	0.0%	2.56	1.0%	88.6%	100.0%	0.0%
PROBIT	GARCH EMP	EM9_10	3.25	0.62%	6.8%	3.48	9.1%	8.3%	100.0%	100.0%	0.0%	2.72	1.1%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	EX0_10	3.34	0.64%	6.9%	3.52	8.4%	7.5%	100.0%	100.0%	0.0%	2.80	1.1%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	EX9_5	3.39	0.65%	6.5%	3.48	22.1%	21.3%	100.0%	100.0%	0.0%	2.81	1.1%	87.3%	100.0%	0.0%
PROBIT	GARCH EMP	EM9_5	3.49	0.67%	6.7%	3.59	9.1%	8.3%	100.0%	100.0%	0.0%	2.89	1.2%	87.3%	100.0%	0.0%
PROBIT	GARCH EMP	WE9_10	3.49	0.67%	7.5%	3.73	8.2%	7.4%	100.0%	100.0%	0.0%	2.91	1.2%	86.1%	98.7%	1.3%
PROBIT	GARCH EMP	LN9_10	3.53	0.67%	7.3%	3.67	8.1%	7.3%	100.0%	100.0%	0.0%	2.97	1.2%	86.1%	98.7%	1.3%
PROBIT	GARCH EMP	GM9_10	3.62	0.69%	7.5%	3.77	8.0%	7.2%	100.0%	100.0%	0.0%	3.04	1.2%	82.3%	97.5%	2.5%
PROBIT	GARCH EMP	LN9_5	3.65	0.69%	7.1%	3.70	8.2%	7.4%	100.0%	100.0%	0.0%	3.01	1.2%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	GM9_5	3.67	0.70%	7.1%	3.72	18.1%	17.2%	100.0%	100.0%	0.0%	3.04	1.2%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	WE9_5	3.70	0.70%	7.1%	3.75	8.3%	7.5%	100.0%	100.0%	0.0%	3.05	1.2%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	EX8_5	3.87	0.74%	7.6%	3.93	7.8%	7.0%	100.0%	100.0%	0.0%	3.19	1.3%	78.5%	100.0%	0.0%
CLOGLOG	GARCH EMP	WE0_10	4.54	0.87%	9.7%	4.55	7.4%	6.6%	93.7%	98.7%	1.3%	3.82	1.5%	73.4%	94.9%	5.1%
-	GARCH EMP	-	4.61	0.88%	9.16%	4.67	7.23%	6.43%	94.9%	97.5%	2.5%	3.73	1.49%	68.4%	96.2%	3.8%
-	GARCH	-	6.42	1.22%	12.48%	6.42	6.58%	5.79%	78.5%	93.7%	6.3%	5.18	2.07%	39.2%	78.5%	21.5%
-	EGARCH	-	6.53	1.24%	12.48%	6.54	6.68%	5.90%	78.5%	92.4%	7.6%	5.19	2.08%	40.5%	81.0%	19.0%

In the table, white fields refer to the EWS-GARCH models with lognormal, Weibull or Gamma distributions, while grey fields to benchmark models. The same abbreviations as in table 1 are used.

Source: own calculations.

Table 4.

The results of the analysis of the quality of Value-at-Risk forecasts models obtained from the EWS-GARCH(1,1) with the amendment to empirical distribution of random error – coverage tests results

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^D_{UC}	Z^G_{UC}
-	GARCH EMP	-	7.59%	8.86%	5.06%	10.13%	5.06%	5.06%
-	GARCH	-	8.86%	8.86%	7.59%	24.05%	2.53%	21.52%
PROBIT	GARCH EMP	EX8_5	10.13%	8.86%	3.80%	10.13%	10.13%	0.00%
CLOGLOG	GARCH EMP	WE0_10	10.13%	15.19%	8.86%	15.19%	8.86%	6.33%
-	EGARCH	-	10.13%	5.06%	8.86%	24.05%	2.53%	21.52%
PROBIT	GARCH EMP	WE9_5	11.39%	10.13%	6.33%	11.39%	11.39%	0.00%
PROBIT	GARCH EMP	GM9_5	13.92%	8.86%	5.06%	13.92%	13.92%	0.00%
PROBIT	GARCH EMP	LN9_5	13.92%	8.86%	5.06%	13.92%	13.92%	0.00%
PROBIT	GARCH EMP	EM9_5	16.46%	8.86%	7.59%	16.46%	16.46%	0.00%
PROBIT	GARCH EMP	EX9_5	17.72%	8.86%	7.59%	17.72%	17.72%	0.00%
PROBIT	GARCH EMP	GM9_10	17.72%	8.86%	7.59%	17.72%	17.72%	0.00%
PROBIT	GARCH EMP	WE9_10	17.72%	7.59%	10.13%	17.72%	17.72%	0.00%
PROBIT	GARCH EMP	LN9_10	18.99%	7.59%	7.59%	18.99%	18.99%	0.00%
PROBIT	GARCH EMP	EX0_10	20.25%	6.33%	8.86%	20.25%	20.25%	0.00%
PROBIT	GARCH EMP	EM9_10	21.52%	6.33%	12.66%	21.52%	21.52%	0.00%
PROBIT	GARCH EMP	EX9_10	22.78%	6.33%	12.66%	22.78%	22.78%	0.00%
-	GARCH-t	-	77.22%	2.53%	51.90%	77.22%	75.95%	1.27%

In the table, white fields refer to the EWS-GARCH models with lognormal, Weibull or Gamma distributions, while grey fields to benchmark models.
The same abbreviations as in table 2 are used.

Source: own calculations.

5. CONCLUSIONS

Given all the results, it can be stated that the EWS-GARCH models provide Value-at-Risk forecasts with sufficient quality and can be used as the Value-at-Risk forecasting models. The EWS-GARCH models with the lognormal, the Gamma or the Weibull distributions are sufficient alternatives for the EWS-GARCH models with the exponential or the empirical distributions. The models with the lognormal, the Gamma or the Weibull distributions have a bit higher excess ratios, but they also cost less in terms of the excess cost.

Among the models with the lognormal, the Gamma or the Weibull distributions the best seems to be the model with the conservative definition of Value-at-Risk quantile, the 5% definition of the state of turbulence and the lognormal distributions. This model has in both cases a very good relation between the Value-at-Risk quality and the excessive costs of using the model.

Even though the EWS-GARCH models provide Value-at-Risk of good quality and may be used to measure the market risk, it could be improved in the future. Firstly, the states forecasting models may be extended by considering the use of additional variables or incorporating an autoregressive process into the model. Secondly, different Value-at-Risk models in both states may be considered (other GARCH models for the tranquil or turbulent states).

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CZY ZASTOSOWANIE ROZKŁADÓW LOGNORMALNEGO, WEIBULLA LUB GAMMA MOŻE POPRAWIĆ PROGNOZY WARTOŚCI NARAŻONEJ NA RYZYKO UZYSKIWANE NA PODSTAWIE MODELI EWS-GARCH?

Streszczenie

W badaniu analizie poddane zostały dwustopniowe modele EWS-GARCH służące do prognozowania wartości narażonej na ryzyko. W ramach analizy rozpatrywane były modele EWS-GARCH zakładające rozkłady lognormalny, Weibulla oraz Gamma w stanie turbulencji oraz modele GARCH(1,1) i GARCH(1,1) z poprawką na rozkład empiryczny w stanie spokoju.

Ocena jakości prognoz Value-at-Risk uzyskanych na podstawie wspomnianych modeli została przeprowadzona na podstawie miar adekwatności (wskaźnik przekroczeń, test Kupca, test Christoffersena, test asymptotyczny bezwarunkowego pokrycia oraz kryteria *backtestingu* określone przez Komitet Bazylejski) oraz analizy funkcji strat (kwadratowa funkcja straty Lopeza, absolutna funkcja straty Abad i Benito, 3 wersja funkcji straty Caporina oraz funkcja nadmiernych kosztów). Uzyskane wyniki wskazują, że modele EWS-GARCH z rozkładem lognormalnym, Weibulla lub Gamma mogą konkurować z modelami EWS-GARCH z rozkładem wykładniczym lub empirycznym. Modele EWS-GARCH z rozkładem lognormalnym, Weibulla lub Gamma są nieco mniej konserwatywne, jednocześnie jednak koszt ich stosowania jest mniejszy niż modeli EWS-GARCH z rozkładem wykładniczym lub empirycznym.

Słowa kluczowe: wartość zagrożona (Value-at-Risk), modele GARCH, modele zmiany stanu, prognozowanie, ryzyko rynkowe

CAN LOGNORMAL, WEIBULL OR GAMMA DISTRIBUTIONS IMPROVE THE EWS-GARCH VALUE-AT-RISK FORECASTS?

A b s t r a c t

In the study, two-step EWS-GARCH models to forecast Value-at-Risk are analysed. The following models were considered: the EWS-GARCH models with lognormal, Weibull or Gamma distributions as a distributions in a state of turbulence, and with GARCH(1,1) or GARCH(1,1) with the amendment to empirical distribution of random error models as models used in a state of tranquillity.

The evaluation of the quality of the Value-at-Risk forecasts was based on the Value-at-Risk forecasts adequacy (the excess ratio, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage and the backtesting criteria defined by the Basel Committee) and the analysis of loss functions (the Lopez quadratic loss function, the Abad & Benito absolute loss function, the 3rd version of Caporin loss function and the function of excessive costs). Obtained results show that the EWS-GARCH models with lognormal, Weibull or Gamma distributions may compete with EWS-GARCH models with exponential and empirical distributions. The EWS-GARCH model with lognormal, Weibull or Gamma distributions are relatively less conservative, but using them is less expensive than using the other EWS-GARCH models.

Keywords: Value-at-Risk, GARCH models, regime switching, forecasting, market risk