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Estimation of the average wage in Polish small companies using the robust approach*

Abstract. *There is a growing demand for multivariate economic statistics for cross-classified domains. In business statistics, this demand poses a particular challenge given the specific character of the population of enterprises, which necessitates searching for methods of analysis that would represent the robust approach to estimation, where auxiliary variables could be utilised. The adoption of new solutions in this area is expected to increase the scope of statistical output and improve the precision of estimates. The study presented in the paper furthers this goal, as it is focused on testing the application of a robust version of the Fay-Herriot model, which makes it possible to meet the assumption of normality of random effects under the presence of outliers. These alternative models are supplied to estimate the parameters of small firms operating in 2012. Variables from administrative registers were used as auxiliary variables, which made the estimation process more comprehensive. The paper refers to small area estimation methods. The variables of interest are estimated at a low level of aggregation represented by the cross-section province and NACE sections.*

Keywords: robust estimation, business statistics, small area estimation, Fay-Herriot model

JEL Classification: C40, C13, C40, C51, M20

1. INTRODUCTION

One of the conditions for economic growth is the development of entrepreneurship. Nowadays, however, in the fast-changing social, economic and legal environment, it is not easy to do business: customer needs keep changing, different markets are undergoing integration and business environment is getting increasingly competitive. To meet these challenges, entrepreneurs have to interact with various actors and exchange information; they need access to detailed information at a low level of aggregation, which enables them to react quickly to market changes.

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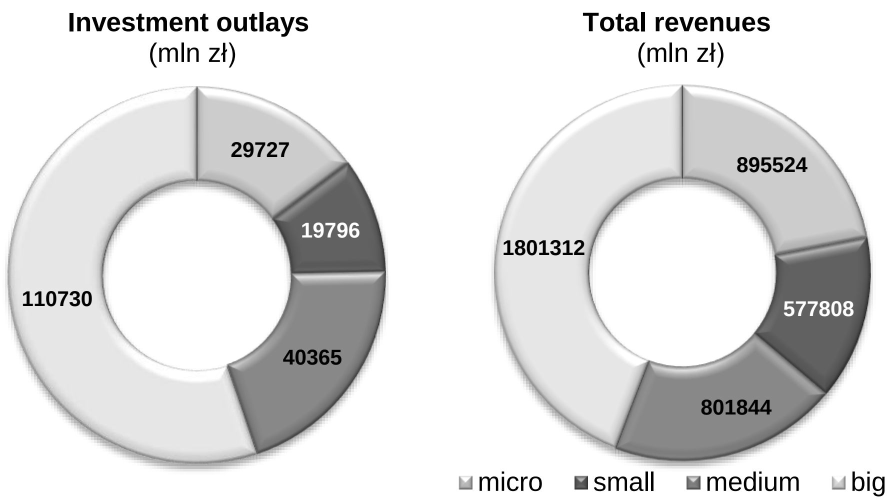
The growing demand for information for small domains has called for new estimation methods that would satisfy consumers' needs in this regard. In the case of economic statistics, the estimation of key variables proves particularly challenging due to problems such as strong asymmetry and high variation and concentration, which make it difficult to retain the properties of classical estimators used in sample surveys. To overcome these problems, there have been attempts to apply robust indirect estimation techniques using auxiliary variables from additional data sources, which could yield more reliable estimates than those obtained by means of direct estimation. This paper contributes to this approach, as its aim is to test the usefulness of the application of one of the methods from the realm of small area statistics to the estimation of the average salary in the enterprise sector according to province and NACE[3] section, utilising information collected in administrative registers.

The paper consists of four parts. The first part is devoted to the characteristics of the Polish small business. The second part describes data sources used for the estimation and provides details of the empirical study. The third part applies methodological considerations to the analysis. The fourth part summarizes the results of the study and presents their interpretation. The study focuses on small enterprises employing from 10 to 49 persons. Its aim is to estimate the average wage in these companies using a robust version of the Fay-Herriot model and auxiliary variables from administrative registers (Fay and Herriot, 1979; Sinha and Rao, 2009). The study is the continuation of the antecedent research on this subject by Dehnel and Wawrowski (2018).

The structure of the Polish business sector has remained stable for many years, where small enterprises have constituted less than 3 percent of the entire sector. Nevertheless, they have played a significant, and, in some respects, a crucial role in the economy. It is because small firms, which are free from corporate connections and dependencies, are able to compete with the largest units. They are legally and economically independent to a considerable extent, and also relatively flexible thanks to tight cost control, quick responsiveness to changing market requirements, and the ability to quickly implement innovations. In 2015, small companies invested almost PLN 20 billion (9.9% of the total value of investments in the enterprise sector), cf. Fig. 1. They acted according to their own strategies, strove to achieve their own goals, often taking financial risk. Their revenues accounted for about a quarter of the revenue of the entire small and medium enterprises sector (SME sector).

[3] NACE – The Statistical Classification of Economic Activities in the European Community

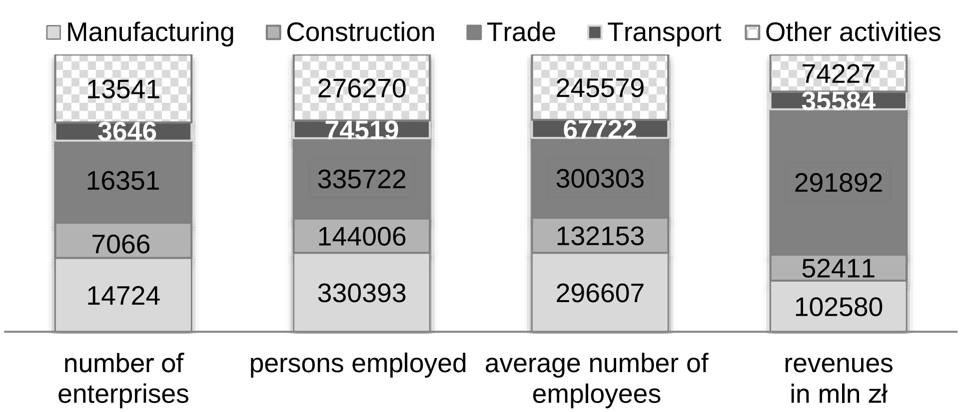
Figure 1. Enterprises' characteristics by size class as of 31 Dec 2015 (millions of PLN)



S o u r c e: based on "Statistics Poland's" study (GUS, 2017).

From the point of view of business classification, the most important sections in this sector are: manufacturing, construction, wholesale and retail trade (trade), and transport and storage (transport). These sections account for over 75% of all small businesses, produce almost 90% of the total revenue of the sector cf. Fig. 2. and also provide 86% of all the jobs in the small business sector (GUS, 2017).

Figure 2. Small enterprises' characteristics by NACE section as of 31 Dec 2015



S o u r c e: based on "Statistics Poland's" study (GUS, 2017).

2. DESCRIPTION OF THE STUDY

Data for the present analysis has been drawn from the DG1[4] survey carried out by the Statistical Office in Poznań. The survey is conducted in the form of reports that all large and medium-size enterprises as well as a 10-percent sample of small companies have to submit every month, and whose objective is to collect updated basic indicators of the economic activity.

For the purpose of the study, the scope of data collected from the DG1[4] survey was limited to the statistics of small enterprises operating in August 2012 – the period determined by the availability of data. The average wage was the target variable, while net revenues in 2011 taken from the Ministry of Finance's register and the number of enterprises per 10,000 population in 2011 taken from the REGON register were the auxiliary variables.

The data concerning the average wage in small companies from the manufacturing, construction, trade and transportation sections, published by Statistics Poland, is available only at the country level. For this reason, as well as being aware of the growing demand for more detailed information voiced by data users, the authors decided to carry out a study whose goal was to estimate certain variables at the level of province (NUTS 2), thus the target domain for estimation in the paper is a province cross-classified by NACE section (Dehnel, 2017).

3. ROBUST FAY-HERRIOT MODEL

The Fay-Herriot model belongs to a class of area-level models, which means that it utilises aggregated data instead of unit-level information. This approach was developed in 1979 as a tool for estimating income for small areas in the United States (Fay and Herriot, 1979). The Fay-Herriot model is constructed in two stages. Firstly, it is assumed that the direct estimator is unbiased and can be written as the sum of the true value of the estimated parameter and the random error:

$$\hat{\theta}_d = \theta_d + e_d, \tag{1}$$

where $e_d \overset{iid}{\sim} N(0, \sigma_{ed}^2)$. In practice, the variance σ_{ed}^2 is unknown and has to be estimated on the basis of the survey data. The direct estimator used most frequently in the Fay-Herriot model is the Horvitz-Thompson (HT) estimator (Horvitz and Thompson, 1952), which has also been used in this study.

In the second stage, the true value of the parameter is treated as a dependent variable in the linear model with an area random effect:

[4] DG1 – the largest survey in Polish short-term business statistics. It collects data from businesses employing over 9 people.

$$\theta_d = x_d^T \beta + u_d, \tag{2}$$

where x_d is a vector of auxiliary information for area d, β is a vector of regression parameters and u_d is an area random effect with the distribution $u_d \overset{iid}{\sim} N(0, \sigma_u^2)$.

By combing equations (1) and (2), we obtain the formula of the Fay-Herriot model:

$$\hat{\theta}_d = {x_d}^T \beta + u_d + e_d. \tag{3}$$

The estimator of the Fay-Herriot model is known as EBLUP (Empirical Best Linear Unbiased Predictor) and is expressed by the following formula:

$$\hat{\theta}_d^{FH} = x_d^T \hat{\beta} + \hat{u}_d = \hat{\gamma}_d \hat{\theta}_d + (1 - \hat{\gamma}_d) x_d^T \hat{\beta}, \; d = 1, \dots, D \tag{4}$$

where

$$\hat{\beta} = \left(\sum_{d=1}^{D} \hat{\gamma}_d x_d x_d^T\right)^{-1} \sum_{d=1}^{D} \hat{\gamma}_d x_d \hat{\theta}_d \text{and } \hat{\gamma}_d = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}_{ed}^2}. \tag{5}$$

EBLUP is a weighted average of the direct estimator and the regression-synthetic estimator. The weight $\hat{\gamma}_d$ measures the uncertainty of the regression component. If the sample variance estimator $\hat{\sigma}_{ed}^2$ is small, then a larger part of the final estimate will be contributed by the direct estimator $\hat{\theta}_d$ (Boonstra and Buelens, 2011). The between-area variance $\hat{\sigma}_u^2$ as a sample variance is also unknown and has to be estimated, which can be done using many techniques, e.g. the Fay-Herriot method, Prasad-Rao method, ML or REML described in Chapter 6 of Rao book (2014, p. 126–129).

The robust version of the Fay-Herriot model uses the Huber (1981) influence function to restrict the influence of u_d and e_d.

Let us replace the estimates of $\hat{\sigma}_{ed}^2$ and $\hat{\sigma}_u^2$ with covariance matrices Σ_e and Σ_u and let $V = \Sigma_e + \Sigma_u$. Then the vector of the fixed effects β is expressed by:

$$\beta = (X^T V^{-1} X)^{-1} X V^{-1} y \tag{6}$$

and random effects vector u is:

$$u = \Sigma_u Z^T V^{-1} (y - X\beta). \tag{7}$$

It is demonstrated that equations (6) and (7) can be transformed into:

$$X^T V^{-1} (y - X\beta) = 0 \tag{8}$$

and

$$\Sigma_u Z^T V - 1(y - X\beta) - u = 0. \tag{9}$$

Sinha and Rao (2009) proposed a robust version of equations (8) and (9):

$$X^T V^{-1} U^{\frac{1}{2}} \psi(U^{\frac{1}{2}}(y - X\beta)) = 0, \tag{10}$$

where $U = diag(V)$. A robust random effects vector is defined by:

$$\begin{aligned}\psi((y - X\beta)^T U^{\frac{1}{2}}) U^{\frac{1}{2}} V^{-1} (\partial V/\partial\theta) V^{-1} U^{\frac{1}{2}} \psi(U^{\frac{1}{2}}(y - X\beta)) \\ = \mathrm{tr}(\mathrm{D}^{\Psi}(\partial \mathrm{V}/\,\partial\theta)),\end{aligned} \tag{11}$$

where $\partial V/\partial\theta$ is the first order partial derivative of V with respect to the variance component θ and for $Z \sim N(0,1)$, $D^{\psi} = E(\psi^2(Z))V^{-1}$.

Moreover, Warnholz (2016) proposed a modification of the above equation in which only diagonal elements of V matrix are used to standardise the residuals. In the robust Fay-Herriot model this matrix is diagonal, but the transformation can be useful in models with correlated random effects, e. g. SAR(1) and AR(1), where calculations are likely to be time-consuming.

Robust EBLUP is expressed by the formula:

$$\hat{\theta}_d^{RFH} = x_d^T \hat{\beta}^{\psi} + \hat{u}_d^{\psi},\ d = 1, \ldots, D. \tag{12}$$

For unsampled domains, and where the between-area variance equals zero, the indirect estimation relies only on the regression component.

To estimate the mean square error (MSE) for Fay-Herriot model, we can use the parametric bootstrap method proposed by González-Manteiga et al. (2008). The algorithm proceeds along the following steps:

1. fit the model to obtain estimates of $\hat{\sigma}_u^2$ and $\hat{\beta}$;
2. generate a vector of u^* with $N(0, \hat{\sigma}_u^2)$ and calculate $\theta^* = X\hat{\beta} + u^*$;
3. generate a vector of e^* with $N(0, \hat{\sigma}_{ed}^2)$;
4. construct a bootstrap data vector of $\hat{\theta}^* = \theta^* + e^* = X\hat{\beta} + u^* + e^*$;
5. fit the model to bootstrap data $\hat{\theta}^*$ to obtain new estimates of $\hat{\sigma}_u^{2*}$ and $\hat{\beta}^*$;
6. calculate $\hat{\theta}^{*B}$ taking into account values obtained in step 5;
7. repeat steps 2-6 B times, assuming that $\theta^{*(b)}$ is the true value, and $\hat{\theta}^{*(b)}$ are EBLUP estimates obtained in b-th bootstrap replication;
8. the MSE estimator of $\hat{\theta}$ is expressed by:

$$MSE(\hat{\theta}) = B^{-1} \sum_{b=1}^{B} [\hat{\theta}^{*(b)} - \theta^{*(b)}]^2. \tag{13}$$

In the case of the robust Fay-Herriot, model parameter estimates are replaced by their robust versions $\hat{\beta}^{\psi}$, $\hat{\sigma}_u^{2\,\psi}$ and $\hat{\sigma}_{ed}^{2\,\psi}$ and the Robust Fay-Herriot model is calculated in step 5 of the above algorithm (Sinha and Rao, 2009).

Given the MSE, one can calculate relative root mean square error, which is a common measure of precision used in all approaches:

$$RRMSE\left(\hat{\theta}\right) = \frac{\sqrt{MSE\left(\hat{\theta}\right)}}{\hat{\theta}}. \tag{14}$$

4. ESTIMATION RESULTS AND ASSESSMENT OF THEIR PRECISION

Out of the total of 21 NACE sections, the following four were selected: manufacturing, construction, trade and transportation − as this particular combination yielded samples of different sizes. Tabl. 1 presents descriptive statistics of the sample size in the selected sections.

TABLE 1. SAMPLE SIZE BY NACE SECTION

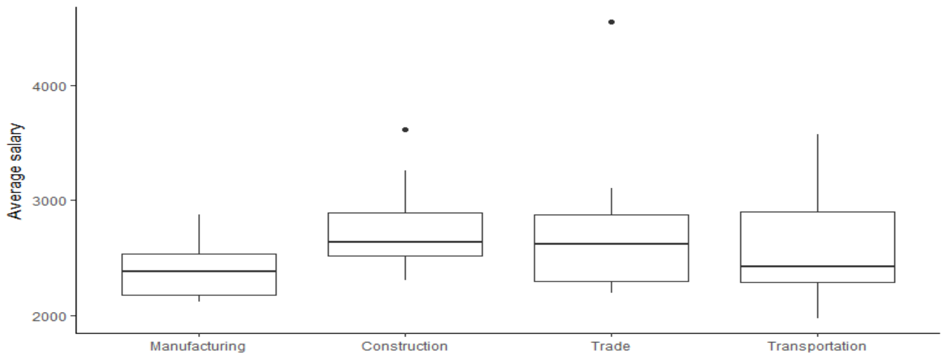
NACE section	Minimum	Median	Mean	Maximum
Manufacturing	129	222	245	440
Construction	41	77	93	197
Trade	131	216	256	562
Transportation	19	31	40	101

S o u r c e: based on data from the DG1 survey.

As the figures demonstrate, the biggest samples were selected for the trade section. The largest of them consisted of 562 enterprises and came from the Śląskie province, whereas the second largest, of 547 companies, came from Mazowieckie. Within the manufacturing section, the Wielkopolskie province provided the biggest sample, of 440 enterprises. The smallest sample of all, which consisted of 19 enterprises, was selected for the transportation section in the Opolskie province. The smallest sample for the construction section consisted of 41 enterprises and was drawn from the Podlaskie province.

The first step in the analysis was to produce direct estimates of the variable of interest for all target domains i.e. province and 4 sections. Fig. 3 presents the distribution of the estimates.

Figure 3. Distribution of the average wage estimates by NACE section



S o u r c e: based on data from the DG1 survey and the administrative register.

Fig. 3 shows two province outliers. In both cases, it is the Mazowieckie province, with the average wage at 3614 PLN for the construction section and 4550 PLN for the trade section. The smallest value of the average wage for the construction section is 2306 PLN, in the Świętokrzyskie province, whereas the smallest value for the trade section is 2194 PLN, in the Lubelskie province. The smallest value of the average wage, 1970 PLN, was estimated for the Podkarpackie for the transportation section, whereas the largest value, in this section, – 3570 PLN, was estimated for the Pomorskie province. The manufacturing section was characterized by the smallest range of wages of all the four sections. The smallest value of the average wage in this section was 2114 PLN (in the Podlaskie province), and the largest amounted to 2875 PLN (in the Mazowieckie). The figures indicate that the average wages in all but one section assume the largest values in the Mazowieckie province. A similar level of the median values of the average wage – 2637 PLN and 2622 PLN – were observed in the construction and trade sections, respectively, as well as in the manufacturing and transportation sections, where they reached 2378 PLN and 2421 PLN, respectively.

In addition to the distribution of direct estimates, it is important to analyse the precision of these estimates. Variances of direct estimates were calculated using the bootstrap method implemented in the survey R package (Lumley 2004). Tabl. 2 presents descriptive statistics of relative root mean square errors (RRMSE) of the direct estimates of the average wage.

TABLE 2. DESCRIPTIVE STATISTICS OF RRMSE OF ESTIMATES BY NACE SECTION (IN %)

NACE section	Minimum	Median	Mean	Maximum
Manufacturing	2.2	3.0	3.0	3.9
Construction	2.9	5.4	5.4	7.2
Trade	2.9	3.4	3.5	4.3
Transportation	5.0	8.7	9.9	21.4

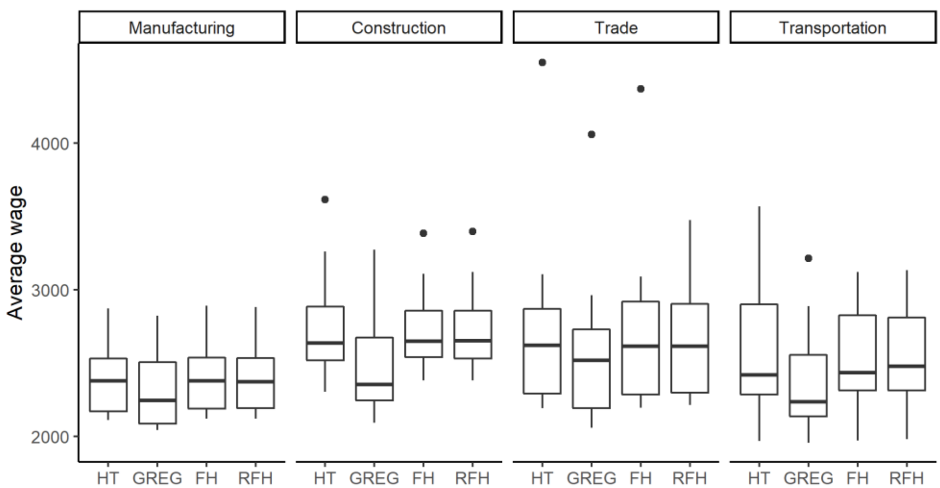
S o u r c e: based on data from the DG1 survey and the administrative register.

Direct estimates of the average wage in all the sections except transportation are relatively precise. The maximum value of the RRMSE for these three sections does not exceed 7.2% (Warmińsko-Mazurskie province). In the case of the transportation section, however, the maximum relative root mean square error amounts to over 20%. This particular value was observed in the Opolskie, where the sample was of the smallest size. According to Statistics Poland's guidelines, estimates can only be published if their RRMSE falls below 10% for planned domains (GUS, 2013; Eurostat, 2013).

To obtain more precise estimates, the authors applied indirect methods of estimation – the Fay-Herriot model (FH) and the robust Fay-Herriot model (RFH). In the modelling process, they used data concerning the average wage in 2011 from the registers of the Ministry of Finance, and concerning the number of enterprises per 10,000 population from the REGON register.

The distributions of the estimates based on the direct estimator (HT – Horvitz and Thompson, 1952), GREG (Dehnel, 2017), the Fay-Herriot model (Dehnel et al., 2017) and Robust Fay-Herriot model are presented in Fig. 4.

Figure 4. Distribution of the average wage estimates by NACE section and estimator



S o u r c e: based on the data from the DG1 survey and the administrative register.

For all the four approaches, the distribution of estimates is similar. The most visible change in the distribution can be observed for the maximum value of the average wage in the trade section. The Horvitz-Thompson estimate was 4550 PLN, the value estimated by the Fay-Herriot model 4367 PLN, and by the robust Fay-Herriot – 3476 PLN.

The precision of estimates can be assessed on the basis of the values of relative root mean square errors presented in Tabl. 3, listed for each section and estimator.

TABLE 3. DESCRIPTIVE STATISTICS OF RRMSE OF ESTIMATES BY NACE SECTION AND ESTIMATOR (IN %)

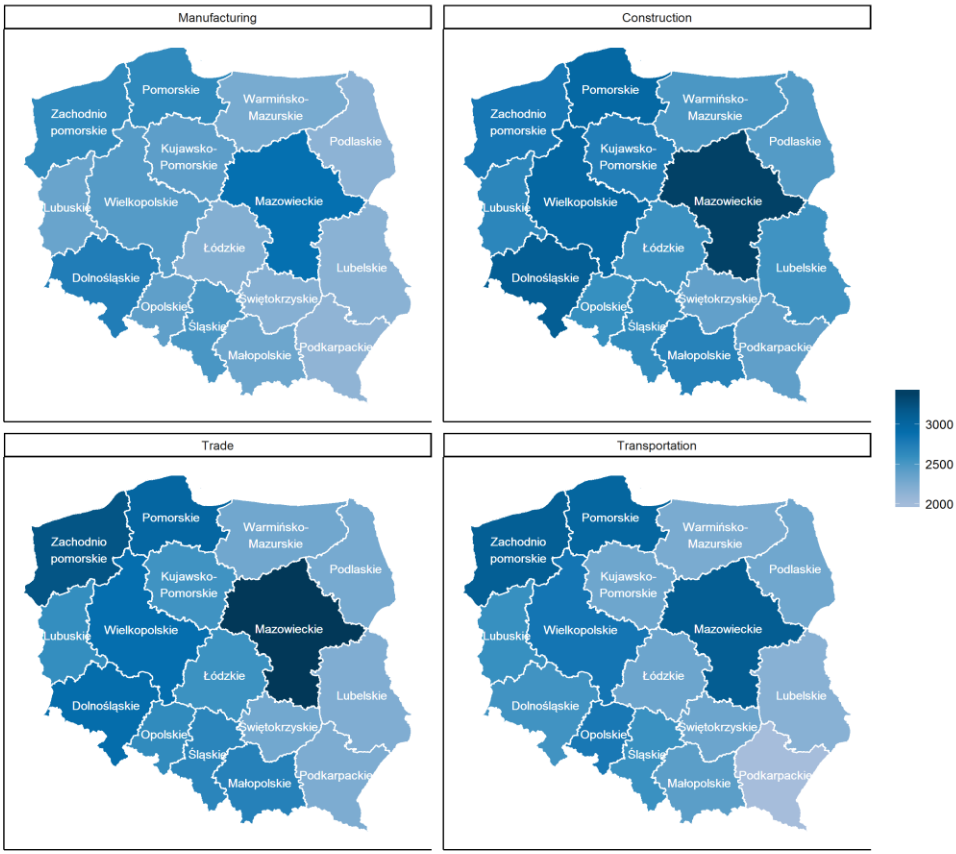
NACE section	Estimator	Minimum	Median	Mean	Maximum
Manufacturing	HT	2.2	3.0	3.0	3.9
Manufacturing	GREG	1.9	2.5	2.6	3.5
Manufacturing	FH	2.0	2.6	2.6	3.2
Manufacturing	RFH	2.0	2.6	2.6	3.2
Construction	HT	2.9	5.4	5.4	7.2
Construction	GREG	2.8	4.8	5.0	7.3
Construction	FH	2.8	4.6	4.6	5.8
Construction	RFH	2.8	4.6	4.6	5.7
Trade	HT	2.9	3.4	3.5	4.3
Trade	GREG	2.4	3.0	3.0	3.6
Trade	FH	2.8	3.3	3.4	4.4
Trade	RFH	2.4	2.8	2.8	3.5
Transportation	HT	5.0	8.7	9.9	21.4
Transportation	GREG	4.8	7.4	8.5	21.3
Transportation	FH	4.1	6.1	6.1	8.6
Transportation	RFH	3.5	5.8	6.0	10.0

S o u r c e: based on data from the DG1 survey and the administrative register.

The application of indirect methods of estimation made it possible to reduce the RRMSE of the average wage for unplanned domains, i.e. provinces cross-classified with NACE sections. The RRMSE of the estimates obtained using the Fay-Herriot model are always lower than the precision of direct estimates. Robust Fay-Herriot estimates for all sections are, on average, either equally or more precise than those based on the Fay-Herriot model. The exception here is the transportation section, where the maximum RRMSE value is higher than that estimated by the Fay-Herriot model. This has been caused by a small sample from the Opolskie province. In general, none of the estimates exceed the 10% threshold set by Statistics Poland.

It is worth mentioning, though, that the MSE estimators are also biased, but this aspect is not analysed in detail in literature on small area estimation (Krzciuk, 2017). The size of the error can be estimated using the Monte Carlo simulation, but to do this, one would have to know the value of the estimated quantity for the whole population (Żądło, 2008, Żądło, 2012). Such information was not available for this study. Another step in the assessment of the obtained results is the analysis of spatial variation. Fig. 5 visualises the average wage across provinces for the four NACE sections.

Figure 5. Average wage estimates by NACE section and province

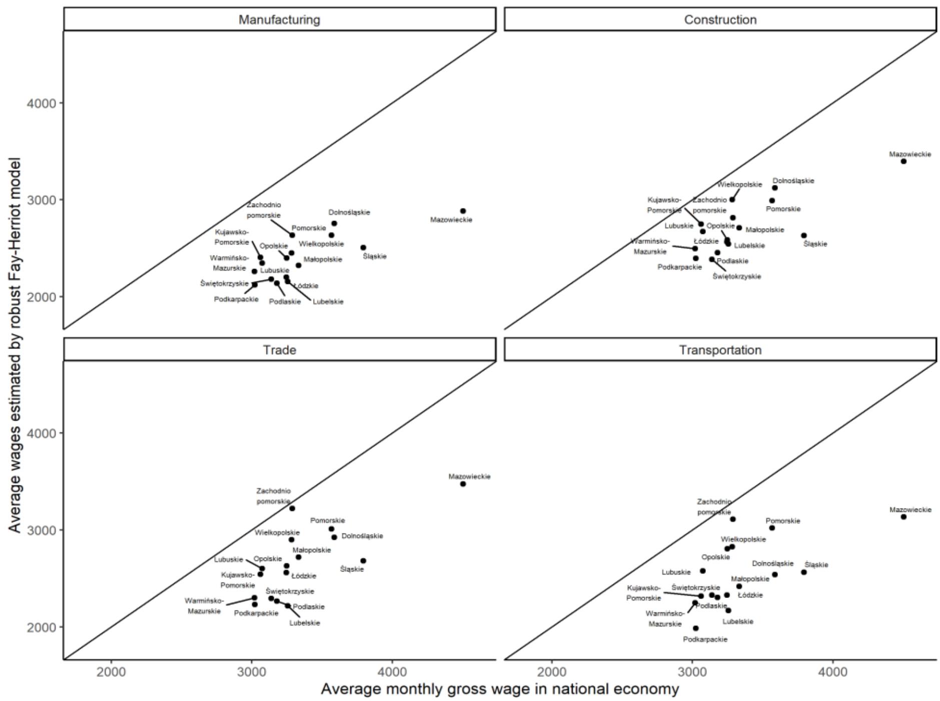


S o u r c e: based on data from the DG1 survey and the administrative register.

As Fig. 5 demonstrates, there is a strong spatial diversity in the average wage across provinces. The Mazowieckie province visibly stands out – average salaries reach the highest values in all the studied sections there. Average salaries reach the second highest values in the Dolnośląskie province (construction section) and in the Zachodniopomorskie (trade and transportation sections), whereas they assume the lowest values in Eastern Poland (in all the sections).

In the last part of the analysis, the obtained estimates are compared with the average gross wage in the national economy, which is presented in Fig. 6, in order to find out to what extent the estimates correspond with wage data from administrative registers.

Figure 6. Estimated average wage in small enterprices vs. average wage in the national economy by NACE sector in 2012



S o u r c e: based on data from the DG1 survey and the administrative register.

Fig. 6 shows a correlation between the estimates and the average wage in the national economy. Values of Pearson linear correlation coefficient vary from $r = 0.61$ for transportation to $r = 0.77$ for manufacturing. It is worth noting that the values of the average wage in the four sections are slightly lower than the national average.

5. ESTIMATION AT THE LOCAL LEVEL

As the estimation at the level of provinces (NUTS 2) was successfully conducted, the authors decided to carry out a similar estimation at the level, i.e. for districts (NUTS 3). Since there are many more territorial units at this level, the minimum sample size in particular domains was much smaller. In addition, there were some districts with no entities suitable for samples. As a result, calculations were made only for one section – manufacturing. Out of all the 379 districts, 350 were represented in the sample. The calculations yielded direct estimates of the average wage in small companies. These estimates ranged from 1258 PLN to 4246 PLN, while relative errors (RRMSE) ranged from 1% to 33%, with a mean of 11%.

After applying the robust Fay-Herriot model, the range of estimates did not change considerably – the minimum remained the same, while the maximum decreased to 3509 PLN. However, this method improved the estimation precision. The application of auxiliary variables made it possible to decrease the maximum RRMSE to 21%, with the mean at 8.7%. The above-described exercise shows that an average wage can also be estimated at the level of districts, but, given the smaller sample size in domains, this approach requires further analysis to test other sources of auxiliary information or other modified robust methods.

6. CONCLUSION

Indirect methods of estimation enable the estimation of the average wage for four NACE sections for the previously unpublished domains. The results obtained by means of the Fay-Herriot model and its robust version are, in most cases, more precise than the direct estimator when measured with the RRMSE. Moreover, robust estimation reduces the impact of outliers on the average wage and limits the range of estimates.

The results also show that the level of average wage varies across the four NACE sections. It assumes highest values in the Mazowieckie province. The size of bias was assessed using general data about the average monthly gross wage in the national economy.

It is worth noting that the application of the robust Fay-Herriot model at the level of districts has generally improved the estimation precision compared to direct estimation method. However, due to the fact samples are too small in some domains (even zero samples), there is a strong need for additional analysis to test other sources of auxiliary information, or other modified robust estimation methods.

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