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Estimation of Yu and Meyer bivariate stochastic volatility model by iterated filtering

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Abstract. In financial applications, understanding the asset correlation structure is crucial to tasks such as asset pricing, portfolio optimisation, risk management, and asset allocation. Thus, modelling the volatilities and correlations of multivariate stock market returns is of great importance.

This paper proposes the iterated filtering algorithm for estimating the bivariate stochastic volatility model of Yu and Meyer. The iterated filtering method is a frequentist-based approach that utilises particle filters and can be applied to estimating the parameters of non-linear or non-Gaussian state-space models.

The paper presents an empirical example that demonstrates the way in which the proposed estimation method might be used to estimate the correlation between the returns of two assets: Standard and Poor's 500 index and the price of gold in US dollars. This is accompanied by a simulation study that proves the validity of the approach.

Keywords: multivariate stochastic volatility, iterated filtering, particle filters

JEL: C32, C58, G15

1. Introduction

The knowledge of correlation structures is vital in many financial applications, because it provides a measurement of the relationship between different financial assets or variables. This information can be used to make informed investment decisions, assess risk, and design and evaluate financial products. In a portfolio construction, knowledge of the correlation structure between assets can help investors create a well-diversified portfolio. If the assets are highly correlated, their returns are likely to move in the same direction, which can lead to a higher portfolio risk. On the other hand, if the assets are uncorrelated or negatively correlated, their returns may offset each other, reducing the portfolio risk. In risk management, understanding the correlation structure between different financial variables can help financial institutions assess the potential for losses and determine how to allocate capital to manage risk. This is particularly important for complex financial products, such as derivatives, which can have non-linear and highly interconnected risk profiles. The knowledge of the correlation structure can also be used to price

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financial products and to develop financial models, e.g. value at risk (VaR) models, which are widely used to measure the risk of financial portfolios.

Numerous applications of the correlation structure generate the need for modelling the volatilities of multivariate stock market returns. Over several past decades, there has been significant progress in the estimation of multivariate volatility models. Nowadays, we can distinguish three main approaches to this problem: multivariate generalised autoregressive conditional heteroskedasticity (GARCH) models (represented by e.g. the BEKK models of Engle and Kroner (1995) and the DCC models of Engle (2002)), multivariate stochastic volatility (MSV) models (see Chib et al., 2009), and realised covariance models (see e.g. Bollerslev et al., 2018; Jin & Maheu, 2013). Recently, machine learning (ML) algorithms have also become increasingly popular for the forecasting of multivariate financial time series (Bejger & Fiszeder, 2021; Fiszeder & Orzeszko, 2021).

The estimation of multivariate stochastic volatility models generates significant difficulties due to both their high-dimensional parameter space resulting from their multidimensional nature, and the absence of a closed-form likelihood in stochastic volatility models. Moreover, the estimation process has to account for the positive semidefiniteness of the covariance matrix.

Despite these challenges, various methods for estimating MSV models have been developed. The first approach involved the application of Kalman-based filtering to the evaluation and maximisation of the quasi-loglikelihood function (Harvey et al., 1994; So et al., 1997). Soon afterwards, the Bayesian approach began to dominate the estimation of MSV models. It was followed by the multi-move sampler proposed by Shephard and Pitt (1997) and modified by Watanabe and Omori (2004), which became a standard technique in the early 2000s. This method was used, among other authors, by Ishihara and Omori (2012) for the estimation of the MSV model with cross-leverage and heavy-tailed errors, Ishihara et al. (2016) for the matrix exponential MSV model with cross-leverage, and Kastner et al. (2017) for the multivariate factor SV model. In the case of MSV models, it is necessary to construct an efficient MCMCM sampler separately in each model (Chib et al., 2009).

Although the Bayesian inferences are very effective, they have some limitations that can restrict their usefulness in certain applications. Firstly, the inferences depend on the choice of prior distributions. Choosing appropriate prior distributions is therefore of key importance in the case of Bayesian methods. Otherwise, estimates might turn out biased, and results incorrect. It is especially advisable to use methods independent of a priori beliefs if the task is to identify the information in the studied data. Secondly, in some cases, the interpretation of Bayesian models can be challenging due to the complexity of the posterior distributions. In addition, Bayesian methods are more difficult to implement and require more computing power than frequentist approaches.

Frequentist approaches, such as maximum likelihood estimation, are often easier to interpret and are computationally simpler, which renders them more accessible than Bayesian methods, and thus widely used. However, they also have limitations, such as problems with taking into account a priori knowledge. In addition, using them, one cannot get a full picture of the uncertainty of estimates. The choice between the Bayesian and the frequentist approaches is often very subjective. Both have strengths and weaknesses, and the preference of one over the other depends on the specific context and the type of problem addressed.

Frequentist-based statistical inference for MSV is very limited compared to the Bayesian analysis. The quasi-maximum likelihood method of Harvey et al. (1994) is restricted only to models with constant correlation. Jungbacker and Koopman (2006) proposed importance-sampling Monte Carlo techniques for the maximum likelihood estimation of the SV of three specific multivariate extensions of the basic SV model.

In this paper, we propose to use an iterated filtering algorithm (Ionides et al. 2006, 2015) for estimating the bivariate SV model of Yu and Meyer (2006). Iterated filtering is a frequentist-based method based on particle filters that can be used to estimate parameters for general non-linear or non-Gaussian state-space models (SSM). Despite being limited to only a bivariate relationship, the Yu and Meyer model has gained popularity due to its ability to model the dynamic correlation between a pair of assets.

Johansson (2010b) used the Yu and Meyer model to study the systematic risk of sovereign bonds in four East Asian countries, and the relationship between stocks and bonds in nine Asian countries (Johansson, 2010a). Du et al. (2011) applied it to the investigation of volatility spillovers in the crude oil and agricultural commodity markets. Hui and Zheng (2012a, 2012b) examined the correlations between housing and retail property markets in Hong Kong by means of it. Gębka and Karoglou (2013) used the Yu and Meyer model to explore the integration of the European peripheral financial markets with Germany, France, and the UK. Kliber (2011) applied it to the study of the correlation between selected sovereign Central European credit default swaps. Recently, Kliber et al. (2019) used the Yu and Meyer model to check whether Bitcoin can act as a hedge, diversifier or safe haven in five countries (Japan, Venezuela, China, Estonia and Sweden), whereas Będowska-Sójka and Kliber (2021) examined by means of it the safe-haven properties of gold and two cryptocurrencies, Bitcoin and Ether, for four main stock indices (S&P500, FTSE, DAX and STOXX600). All the above-mentioned applications of the Yu and Meyer model utilised the Bayesian approach for parameter estimation.

The most valuable contribution of this paper to the existing body of research is the proposed frequentist-based estimation method for the Yu and Meyer model. The frequentist-based approach may also be applied to the estimation of the filtering distribution of log-volatilities and the dynamic correlation using the standard bootstrap particle filter. The paper further consists of: Section 2 introducing the Yu and Meyer model and categorising it as a special case within the broader class of state-space models, Section 3 presenting the estimation methodology, Section 4 featuring an empirical example, Section 5 where a simulation study is conducted, and Section 6 presenting the conclusions of the study.

2. Yu and Meyer model

Yu and Meyer (2006) proposed a bivariate SV model for which not only volatilities but also correlation coefficients are time-varying. This model describes the evolution of two asset returns through time. Let us denote the observed mean-centred log-returns at time t as $y_t = (y_{1t}, y_{2t})'$ for $t = 1, \dots, T$. Let $h_t = (h_{1t}, h_{2t})'$ be a vector of log-volatiles, $\mu = (\mu_1, \mu_2)'$ a vector of long-term means of log-volatiles, $\Omega_t = diag(\exp(h_{1t}/2), \exp(h_{2t}/2))$ a diagonal matrix of log-return standard deviations, and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ and $\eta_t = (\eta_{1t}, \eta_{2t})'$ two vectors of error terms. The model might then be written as:

$$
\begin{cases}
y_t = \Omega_t \varepsilon_t, \qquad \varepsilon_t | \Omega \sim iid\, N(0, \Sigma_{\varepsilon,t}), \\
\Sigma_{\varepsilon,t} = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix}, \\
h_{t+1} = \mu + diag(\phi_1, \phi_2)(h_t - \mu) + \eta_t, \eta_t \sim iid\, N\left(0, diag(\sigma_{\eta 1}^2, \sigma_{\eta 2}^2)\right), \\
q_{t+1} = \psi_0 + \psi_1(q_t - \psi_0) + \sigma_\eta v_t, \qquad v_t \sim iid\, N(0,1),\ \rho_t = \dfrac{\exp(q_t) - 1}{\exp(q_t) + 1},
\end{cases} \tag{1}
$$

with initial conditions: $h_0 = \mu$ and $q_0 = \psi_0$. Error terms ε_t, η_t and v_t are independent. Correlation matrix $\Sigma_{\varepsilon,t}$ is well-defined, because the inverse Fisher transformation for q_t constrains ρ_t to interval (–1,1). Log-volatiles h_t follow reverting autoregressive processes to the order one mean. The model is equivalent to the bivariate GARCH model with dynamic conditional correlation (DCC-MGARCH). One significant limitation of this model is that it is difficult to extend it to higher dimensions. The main challenge is to ensure the positive definiteness of correlation matrix $\Sigma_{\varepsilon,t}$. Asai et al. (2006) suggested the following solutions for situations where the dimension of log-returns is larger than two: the Cholesky decomposition (Tsay, 2005), the matrix exponential (Chiu et al., 1996), and the

Wishart models (Gouriéroux, 2006). However, in many practical situations, the principal goal is to examine the temporal correlation between a pair of assets.

There are nine parameters to be estimated in the Yu and Meyer model, namely $\mu_1, \mu_2, \phi_1, \phi_2, \sigma_1, \sigma_2, \psi_0, \psi_1$ and σ_η. The authors employed a Bayesian approach, defining separate prior distributions for each of the considered parameters. They used the WinBUGS, which enables a convenient and efficient implementation of the single-move Gibbs sampler. All the examples of the application of the Yu and Mayer model mentioned in the introduction also use the Bayesian approach through WinBUGS[1] or OpenBUGS (often using the R2WinBUGS and R2OpenBUGS R packages, respectively (Sturtz et al., 2005)). To the author's best knowledge, there has been no attempt to estimate the Yu and Meyer model in the classical inference paradigm so far.

In fact, the Yu and Meyer model is, like most (multivariate) SV models, an example of a broader class of statistical models called state-space models (SSMs). This class provides a general framework for analysing a hidden stochastic process that is measured or observed through another stochastic process. SSMs are very flexible and have been widely applied in economics, ecology, epidemiology, medicine (mainly neuroscience), signal processing and mechanical system monitoring (see Chapter 1 in Cappé et al. (2005) and Chapter 2 in Chopin and Papaspiliopoulos (2020) for details).

More specifically, an SSM consists of a pair of discrete-time processes: $\mathbb{Y}_t = (Y_t)_{t\geq 0}$, i.e. the measurement process, and $\mathbb{X}_t = (X_t)_{t\geq 0}$, i.e. the latent state process. The observable random variables $\mathbb{Y}_t$ are assumed to be conditionally independent given $\mathbb{X}_t$. According to the definition of SSMs, the latent process model is determined by the set of densities $\left(f_t(x_{t+1}|x_t;\theta)\right)_{t\geq 0}$ and the initial density $f_0(x_0;\theta)$ (i.e. the state process, $\mathbb{X}_t$, is Markovian). The measurement process is determined by the set of densities $\left(g_t(y_t|x_t;\theta)\right)_{t\geq 0}$ (see Chapter 2 of Chopin and Papaspiliopoulos (2020) for a detailed examination of this definition of SSMs and two alternative ones).

In the case of the Yu and Meyer model at time *t*, the latent state process is $X_t = (h_{1t}, h_{2t}, q_t)'$, and the measurement process is $Y_t = (y_{1t}, y_{2t})'$. The latent process density (at time *t*) $f_t(x_{t+1}|x_t;\theta)$, due to independence of error terms η_t and υ_t, may be decomposed as:

$$h_{t+1}|h_t \sim N\left(\mu + diag(\phi_1, \phi_2)(h_t - \mu), diag\left(\sigma_{\eta 1}^2, \sigma_{\eta 2}^2\right)\right), \quad (2)$$

[1] WinBUGS 1.4.3 is available for routine use, but is no longer being developed (Lunn et al., 2009).

$$q_{t+1}|q_t \sim N\left(\psi_0 + \psi_1(q_t - \psi_0), \sigma_\eta^2\right), \tag{3}$$

with the initial densities:

$$h_0 \sim \delta(h_0 - \mu), q_0 \sim \delta(q_0 - \psi_0), \tag{4}$$

where δ is delta Dirac function (i.e. $P(h_0 = \mu) = 1$, $P(q_0 = \psi_0) = 1$). The measurement process density (at time t) may be written as

$$y_t|h_t, q_t \sim N\left(0, \Omega\Sigma_{\varepsilon,t}\Omega'\right), \tag{5}$$

where:

$$\Omega_t = diag(\exp(h_{1t}/2), \exp(h_{2t}/2)), \Sigma_{\varepsilon,t} = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix}, \rho_t = \frac{\exp(q_t) - 1}{\exp(q_t) + 1}. \tag{6}$$

The SSM resulting from the Yu and Meyer model is non-linear (exponential transformation for h_t and the inverse Fisher transformation for q_t) and Gaussian. Strong non-linearity excludes the direct use of the Kalman filter. Particle filters are an established method of estimating the latent state in a nonlinear/non-Gaussian SSM when the parameters are fixed. Particle filters also provide an unbiased likelihood estimator, but they cannot be used directly to estimate likelihood, because the estimator of likelihood is not continuous as a function of parameters θ (Malik & Pitt, 2011). Xu and Jasra (2019) proposed a particle filter technique for the inference of high-dimensional SV models with a constant correlation matrix, but the parameter estimation is based on the Bayesian particle marginal Metropolis-Hasting algorithm (Andrieu et al., 2010).

3. Estimation method

Estimating parameters for state-space models (SSMs) is a complex task. The primary obstacle is that the exact computation of likelihood functions is not possible, as it requires evaluating multiple integrals. Another significant difficulty is that SSMs often generate log-likelihoods that are awkward to optimise numerically, for example non-concave, multi-modal, or flat (in certain directions) ones (Chopin & Papaspiliopoulos, 2020, p. 260).

Iterated filtering was pioneered by Ionides et al. (2006), and theoretically substantiated by Ionides et al. (2011). The second generation of iterated filtering, IF2, was introduced by Ionides et al. (2015) and developed by a theoretical study of

Nguyen (2016). Even though both generations of iterated filtering employ a recursive filtering approach through an augmented model, their theoretical foundations differ. The first generation (IF1) approximates the Fisher score function, while the second one (IF2) combines the concept of data cloning (Lele et al., 2007) with the convergence of an iterated Bayes map (Nguyen, 2016). Empirical studies showed that the IF2 outperforms the IF1 (Ionides et al., 2015). The calculations in this article used exclusively the second generation of the algorithm.

Iterated filtering has been successfully applied to many SSMs, mostly in the context of epidemiology (Bhadra et al., 2011; He et al., 2009; King et al., 2008; Stocks et al., 2020; You et al., 2020), but also in economic modelling, especially for univariate SV models (Bretó, 2014; Szczepocki, 2020).

Iterated filtering is a technique that utilises particle filters and involves replacing the model we are interested in with a similar model, but with parameters that take a random walk in time. This extra variability smooths the likelihood surface (which is the main impediment for particle filters in parameter estimation) and counteracts particle depletion. Over multiple repetitions of the filtering procedure (each made using a particle filter), the variance of this random walk goes to zero and the augmented model approaches the original one. As a result, iterated filtering provides a sequence of iteratively updated parameter estimates that converge to the maximum likelihood estimate (see Ionides et al., 2015; Nguyen, 2016, for details). Thus the algorithm is *likelihood-based*.

In practical applications, the convergence of the algorithm is often assessed via diagnostic plots (see e.g. Bretó, 2014; King et al., 2008; Szczepocki, 2020). Iterated filtering uses only a basic bootstrap particle filter (Gordon et al., 1993), and thus it does not have to evaluate the transition density $f_t(x_{t+1}|x_t;\theta)$. It only requires the capability to simulate from this density (*simulation-based*). This *simulation-based* methodology has developed fast because of the relatively non-restrictive requirements, but its main representatives follow the Bayesian paradigm, i.e. the Approximate Bayesian Computation (Toni et al., 2009) and the Particle Markov Chain Monte Carlo (Andrieu et al., 2010), SMC^2 (Chopin et al., 2013).

To sum up, iterated filtering is one of few, if not the only method for the maximum likelihood inference in general state-space models (SSMs) that satisfy the three following conditions: it is *likelihood-based* (applies full data-likelihood inference), *simulation-based* (captures dynamics of the model only via the simulation of $f_t(x_{t+1}|x_t;\theta)$), and *frequentist-based* (based on a frequency interpretation of probability).

The necessary conditions for the application of the iteration algorithm to a specific SSM include the ability to:

- simulate from the initial density $f_0(x_0;\theta)$;
- simulate from the transition density $f_t(x_{t+1}|x_t;\theta)$;
- evaluate the measurement density $g_t(y_t|x_t;\theta)$.

In the case of the Yu and Meyer model, all the above conditions are fulfilled. The initial conditions are with probability one, which in practical implementations are treated as not random. However, there is also a possibility to initially draw conditions from stationary distributions:

$$h_{0i} \sim N\left(\mu_i, \frac{\sigma_i^2}{1-\rho_i^2}\right), i = 1,2, \tag{7}$$

$$q_0 \sim N\left(\psi_0, \frac{\sigma_\eta^2}{1-\psi_1^2}\right) \tag{8}$$

Simulating from the transition density is straightforward because it requires drawing from a normal distribution (Equations (2) and (3)). Similarly, evaluating the measurement density is immediate, as it is based on the bivariate normal distribution (Equations (5) and (6)).

All the computations presented in the article were made using the *POMP* package (Partially Observed Markov Processes, King et al., 2016) written for the R statistical computing environment (R Development Core Team, 2010). To make the calculations faster, the code of initial, transition and measurement density was written in the C programming language.

4. Empirical example

In this section, we will apply our Yu and Meyer model-estimation method to the estimation of the correlation between the returns of two assets, i.e. Standard and Poor's 500 index (S&P500) and the price of gold in US dollars. The sample period from 22 July 2014 to 3 March 2022 yielded a total of 1,751 observations of logarithmic returns multiplied by 100 (we excluded those days when at least one of two observations was not reported). The data came from Eikon Refinitiv Database. Figure 1 presents time series of assets prices (top row) and returns (bottom row). Table 1 shows summary statistics of returns which demonstrate that the S&P500 returns are more leptokurtic and more left-skewed than the gold returns. As in Yu and Meyer (2006), the returns were mean-corrected before the estimation.

Gold is often considered a safe haven asset, which means that it is expected to retain or increase its value during periods of economic or political uncertainty. This is because for centuries gold has been the carrier of value and was able to retain purchasing power over time. During crises, investors tend to resort to gold as a way to protect their assets from inflation or currency devaluation.

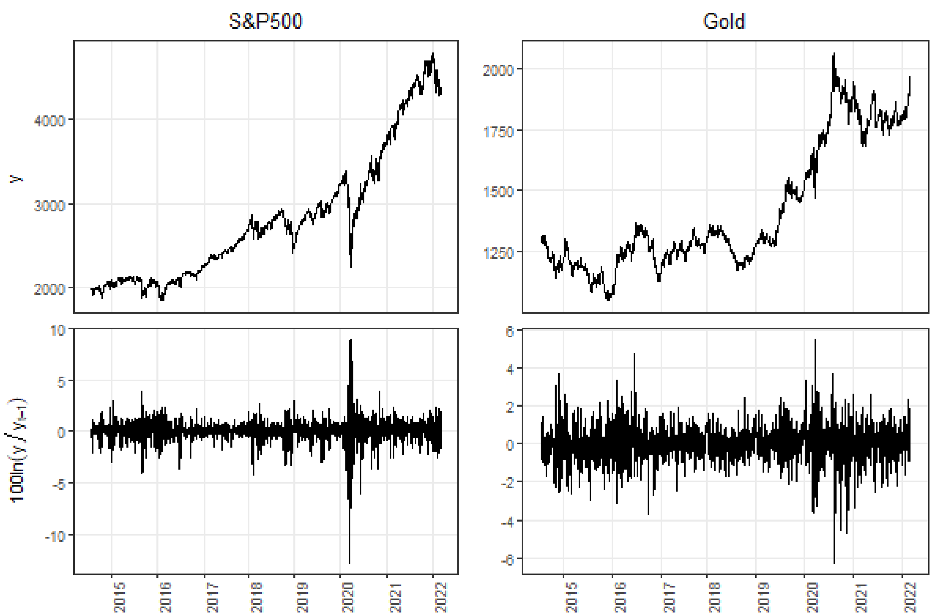
Baur and Lucey (2009) introduced a precise conceptual distinction between a ‘safe haven’ and a ‘hedge’. A safe haven asset is defined as a security that is uncorrelated with stock market returns in the case of market crash, and a hedge as a security that is uncorrelated with the stock market on average. Baur and McDermott (2010) distinguished between strong and weak safe haven effects. A strong safe haven is an asset that is negatively correlated, and a weak safe haven is one that is uncorrelated with another asset or portfolio at a time of the falling stock prices.

Table 1. Summary statistics of logarithmic returns multiplied by 100 calculated for Standard and Poor's 500 index (S&P500) and the price of gold in US dollars in the period from 22 July 2014 to 3 March 2022

Asset	Summary statistics								
	Mean	St. Dev.	Skewness	Kurtosis	Minimum	Q1	Median	Q3	Maxi-mum
S&P500	0.045	1.170	−1.168	22.808	−12.765	−0.342	0.069	0.550	8.968
Gold	0.023	0.928	-0.207	7.157	−6.254	-0.452	0.033	0.500	5.477

Source: author’s work.

Figure 1. Time series of Standard and Poor's 500 index (S&P500) and the price of gold (top row) and corresponding logarithmic returns multiplied by 100 (bottom row) in the period from 22 July 2014 to 3 March 2022



Source: author's work.

Table 2 presents the maximum likelihood estimation results for the analysed data. We used the following algorithmic settings: 200 iterations, 1,000 particles, random-walk perturbations with the initial 0.01 perturbations, and a geometric decay of perturbations of α = 0.5 (the perturbations at the end of 50 iterations are a fraction α smaller than they are at first) for all the parameters. The estimation of the log-likelihood results from taking the average of nine likelihood evaluations of a bootstrap particle filter with 1,000 particles, from which we also calculate the Monte Carlo standard error. Sandard errors of the parameters were calculated via numerical approximation to the Hessian (see supporting text by Ionides et al. (2006) for details of the procedure). Figures A1 and A2 in the Appendix show diagnostics plots.

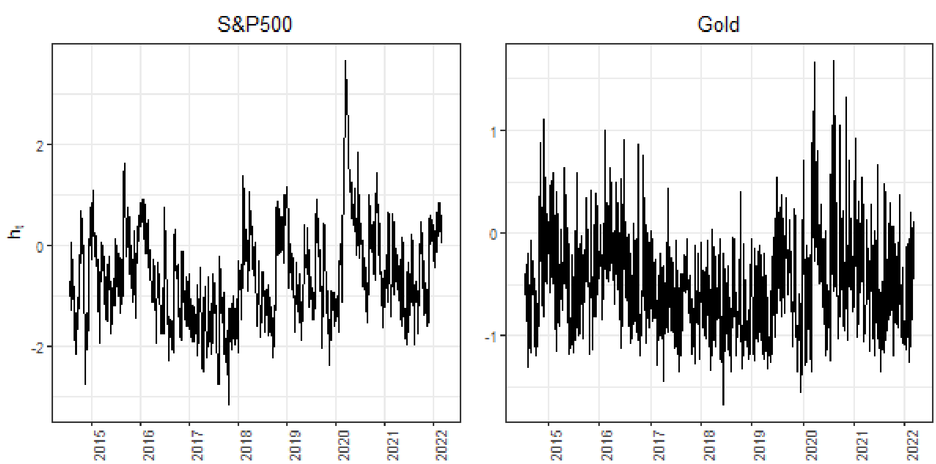
Table 2. Parameter and log-likelihood estimates and standard errors (in parentheses) of the Yu and Meyer model obtained using iterated filtering for the analysed data

Log-likelihood	S&P500			Gold			Conditional correlation		
	μ_1	ϕ_1	σ_1	μ_2	ϕ_2	σ_2	ψ_0	ψ_1	σ_η
−4344.984 (1.139)	−0. 6408 (0.0767)	0.9313 (0.0067)	0.3939 (0.0085)	−0. 5003 (0.0512)	0.7668 (0.0090)	0.5236 (0.0089)	−0.3769 (0.0768)	0.9746 (0.0055)	0.1425 (0.0089)

Note. Algorithm settings: 200 iterations, 1,000 particles, random-walk perturbations with the initial 0.01 perturbations, and a geometric decay of perturbations of α = 0.5 for all the parameters.
Source: author's work.

Comparing the estimation results of log-volatilities for S&P500 and gold, we can see that the stock index has the value of persistency parameter ϕ closer to one and a smaller value of conditional volatility parameter σ. The former parameter controls the persistency of log-volatility (1- ϕ is the strength of a mean-reversion towards the unconditional mean μ after a shock in log-volatility), while the latter regulates its variability. Consequently, the volatility of the S&P500 index is more clustered and less time-varying than the volatility of gold. These differences are visible on the plots of logarithmic returns (bottom row of Figure 1) and log-volatilities (Figure 2).

Figure 2. The mean of the filtering distributions of the log-volatilities.



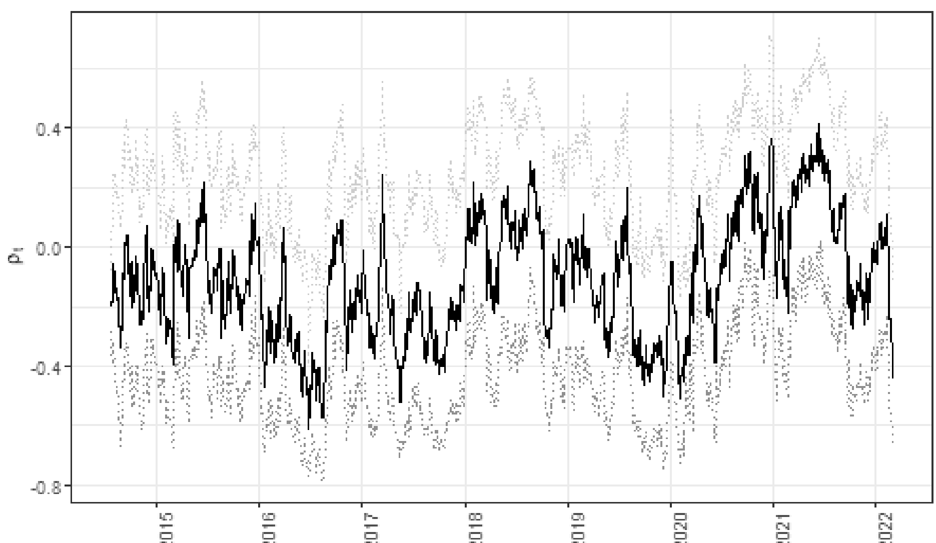
Note. The results were obtained by using a bootstrap particle filter with 1,000 particles.
Source: author's work.

Parameters ψ_0, ψ_1 and σ_η control the Fisher-transformed conditional correlation process q_t. Similarly to log-volatilities, this is also the mean-reverted autoregressive process of order 1, and consequently, the parameters have similar interpretations: ψ_0

is the long-term mean, and ψ_1 and σ_η regulate the persistency and the variability of the transformed conditional correlations, respectively. On the basis of the estimated value of ψ_0, after the inverse Fisher transformation, we obtain the long-term value of correlation –0.186. This value can be interpreted as a slightly or moderately negative long-term correlation. The strength of the mean-reversion is very high – higher than in the case of both log-volatilises.

Figure 3 presents a plot of conditional correlation process ρ_t. For most of the sample period, the conditional correlation had a negative value, but the trajectory was volatile with several picks above zero. For the period of the COVID-19 pandemic (2020–2022), the relationship between the S&P500 index and gold was mainly positive. Generally, in the sample period, gold acted as a hedge for the S&P500 index, but failed to be a safe haven asset during the COVID-19 crisis. Będowska-Sójka and Kliber (2021) arrived at a similar conclusion, as they analysed, among other things, the safe haven characteristics of gold versus the S&P500 index: 'gold tends to take the role of a safe haven asset in relatively short periods, yet the recent COVID crisis does not belong to them'.

Figure 3. The mean (solid black line), the 0.05 quantile and the 0.95 quantile (dotted lines) of the filtering distributions of the dynamic conditional correlation



Note. The results were obtained by using a bootstrap particle filter with 1,000 particles.
Source: author's work.

5. Simulation study

As the convergence of the iterated filtering algorithm to the maximum likelihood estimates is difficult to demonstrate analytically, we assessed its performance through a simple simulation study. In order to evaluate the precision of the parameter estimation, we conducted 100 time-series simulations of the Yu and Meyer model, using identical parameter values as those derived from the empirical study detailed in Section 4 (see Table 2). Each simulation has the same length as the analysed time series of the S&P500 index and gold (1,751 observations). We followed the same estimation procedure as in the empirical study for each simulation. The results of the study are presented in Table 3, which consists of the mean errors (ME) and the root of mean square errors (RMSE) of parameter estimates. The obtained results show that there is no such type of a parameter (long-term mean, persistency, volatility) that would be biased in one direction. The obtained RMSE are comparable with standard errors from the empirical study. The results indicate that the proposed method is reliable to a sufficient degree.

Table 3. Results of the simulation study based on a 100-time-series simulation of the Yu and Meyer model with the same parameter values as those obtained in the empirical study and the same length of time series as the analysed time series of the S&P500 index and gold

Measure of errors	S&P500			Gold			Conditional correlation		
	μ_1	ϕ_1	σ_1	μ_2	ϕ_2	σ_2	ψ_0	ψ_1	σ_η
ME	0.0294	0.0154	–0.0446	–0.0326	–0.0403	0.0851	0.0025	0.0182	–0.0175
RMSE	0.0399	0.0009	0.0049	0.0104	0.0039	0.0101	0.1006	0.0057	0.0134

Note. Algorithm settings: 200 iterations, 1,000 particles, the random-walk perturbations with initial 0.01 perturbations, and geometric decay of perturbations of $\alpha = 0.5$ for all parameters.

Source: author's work.

6. Conclusions

The main contribution of this paper to the is the existing body of research is the proposed iterated filtering algorithm for estimating the bivariate SV model of Yu and Meyer. It allows the estimation of parameters in the frequentist-based approach. Consequently, the filtering distribution of log-volatilities and the dynamic correlation might be estimated in the frequentist-based approach using a standard bootstrap particle filter. In order to show the effectiveness of the proposed estimation method, an empirical example was presented, in which the Yu and Meyer model was used to estimate the dynamic correlation between the returns of two assets, i.e. Standard and Poor's 500 index (S&P500) and the price of gold in US dollars. The results indicated that gold acted as a hedge within the observed period, but during the COVID-19 pandemic, it failed to perform as a safe haven. The obtained parameter estimates from the empirical study were confirmed in the simulation experiment. One of the limitations of our study was its restriction to a bivariate model of stochastic volatility. Further research in this area should determine whether the proposed approach proves effective in multivariate models of stochastic volatility based on the Cholesky decomposition or a matrix exponential.

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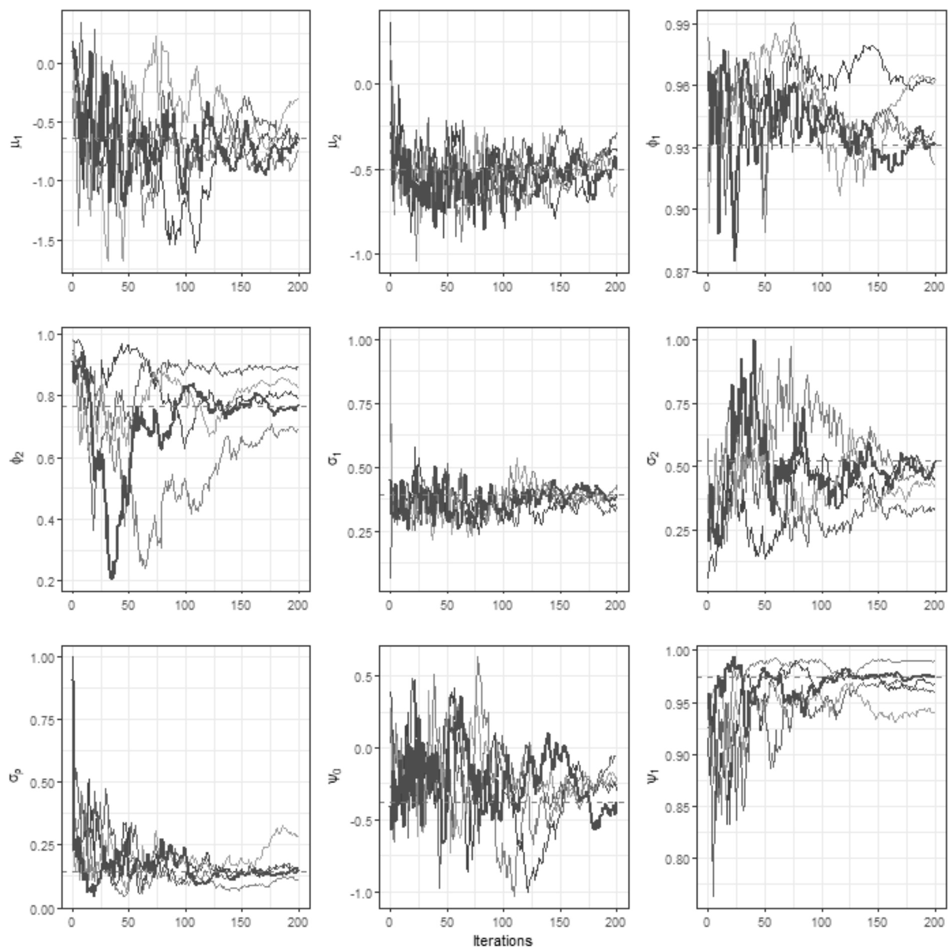
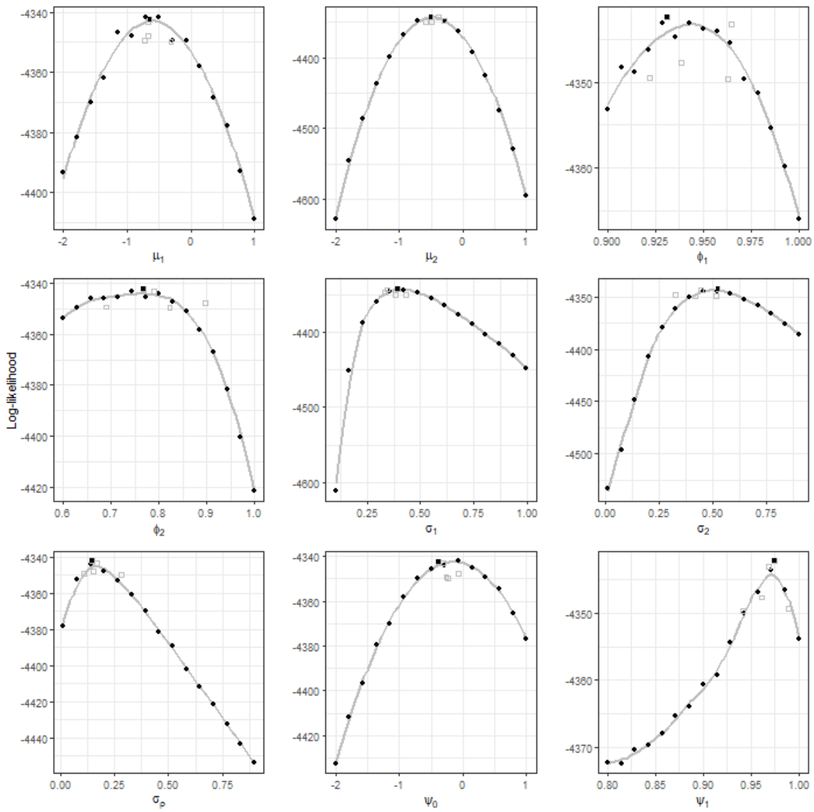
Appendix

Figure A1. Five trajectories of the algorithm with random start points (updates of estimation values during iteration steps)



Note. The bolded trajectory converges to the highest value of log-likelihood.
Source: author's work.

Figure A2. Sliced log-likelihoods for the analysed parameters



Note. The log-likelihood surface is explored along one of the parameters, keeping the others fixed at the point to which iterated filtering converges (see Table 2 for values). Each black circle shows the log-likelihood estimation obtained with 2,000 particles. The grey curve results from smoothing the log-likelihood evaluations with local quadratic regression (R base function *loess*). The empty squares correspond to the ends of the five trajectories of the iterated filtering algorithm (see Figure A1 for the trajectory plots). The black squares correspond to the estimated values.

Source: author's work.