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Reduce extreme losses and retain extreme profits through hedging with gold and cryptocurrencies: A global stock market perspective

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Abstract. The study focuses on the safe-haven and hedging properties of gold and selected cryptocurrencies against stock markets' extreme risk observed during the COVID-19 pandemic and the Russian invasion of Ukraine. The loss reduction is compared with the profit sacrifice obtained through hedging in terms of the tail thickness of the return distribution. The findings show that gold is able to reduce extreme losses more intensively than extreme profits. Tether reduces volatility and tail risk the most effectively but it is characterised by the worst profit/risk ratio. Bitcoin and Ether increase investment risk; thus, they fail to act as an effective hedge or a safe haven. On the other hand, these cryptocurrencies added to the stock portfolio increase the probability of extreme profits more than extreme losses. The paper provides new insights into the benefits of safe-haven or hedging strategies.

Keywords: gold, cryptocurrencies, conditional value at risk, distribution tail, hedging, safe haven

JEL: C13, C58, G11, G15

1. Introduction

In December 2019, a new virus called SARS-CoV-2 had started to spread rapidly all over the globe, causing the COVID-19 disease and about seven million deaths. A series of unprecedented government interventions to control the infection brought the economy to a standstill. Financial markets also replied with crashes to an extent that had not been observed since the global financial crisis of 2008. In February 2022, the Russian invasion of Ukraine threw global financial and commodity markets into further turmoil. At times like these, investors avoid risky stocks searching for safe-haven assets which, when added to their investments, protect the portfolio from enormous losses. The term ‘safe haven’, introduced by Baur and Lucey (2010), delineates an instrument that exhibits either uncorrelated or negatively correlated behaviour with the held assets during a market crash. If it is uncorrelated or negatively correlated with the held assets on average, it is called

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a hedge. In this paper, the term 'hedging' is employed in a broad sense, encompassing events occurring in extreme market conditions. Finally, if an instrument is positively (but not perfectly) correlated with the held assets on average, it is called a diversifier.

Gold is most commonly considered as a safe-haven asset for stock markets (Baur & McDermott, 2010; Beckmann et al., 2015; Boubaker et al., 2020; Gürgün & Ünalmiş, 2014) due to its relatively low volatility and low correlation with stock markets. Additionally, the fundamental price of gold depends on the economic state of other markets (Baur & Glover, 2014). However, its ability to become a shelter against stock market risk weakened during the COVID-19 period (Akhtaruzzaman et al., 2021; Al-Nassar et al., 2023; Chemkha et al., 2021; Echaust & Just, 2022; Hasan et al., 2021; Salisu et al., 2021). At that time, cryptocurrencies have gained enormous popularity as candidates for 'a port in the storm' (Corbet et al., 2020). The potential of cryptocurrencies stems from the fact that they are independent from central authorities. Mariana et al. (2021) find that the two largest cryptocurrencies (Bitcoin and Ether) are suitable as short-term safe-haven assets as their daily returns tended to be negatively correlated with S&P 500 returns during the pandemic. Bouri et al. (2020) indicate cryptocurrencies as safe-haven assets for the aggerate US equity index and selected sectors, whereas Będowska-Sójka and Kliber (2021) as weak safe havens for selected equity indices. On the other hand, the opposite results were presented by Conlon et al. (2020), Conlon and McGee (2020) and Long et al. (2021). They found that Bitcoin and Ether failed in the role of a safe-haven asset. Baur et al. (2022) show that for extreme levels of volatility, Bitcoin does not reduce the risk when added to a benchmark stock portfolio. They consider portfolios consisting of an optimal allocation of Bitcoin (that aims for a minimum variance or a maximum Sharpe ratio) relative to holding the underlying S&P 500. Moreover, Conlon et al. (2020) show evidence of increased downside risk for portfolios consisting of any weight of Bitcoin and Ether relative to the stock index (MSCI World, S&P 500, FTSE 100, FTSE MIB, IBEX). Recently, Just and Echaust (2024) examined the role of five cryptocurrencies as safe havens against the G7 and BRICS stock market risk. The authors find that the conditional probability that Bitcoin can reduce at least 10% of volatility given that index returns fell below the 1st percentile ranges only from 2% to 28% for various stock indices. Moreover, the probabilities calculated for other cryptocurrencies are lower. Xu and Kinkyo (2023) suggest that investing in G7 stocks and Bitcoin in the short term as well as investing in stocks and gold in the long term are reasonable investments for investors. Gold provides higher hedging effectiveness and downside risk reduction than Bitcoin in the long term. The third cryptocurrency considered in our empirical study whose safe-haven properties are widely discussed is Tether. It belongs to a category of cryptocurrencies called

stablecoins which aim to keep its valuation stable. The low volatility of Tether is perceived as a desired property of safe havens. Cheema et al. (2022) show that during a pandemic, investors should look for liquid and stable assets rather than gold. Meanwhile, Tether could act as a safe-haven investment for global stock markets (Conlon et al., 2020; Kliber, 2022).

Although cryptocurrencies may potentially yield high returns, which is encouraging for investors, they also entail high volatility and downside risk compared to gold and other conventional asset classes (Iqbal et al., 2023). Indeed, investors with a limited risk-aversion do not restrict their perception to risk reduction and they also consider profits for investment in their hedging decisions. Given the limited possibility of Bitcoin and Ether to reduce risk, their ability to generate profits in extreme market conditions may be the main argument for their application in hedging strategies. The implementation of any such strategy, which is essentially intended to protect the investment from losses, is bound to have an unfavourable impact on the profit potential. The prospect theory of Kahneman and Tversky (1979) indicates that people are more sensitive to losses than gains. Loss aversion means that investors perceive more disutility from losses than utility from gains of equal size. The preferable hedge strategy should offer asymmetry between risk and profit reductions. Based on stocks traded in China, Japan, Korea and Taiwan, Eom et al. (2021) investigate a trade-off relationship between loss avoidance and a profit sacrifice through a portfolio diversification strategy. According to their results, investors reduce the likelihood of high losses through portfolio diversification; however, their potential for higher profits is thus sacrificed. We adopt this concept to verify the relative benefits from a hedging strategy against stock market tail risk using gold and cryptocurrencies. We study the unexplored relationship between high loss avoidance and high profit sacrifice relative to the hedging strategy.

The first aim of this research is to compare the ability of gold and cryptocurrencies (Bitcoin, Ether and Tether) to act as safe-haven assets against global stock market risk in the context of the COVID-19 pandemic and the Russia–Ukraine war. The second and most important aim is to verify the asymmetry between risk reduction and profit sacrifice. The third aim is to compare two methods of the asymmetry analysis. The approach based on tail thickness is compared to the conditional value at risk (CVaR). To achieve these objectives, we first follow Kroner and Ng's (1998) optimisation procedure based on Engle's (2002) dynamic conditional correlation (DCC) model. We thus estimate the optimal weights of a hedging portfolio. We then calculate the reduction in volatility offered by pair-wise portfolios and estimate the asymmetry between downside and upside risk. To obtain a comprehensive view of the extrema, we focus on the tails of return

distribution which represent extreme losses and extreme profits on investment. We propose to use the peaks over threshold (POT) method to compare the tail behaviour of hedged and unhedged trading positions as well as the upper (profits) and lower (losses) tails of the return distribution. Subsequently, we compare the tail behaviour approach with the estimates of the CVaR, which is recognised as a credible tool for the assessment of the diversification benefits (Conlon et al., 2020; Conlon & McGee, 2020).

Our main findings can be summarised as follows. Firstly, in line with the literature, gold and Tether are found to have been effective hedges in the research period, whereas Bitcoin and Ether increased the investment risk, thus failing to act as an effective shelter against stock market risk. Secondly, and most importantly, gold is the only asset which is able to reduce extreme losses more intensively than extreme profits for chosen indices. Bitcoin and Ether, added to the stock portfolio, increase the probability of extreme profits more than extreme losses. The return distributions of the hedged portfolios consisting of Bitcoin or Ether indicate much fatter upper tails relative to the lower tails. Therefore, cryptocurrencies provide investors with a valuable profit–loss relationship in terms of extreme values. Thirdly, we found that an inference based on CVaR may lead investors to misleading conclusions and decisions. The investigation of the tail behaviour outperforms the approach based on CVaR offering a broader and more reliable view of extreme losses in relation to extreme profits in a trading strategy.

The remainder of the paper is organised as follows. The next section provides data and an in-depth description of the methodological approach adopted in the empirical part of the paper. Section 3 shows detailed results relating to the research objectives. The final section summarises the main findings and presents the conclusions.

2. Data and methodology

2.1. Data

We analyse the log-returns of global stock indices, gold and three cryptocurrencies. Our sample includes stock indices from the world's largest exchanges by market capitalisation (World Federation of Exchanges, 2023). We selected two indices from each of the three regions: the Americas (S&P 500, SPX – United States; S&P/TSX Composite Index, TSX – Canada), the Asia-Pacific region (Shanghai Composite Index, SHC – China; Nikkei 225, NKX – Japan), and Europe, the Middle East and Africa (CAC 40, CAC – France; FTSE 250, FTM – Great Britain). We focus on the leading stock markets to represent viable investors' interests. Our sample includes

three cryptocurrencies with the largest market capitalisation: two non-stable (Bitcoin, BTC-USD; Ether, ETH-USD) and one stable (Tether, USDT-USD). These cryptocurrencies are most often considered as hedging assets or safe havens. The analysis is based on daily data for the period from 2nd January 2019 to 14th August 2023. Gold is quoted on the London Metal Exchange and its prices are sourced from kitco.com. The cryptocurrency prices and the stock indices are obtained from finance.yahoo.com and stooq.pl, respectively.

The descriptive statistics in Table 1 show that Ether has the highest mean return. On the other hand, Tether is the only asset which has a negative mean/median return. Tether and gold have the lowest volatility among the considered assets, whereas the volatility of Bitcoin and Ether exceeds the volatility level of the indices several times over. Gold, Bitcoin and Ether are positively correlated with indices, while Tether negatively (Table 2). Tether appears to have the desired characteristics of a hedging instrument with low volatility and a negative correlation. However, at the same time, it shows a negative mean return, much lower than the other candidates for hedging instruments. It can be a good hedge for risk-averse investors but its usefulness seems to be undermined when return on investment gains importance.

Table 1. Descriptive statistics of asset returns

Asset	Min	Median	Mean	Max	SD
SPX	-12.77	0.0954	0.0511	8.97	1.40
TSX	-13.18	0.1001	0.0305	11.29	1.19
SHC	-8.04	0.0302	0.0233	5.55	1.09
NKX	-6.27	0.0829	0.0453	7.73	1.24
CAC	-13.10	0.1062	0.0387	8.06	1.35
FTM	-9.82	0.0440	0.0056	8.04	1.24
Gold	-5.26	0.0531	0.0339	5.13	0.96
USDT	-5.26	-0.0010	-0.0021	5.34	0.35
BTC	-46.47	0.1048	0.1726	20.30	4.42
ETH	-55.07	0.1205	0.2127	34.35	5.65

Source: authors' work.

Table 2. Correlation between asset returns

Asset/ Index	SPX	TSX	SHC	NKX	CAC	FTM
Gold	0.1420***	0.2179***	0.1097***	0.0309	0.0826***	0.1073***
USDT	-0.2034***	-0.2428***	-0.0051	-0.0067	-0.1498***	-0.0839***
BTC	0.3242***	0.3492***	0.0459	0.0709**	0.2516***	0.2205***
ETH	0.3459***	0.3630***	0.0959***	0.0980***	0.2680***	0.2401***

Note. Correlation means the Pearson correlation between indices and assets; *** and ** indicate significance at the level of 1% and the level of 5%, respectively.

Source: authors' work.

2.2. Methodology

Hedging stocks implies a combined position consisting of stocks and a hedging instrument. In our setting, investors hold stocks and wish to hedge the stock market risk by adding a hedge instrument in a long position. We can model the hedge portfolio as:

$$r_{p,t} = (1 - w_{i,t})\, r_{Index,t} + w_{i,t} r_{i,t}, \tag{1}$$

where $r_{Index,t}$ is the return of the stock market at time t, and $r_{i,t}$ is the return of the i-th hedge price at time t, $w_{i,t}$ represents the time-varying weights of the i-th hedging instrument.

If investors add gold or cryptocurrencies to their stock portfolios with the aim to reduce risk, the optimal weights of the individual assets ($w_{i,t}, i$ = GOLD, USDT, BTC, ETH) to obtain minimum risk portfolios are calculated from the following formula (Kroner & Ng, 1998):

$$w_{i,t} = \frac{h_{Index,t} - h_{Index/i,t}}{h_{Index,t} - 2h_{Index/i,t} + h_{i,t}}, \tag{2}$$

where $h_{Index,t}$ and $h_{i,t}$ are the conditional variances of the index returns and safe-haven candidate returns, respectively. $h_{Index/i,t}$ is the covariance between index and gold, Bitcoin, Ether and Tether returns on the t-th day. We use the DCC model[1] (Engle, 2002) to compute the conditional variance and covariance. Moreover, we assume no short selling of the assets.

We employ the GARCH(1,1) model[2] (Bollerslev, 1986) to obtain conditional volatility ($h_{i,t}$) of asset returns ($r_{i,t}$):

$$r_{i,t} = \mu_i + e_{i,t},\ e_{i,t} = \sqrt{h_{i,t}}\varepsilon_{i,t},\ \varepsilon_{i,t} \sim \mathrm{N}(0,1), \tag{3}$$

$$h_{i,t} = \omega_i + \alpha_i e_{i,t-1}^2 + \beta_i h_{i,t-1}, \tag{4}$$

where $\omega_i, \alpha_i, \beta_i > 0, \alpha_i + \beta_i < 1$.

[1] Akhtaruzzaman et al. (2021) used four models: DCC, asymmetric DCC, corrected DCC and DCC–DECO to describe the relationship between gold and major stock indices during the COVID-19 pandemic, and obtained consistent results. Therefore, we use the simple DCC model.

[2] Although the GARCH(1,1) model is relatively simple, it provides relatively good estimates and predictions of volatility compared to much more complex models (see e.g. Hansen & Lunde, 2005).

Then, we estimate the bivariate DCC model parameters. Let us denote the two-dimensional vector by $\boldsymbol{e}_t = \left(e_{1,t}, e_{2,t}\right)'$. The DCC model assumes that (Engle, 2002):

$$\boldsymbol{e}_t | I_{t-1} \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{H}_t), \boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t, \tag{5}$$

where I_{t-1} is the information set available at time t-1, $\boldsymbol{D}_t = \mathrm{diag}\left(\sqrt{h_{1,t}}, \sqrt{h_{2,t}}\right)$ and conditional variance $h_{i,t}$ is modelled using the GARCH model. In turn, conditional correlation matrix $\boldsymbol{R}_t$ is given by

$$\boldsymbol{R}_t = \left(\mathrm{diag}(\boldsymbol{Q}_t)\right)^{-1/2} \boldsymbol{Q}_t \left(\mathrm{diag}(\boldsymbol{Q}_t)\right)^{-1/2} \tag{6}$$

with

$$\boldsymbol{Q}_t = (1 - a - b)\bar{\boldsymbol{Q}} + a\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1} + b\boldsymbol{Q}_{t-1}\,, \tag{7}$$

where $\bar{\boldsymbol{Q}}$ is the unconditional covariance matrix of $\boldsymbol{\varepsilon}_t$ $\left(\varepsilon_{i,t} = e_{i,t}/\sqrt{h_{i,t}}\right)$; a, b are parameters such that $a, b \geq 0$ and $a + b < 1$.

In the second stage of the analysis, we investigate how adding gold or cryptocurrencies to a portfolio can reduce the risk of a stock portfolio. We examine the conditional variance of portfolio returns by applying the following formula:

$$h_{Portfolio_i,t} = (1 - w_{i,t})^2 h_{Index,t} + {w_{i,t}}^2 h_{i,t} + 2(1 - w_{i,t}) w_{i,t} h_{Index/i,t}. \tag{8}$$

In the third stage, we focus on extreme returns, i.e. using an extreme-value-theory-based method, we compare the distribution tails and the values of the tail risk measure of stock indices and two-component portfolios consisting of these indices and hedging assets. We need to adequately fit the tails of the return distributions to compare the relationship between high loss avoidance and high profit sacrifice. They can be easily modelled using the peaks over threshold method. This method is an approach of the extreme value theory (EVT) that allows modelling all observations in a sample that exceed a high threshold using the generalised Pareto (GP) distribution.

Let R be a random variable of returns with unknown cumulative distribution function (cdf) F, and excess distribution function F_u over high threshold u is defined by

$$F_u(y) = P(R - u \leq y | R > u) = \frac{F(y+u) - F(u)}{1 - F(u)} \text{ for } 0 \leq y \leq r_0 - u, \tag{9}$$

where $r_0 \leq \infty$ is the right endpoint of F.

According to the Pickands-Balkema-de Haan Theorem (Balkema & de Haan, 1974) for a large class of underlying distribution functions F and high enough threshold u, a function $\beta(u)$ exists so that:

$$\lim_{u \to r_0} \sup_{0 \leq y \leq r_0 - u} \left| F_u(y) - G_{\xi,\beta(u)}(y) \right| = 0. \tag{10}$$

As an after-effect of the theorem, F_u can be approximated by a GP distribution, which is defined as:

$$G_{\xi,\beta}(y) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}} \text{ for } \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\beta}\right) \text{ for } \xi = 0, \end{cases} \tag{11}$$

where $\beta > 0$, $y \geq 0$ for $\xi \geq 0$ and $0 \leq y \leq -\beta/\xi$ for $\xi < 0$. Parameters of the GP distribution are scale parameter β and shape parameter ξ.

An approximation of cdf for returns exceeding a sufficiently high threshold can be obtained by transforming (9) and (11):

$$F(r) = G_{\xi,\beta}(r - u)(1 - F(u)) + F(u) \text{ for } r > u. \tag{12}$$

In order to obtain a useful closed form of distribution (12), $F(u)$ can be simply replaced with the empirical estimator of exceedance over threshold u. The estimator is given by $(n - k_u)/n$, where n represents the total number of observations (returns), and k_u is the number of observations exceeding threshold u.

The log-likelihood method is used to estimate the parameters of the GP distribution. The estimator of cumulative distribution F is then given as:

$$\hat{F}(r) = \begin{cases} 1 - \frac{k_u}{n}\left(1 + \hat{\xi}\frac{r - u}{\hat{\beta}}\right)^{-\frac{1}{\hat{\xi}}} \text{ for } \xi \neq 0 \\ 1 - \frac{k_u}{n}\exp\left(-\frac{r - u}{\hat{\beta}}\right) \text{ for } \xi = 0. \end{cases} \tag{13}$$

Value at risk (VaR) and conditional value at risk (CVaR) are the most commonly used measures of tail risk. These measures differ in terms of their mathematical

properties, stability of statistical estimation, simplicity of optimisation procedures and acceptance by regulators (Sarykalin et al., 2008), which determine the choice of their application. We use CVaR to measure the tail risk for long and short positions at a 95% confidence level as it provides an adequate picture of the risks reflected in the extreme tails (Sarykalin et al., 2008). Since CVaR is defined in terms of VaR, we begin by presenting VaR, which can be seen as a quantile of F. Therefore, the q-quantile of the GP distribution for a sample size of length n is calculated as:

$$\widehat{VaR}_q = \begin{cases} u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{k_u}(1-q)\right)^{-\hat{\xi}} - 1\right) \text{ for } \xi \neq 0 \\ u + \hat{\beta}\ln\left(\frac{n}{k_u}(1-q)\right) \text{ for } \xi = 0. \end{cases} \tag{14}$$

CVaR provides the expected size of return that exceeds VaR:

$$CVaR_q = \mathrm{E}\left(R \middle| R \geq VaR_q\right). \tag{15}$$

Hence, CVaR is given as (Dowd, 2005):

$$\widehat{CVaR}_q = \frac{\widehat{VaR}_q}{1-\hat{\xi}} - \frac{\hat{\beta}-\hat{\xi}u}{1-\hat{\xi}} \text{ for } \xi \neq 0 \tag{16}$$

and

$$\widehat{CVaR}_q = \widehat{VaR}_q + \hat{\beta} \text{ for } \xi = 0. \tag{17}$$

3. Empirical results

3.1. Optimal hedging

Figure 1 displays the time-varying weights for all the portfolios considered in this study. The optimal weight for gold behaves in a different way than for cryptocurrencies. It rose significantly in the first quarter of 2020, when global financial markets suffered high losses and then displayed a significant downward trend. The same phenomenon occurred during the Russian invasion of Ukraine in February 2022. The result indicates that in the first phase of the COVID-19 pandemic and later, at the beginning of the war in Europe, investors should have held more gold to reduce risk, thereby the cost of an optimal hedging strategy was

relatively high during those periods. Optimal weights for Ether and Bitcoin are small and often equal to zero. Both cryptocurrencies are not able to reduce risk effectively when added to an equity portfolio. Such a result confirms the findings of Baur et al. (2022). The authors prove that the benefits of Bitcoin in the portfolio come from the expected returns and can enhance the risk-return relationship but do not substantially lower the risk. Tether has different characteristics compared to the foregoing assets. It negatively correlates to most indices and demonstrates low volatility. The effect is that the optimal weight in the portfolio is close to one for most of the time from the second half of 2020. Such a result shows that investors should sell off equity portfolios and replace them with Tether. However, the benefit of this strategy is questionable since it generates a high cost of hedging and entirely changes the investment profile. Additionally, we must not overlook the importance of Tether's lowest median returns compared to other assets.

Figure 1. Optimal weights for hedging instruments

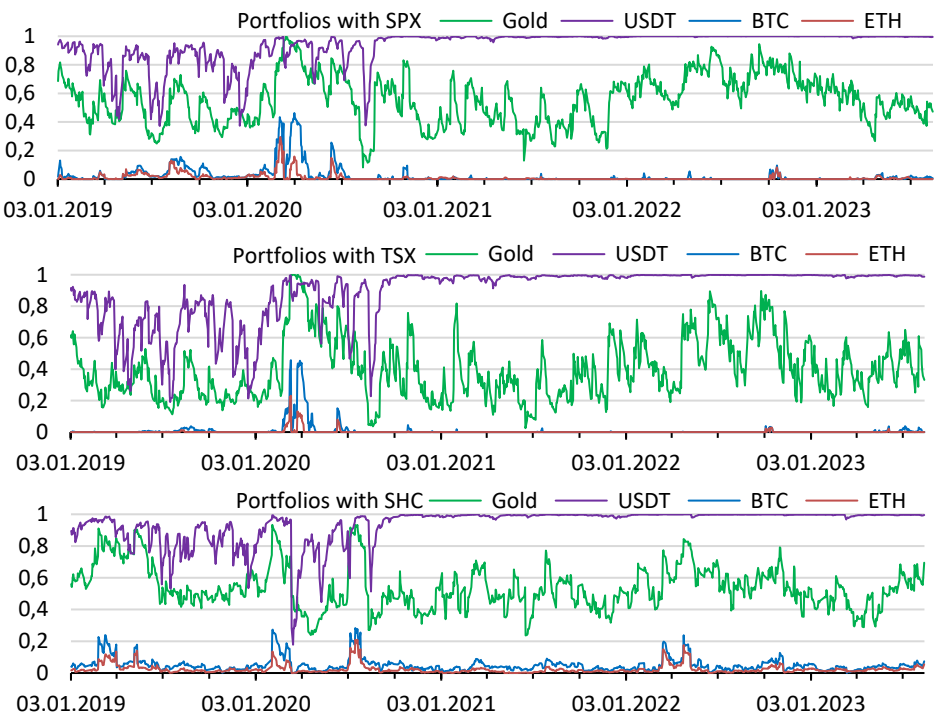
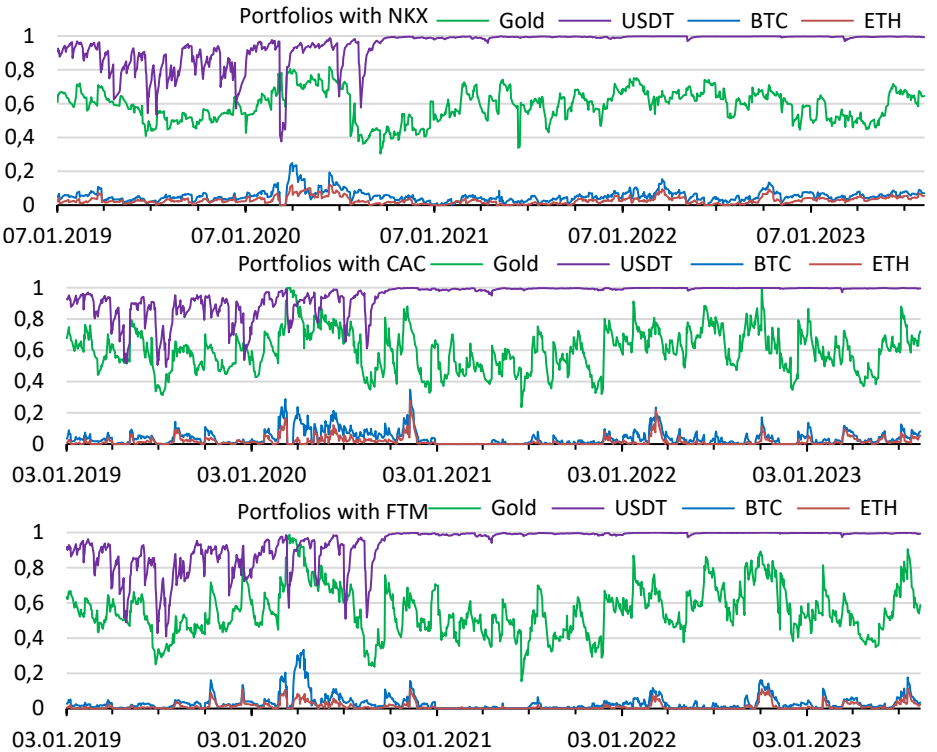


Figure 1. Optimal weights for hedging instruments (cont.)



Source: authors' work.

Table 3 shows the descriptive statistics for optimal portfolios. Adding Bitcoin or Ether to the base portfolio significantly increases downside risk expressed in the minimum return. Gold in the portfolio substantially lowers volatility and range. Note that the portfolio that consists of gold is the only one which can improve the ratio of index performance to volatility (mean/SD) for all the considered assets. On the other hand, Tether is the only hedge instrument which destroys the ratio for all assets. The low risk of the portfolio corresponds to a negative median return and finally makes the mean/risk ratio unattractive. Statistics for the portfolio with Tether are close to those for Tether itself since it almost replaces equities in the optimal hedge strategy.

Table 3. Descriptive statistics of optimal portfolio returns

Portfolio	Min	Median	Mean	Max	SD	Mean/SD
SPX	-12.77	0.0954	0.0511	8.97	1.40	0.0366
SPX + Gold	-5.51	0.0846	0.0469	5.34	0.77	0.0612
SPX + USDT	-3.04	-0.0006	0.0050	5.28	0.32	0.0157
SPX + BTC	-24.24	0.1054	0.0421	8.88	1.52	0.0277
SPX + ETH	-18.82	0.1015	0.0455	8.96	1.48	0.0307

Table 3. Descriptive statistics of optimal portfolio returns (cont.)

Portfolio	Min	Median	Mean	Max	SD	Mean/SD
TSX	-13.18	0.1001	0.0305	11.29	1.19	0.0257
TSX + Gold	-5.41	0.0567	0.0352	5.30	0.70	0.0501
TSX + USDT	-3.52	-0.0008	0.0044	5.26	0.31	0.0140
TSX + BTC	-28.40	0.0961	0.0190	10.36	1.38	0.0138
TSX + ETH	-22.86	0.1001	0.0214	11.29	1.31	0.0164
SHC	-8.04	0.0302	0.0233	5.55	1.09	0.0213
SHC + Gold	-4.17	0.0420	0.0218	3.67	0.74	0.0293
SHC + USDT	-2.89	-0.0002	0.0035	5.09	0.33	0.0106
SHC + BTC	-6.49	0.0224	0.0328	5.30	1.08	0.0303
SHC + ETH	-7.05	0.0233	0.0342	5.64	1.09	0.0313
NKX	-6.27	0.0829	0.0453	7.73	1.24	0.0364
NKX + Gold	-5.01	0.0213	0.0407	5.87	0.78	0.0522
NKX + USDT	-5.85	-0.0013	0.0030	5.02	0.36	0.0084
NKX + BTC	-7.77	0.0740	0.0499	7.48	1.24	0.0404
NKX + ETH	-6.64	0.0644	0.0514	7.58	1.24	0.0413
CAC	-13.10	0.1062	0.0387	8.06	1.35	0.0286
CAC + Gold	-5.98	0.0402	0.0321	5.31	0.77	0.0418
CAC + USDT	-4.12	-0.0005	0.0013	5.11	0.33	0.0038
CAC + BTC	-19.80	0.1093	0.0452	8.06	1.42	0.0319
CAC + ETH	-17.71	0.1148	0.0404	8.06	1.40	0.0288
FTM	-9.82	0.0440	0.0056	8.04	1.24	0.0045
FTM + Gold	-5.94	0.0277	0.0221	5.30	0.75	0.0296
FTM + USDT	-4.12	-0.0009	-0.0014	5.06	0.35	-0.0040
FTM + BTC	-14.41	0.0454	0.0073	7.87	1.27	0.0057
FTM + ETH	-12.42	0.0415	0.0064	8.04	1.26	0.0050

Source: authors' work.

Figure 2. Reduction in conditional volatility (in pp.) by optimal portfolios relative to indices

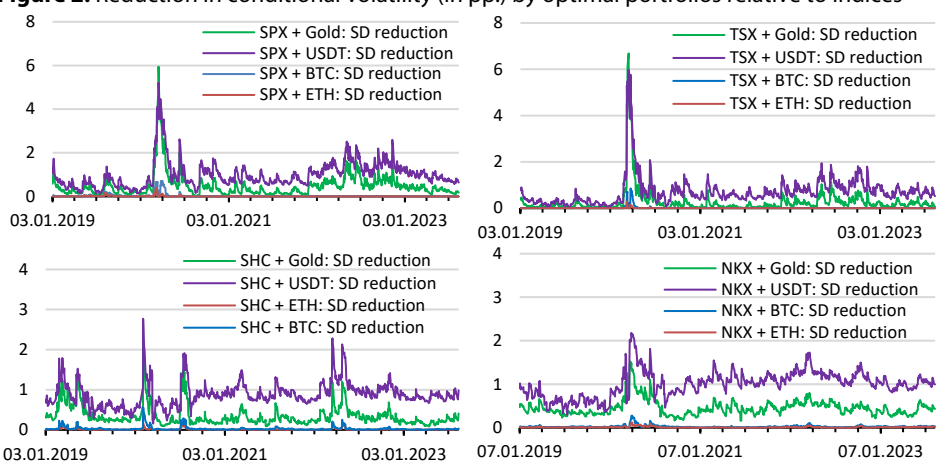
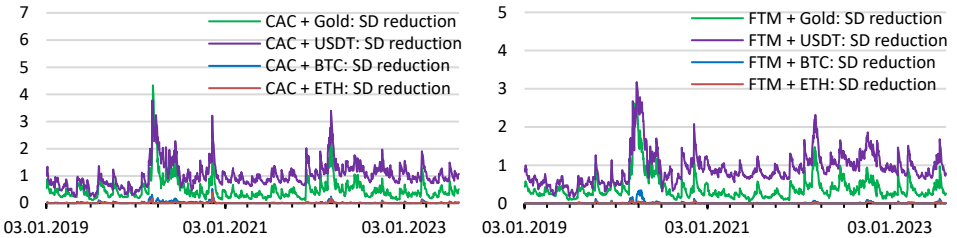


Figure 2. Reduction in conditional volatility (in pp.) by optimal portfolios relative to indices (cont.)



Source: authors' work.

Figure 2 demonstrates how much the conditional volatility of an equity portfolio is reduced by applying an optimal hedging strategy. Gold and Tether can significantly decrease volatility (the Wilcoxon signed rank test for median equality with significance level of 1%), especially in times when the financial market collapsed. Bitcoin and Ether can decrease the risk only to a small extent. Even in periods of a volatility explosion, in March 2020, when optimal weights were relatively high, both cryptocurrencies were not able to significantly reduce the risk. This result proves that Bitcoin and Ether cannot be considered as safe-haven assets.

3.2. Extreme losses versus extreme profits in optimal hedging

Risk-averse investors have no motivation to apply Bitcoin or Ether in the hedging role. This result confirms the findings of Echaust et al. (2024) who compared the hedging effectiveness of cryptocurrencies in a short hedge strategy with favoured index futures contracts. However, the ability of cryptocurrencies to generate abnormal profits may be the key argument to consider in hedging decisions. Their independence from financial market fundamentals might on the one hand provide diversification benefits and generate high returns during financial crashes on the other. The aim is to indicate to what extent hedging strategies reduce extreme risk compared to extreme profits. We propose the comparison of the tail behaviour of the considered assets and portfolios, and compare the findings with the results based on the CVaR. The tails of return distributions are estimated using the peaks over threshold method. The first task involves choosing the appropriate tail threshold which separates the extrema from the middle part of the return distribution. The appropriate selection of a threshold level is considered to be a complex and challenging task. While there are many concepts and approaches presented in the literature, none of them have been indicated to be suitable, and there is no single answer as to where the distribution tail begins. Searching for the tail of the distribution is always a trade-off between bias and variance. If the chosen threshold is too low, the tail estimates indicate a high bias. The more the threshold is away

from the tail, the more the empirical distribution of extrema deviates from the theoretical model. On the other hand, a too high threshold results in high variance of the model estimates since not much data exceeds the threshold. Numerous authors applied a fixed percentile of the total sample size as the threshold, usually 10%, 5% or 1% of the upper statistics (Bee et al., 2016; Echaust, 2021; Fernandez, 2005; Gençay et al., 2003; Longin, 2000; McNeil & Frey, 2000; Totić & Božović, 2016). More sophisticated approaches use a threshold selection based on graphical techniques based on a mean excess plot (Aboura, 2014; Cifter, 2011; Gilli & Këllezi, 2006; Łuczak & Just, 2020) or the graphical representation of the Hill estimator (Hill, 1975). However, the choice procedures of the graphical-based threshold require the identification of the stable regions in the graphs; therefore, they are highly subjective and difficult to apply in empirical studies. Finally, in some studies, the choice of the threshold is based on optimisation procedures. An extensive overview of such methods is provided in Caeiro and Gomes (2016) and Danielsson et al. (2016). A simulation study of Just and Echaust (2021) showed that most of the optimisation algorithms return a too high threshold to calculate tail risk measures according to the requirements of the Basel Committee. We decided to apply the widely accepted in the literature thresholds equal to the 5th percentile for the lower tail and the 95th for the upper tail. Parameters of the general Pareto distribution (Formula 11) for threshold exceedances are computed with the evir R package. For the sake of brevity, we do not present the estimations of the parameters and they are available from the authors upon request. Finally, according to Formula (13), we obtained the estimate of the upper tail of the unconditional distribution for returns. A lower tail can be considered in the same way as the upper tail after the multiplication of returns by minus one.

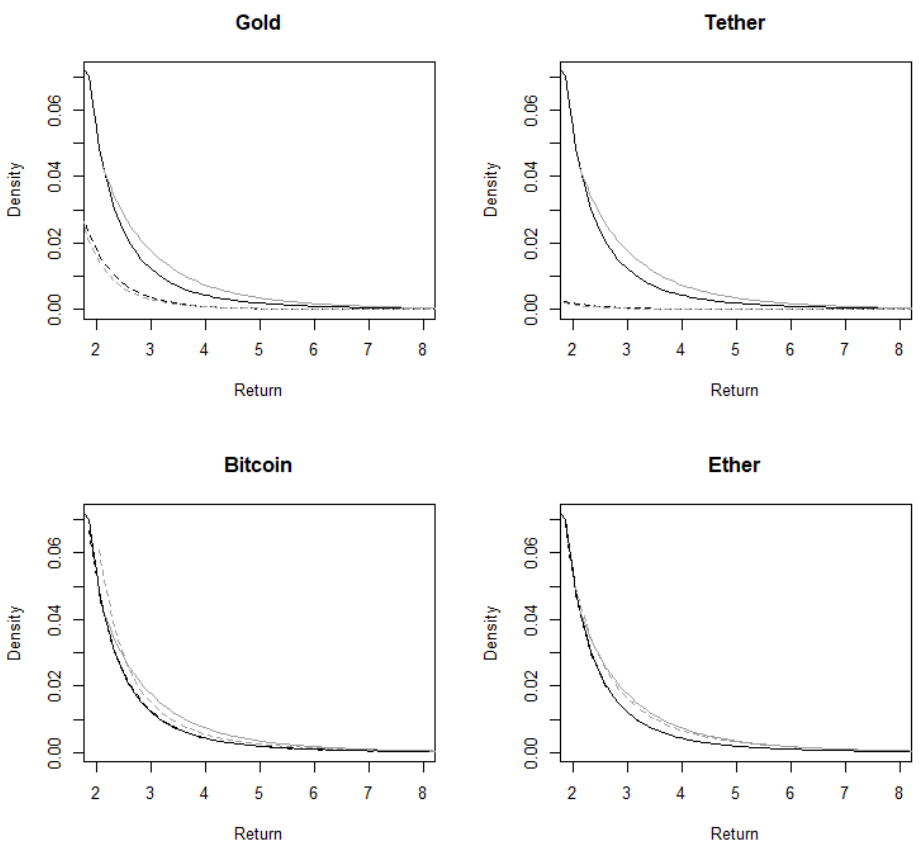
Figure 3 shows the return distribution tails for the S&P 500 index as well as the tails of an optimal pair-wise portfolio combined with an index and a hedging instrument. The solid black line represents the upper tail of the index returns, the solid grey line represents the lower tail of the index, while the dashed lines show the tails of the portfolio distribution (in black for the upper tail and in grey for the lower tail). Since the tails for different assets represent different data ranges, we decided to show the entire upper tail area of 5% for each index and the other tails against its background. Figures A1–A5 in the Appendix show the same for the other considered stock indices. As expected, most index distributions exhibit a fatter lower tail than the upper one. The highest differences are evident for the S&P 500 and CAC indices. From an intuitive point of view, it seems to be clear that the lower tails for financial markets must be heavier than the upper tails. This is due to the fact that the growth trends are built over long-time horizons but the crashes are more volatile and the price movements in absolute value are much larger. However, from

a statistical point of view, the problem is much more complex. In empirical studies, the differences are not significant or distributions exhibit symmetry as the Canadian TSX returns distribution has shown in Figure A1. For instance, Longin (1996) found the equality of tail thickness for the S&P 500 returns. Similarly, Jondeau and Rockinger (2003) did not find statistically significant asymmetry in mature, Asian, Eastern European and Latin American markets. The similarity between the lower and upper tails of returns has been reported in e.g. Chen and Ibragimov (2019), Daníelsson and de Vries (1997) and Koedijk and Kool (1992). Only the minority of studies report heavier lower tails than upper tails, e.g. Gregory-Allen et al. (2012) or Hartmann et al. (2004). Contrary to those studies, we do not compare the estimates of the tail index which is a measure of the tail behaviour. The tail index cannot measure the extreme risk level independently. To properly quantify the probability of extreme events, both threshold and tail behaviour must be taken into account. Graphical analysis enables us to consider the tails in a more complex way than any estimate of tail fatness. Tails for hedge portfolios are presented in Figure 3 and Figures A1–A5 as the dashed lines demonstrate symmetry between lower and upper tails, especially for the highest extrema. The differences between solid and dashed lines represent the effectiveness of the hedge. We can notice significant differences between the tails of the hedged and unhedged trading positions for gold and Tether, which confirms the ability of both assets in the extreme risk reduction. The result supports our findings for volatility presented in Section 3.1. Moreover, Tether reduces extreme risk the most effectively. The highest reduction in the probability of extreme losses compared to the reduction in the probability of extreme profits is noticeable for the S&P 500 and CAC indices. The result is the effect of heavier lower tails of return distribution for these indices. The tails for the portfolios with Bitcoin and Ether do not indicate any differences in relation to the index. Weight close to zero for these hedge instruments makes them have only a minor effect on the extreme returns of the pair-wise portfolios. A more interesting issue is to check their usefulness in generating profits in the suboptimal portfolios described in Section 3.4.

For comparison, we carried out a similar analysis based on the CVaR measure, which takes an average of returns in the tail of distribution. We calculate the CVaR under the generalised Pareto distribution (Formulas 16 and 17) with a threshold equal to the 5th percentile. The results of the computations are presented in Table 4; the second and third column shows the CVaRs for the lower and upper tails, respectively, while the next two columns exhibit the change in CVaR as an effect of the hedging strategy. The inference based on the CVaR is mostly the same as that based on the distribution tails. However, findings regarding the asymmetry between the reduction in losses and profits indicate differences. Based on CVaR, we find that both gold and Tether reduce losses more effectively than profits for all the

considered indices, excluding NKX. Both hedging instruments against the TSX index demonstrate the highest asymmetry of reduction in the extreme returns, which was not reflected in tail plots. TSX returns have the highest extrema among the considered returns and have not too many outliers; therefore, the CVaR defined as an expected value of VaR exceedances is highly affected by extreme returns. Bitcoin and Ether in an optimal portfolio only slightly change CVaR and rather increase the risk in the lower tail.

Figure 3. Distribution tails (pdf) of the SPX returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio



Source: authors' work.

Table 4. Reduction in 5% CVaR (in pp.) by optimal portfolios relative to indices

Portfolio	CVaR Lower tail	CVaR Upper tail	ΔCVaR Lower tail	ΔCVaR Upper tail
SPX	3.47	3.04		
SPX + Gold	1.74	1.76	1.73	1.28
SPX + Tether	0.69	0.76	2.78	2.28
SPX + Bitcoin	3.57	2.91	-0.10	0.13
SPX + Ether	3.60	3.01	-0.14	0.03
TSX	3.09	2.33		
TSX + Gold	1.68	1.57	1.41	0.76
TSX + Tether	0.65	0.73	2.44	1.60
TSX + Bitcoin	3.14	2.27	-0.05	0.06
TSX + Ether	3.29	2.30	-0.20	0.03
SHC	2.62	2.35		
SHC + Gold	1.80	1.61	0.82	0.74
SHC + Tether	0.94	0.79	1.68	1.56
SHC + Bitcoin	2.58	2.40	0.05	-0.05
SHC + Ether	2.61	2.38	0.01	-0.03
NKX	2.77	2.82		
NKX + Gold	1.76	1.81	1.01	1.01
NKX + Tether	0.74	0.83	2.03	2.00
NKX + Bitcoin	2.82	2.74	-0.05	0.08
NKX + Ether	2.81	2.80	-0.04	0.03
CAC	3.41	2.94		
CAC + Gold	1.80	1.71	1.61	1.23
CAC + Tether	0.75	0.78	2.66	2.16
CAC + Bitcoin	3.50	2.92	-0.10	0.02
CAC + Ether	3.50	3.00	-0.09	-0.07
FTM	3.03	2.82		
FTM + Gold	1.71	1.73	1.32	1.09
FTM + Tether	0.78	0.80	2.24	2.03
FTM + Bitcoin	3.07	2.77	-0.05	0.05
FTM + Ether	3.07	2.85	-0.04	-0.03

Source: authors' work.

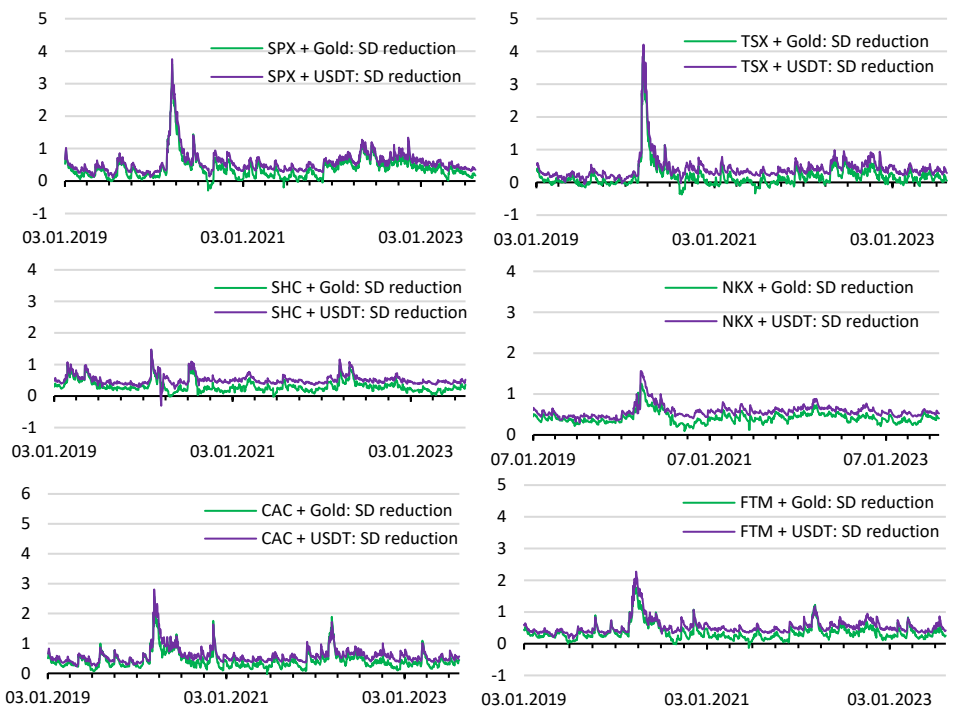
3.3. Hedging for an equal-weighted portfolio

As indicated in Section 3.1, optimal hedging seems to be a reasonable investment strategy only for gold. This section provides a similar analysis for equal-weighted portfolios. There is no consensus in the literature on the proportion of hedge assets that should be included in a strategic portfolio allocation (Akhtaruzzaman et al., 2021; Lucey et al., 2021). An equal-weighted portfolio is only one example portfolio chosen for analysis; however, such a choice enables capturing the important features of the considered instruments. Table 5 shows the descriptive statistics for these portfolios. Statistics for gold do not differ substantially from the statistics presented in Table 3 for the optimal portfolio since the optimal weight oscillates around 50%. Tether is still the most effective hedge and reduces volatility and extrema the most efficiently. Nevertheless, Tether is the only hedge that performs worse than the index in terms of the mean/SD ratio. Bitcoin and Ether added to an index significantly

increase the volatility and extrema. The overall and extreme risks of the portfolio far exceed the risks of the base investment. However, the profit-to-risk ratio has improved. Therefore, it seems reasonable to check the relative benefits from the trading strategy.

Figure 4 shows the reduction in conditional volatility of indices hedged with gold and Tether. There is no reason to present the results for Bitcoin and Ether since their application in the portfolio substantially increases the volatility relative to index. The results coincide with the descriptive statistics. Tether significantly outperforms gold in the hedging role since it reduces volatility to a greater extent. We confirmed the finding with the Wilcoxon signed rank test for the medians at the significance level of 1%. The analysis leads us to the conclusion that Tether is a better hedge and safe-haven asset than gold for all stock markets. Risk reduction may be the primary criterion for investment strategy in turbulent times; however, Tether is the worst option in normal market conditions when profits become the goal of an investment.

Figure 4. Reduction in conditional volatility (in pp.) by equal-weighted portfolios relative to indices



Source: authors' work.

Table 5. Descriptive statistics of equal-weighted portfolio returns

Portfolio	Min	Median	Mean	Max	SD	Mean/SD
SPX	-12.77	0.0954	0.0511	8.97	1.40	0.0366
SPX + Gold	-8.84	0.0738	0.0429	7.05	0.90	0.0475
SPX + USDT	-6.51	0.0409	0.0245	4.67	0.68	0.0358
SPX + BTC	-28.23	0.1288	0.1139	10.07	2.54	0.0448
SPX + ETH	-32.53	0.1479	0.1345	16.43	3.17	0.0425
TSX	-13.18	0.1001	0.0305	11.29	1.19	0.0257
TSX + Gold	-9.16	0.0617	0.0326	8.21	0.84	0.0387
TSX + USDT	-5.39	0.0435	0.0142	5.83	0.58	0.0246
SPX + BTC	-29.82	0.1038	0.1037	10.24	2.49	0.0417
SPX + ETH	-34.12	0.1305	0.1242	17.45	3.12	0.0399
SHC	-8.04	0.0302	0.0233	5.55	1.09	0.0213
SHC + Gold	-4.19	0.0550	0.0298	3.72	0.77	0.0385
SHC + USDT	-3.99	0.0188	0.0105	3.05	0.57	0.0183
SPX + BTC	-24.00	0.0490	0.1038	10.78	2.37	0.0437
SPX + ETH	-28.30	0.0574	0.1252	17.60	3.03	0.0413
NKX	-6.27	0.0829	0.0453	7.73	1.24	0.0364
NKX + Gold	-4.83	0.0229	0.0408	6.01	0.81	0.0506
NKX + USDT	-5.77	0.0262	0.0218	3.77	0.65	0.0338
SPX + BTC	-25.49	0.0831	0.1157	10.62	2.43	0.0476
SPX + ETH	-29.79	0.1261	0.1363	15.91	3.09	0.0441
CAC	-13.10	0.1062	0.0387	8.06	1.35	0.0286
CAC + Gold	-9.13	0.0558	0.0363	6.59	0.86	0.0422
CAC + USDT	-4.35	0.0587	0.0183	4.21	0.67	0.0272
SPX + BTC	-29.79	0.1338	0.1058	9.81	2.47	0.0428
SPX + ETH	-34.09	0.1738	0.1259	17.51	3.08	0.0409
FTM	-9.82	0.0440	0.0056	8.04	1.24	0.0045
FTM + Gold	-7.49	0.0509	0.0197	6.59	0.82	0.0240
FTM + USDT	-4.19	0.0188	0.0017	4.20	0.63	0.0027
SPX + BTC	-28.15	0.0978	0.0891	9.62	2.42	0.0368
SPX + ETH	-32.45	0.1340	0.1091	17.30	3.03	0.0360

Source: authors' work.

3.4. Extreme losses versus extreme profits in an equal-weighted portfolio

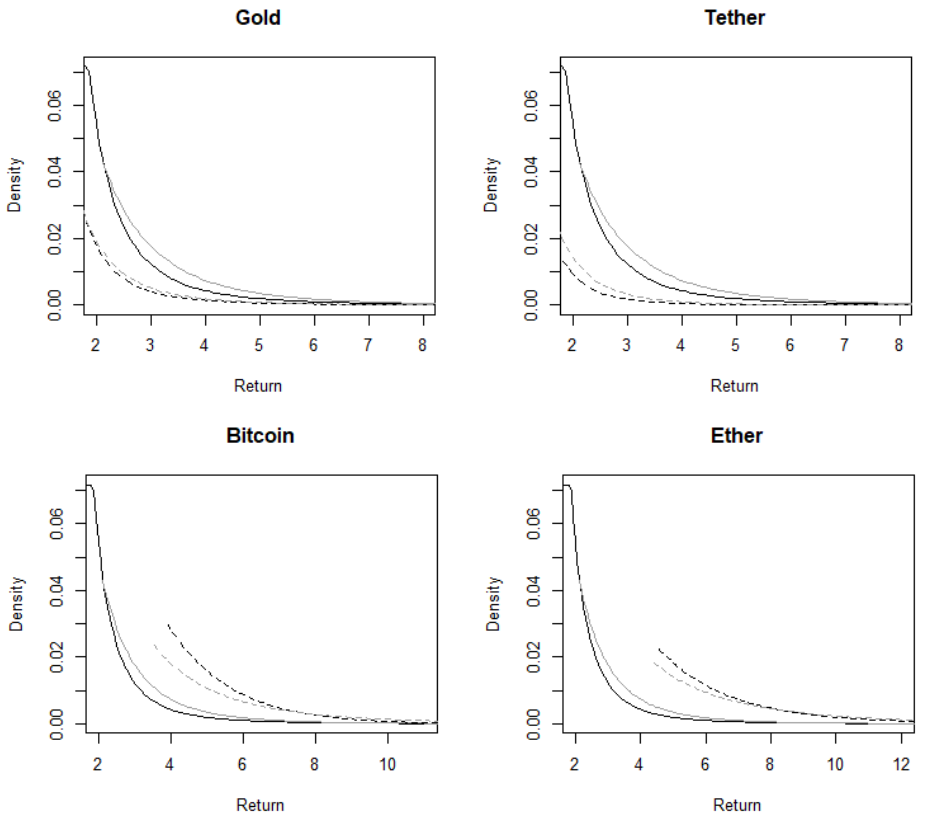
In Section 3.2, we take an optimal weight which is close to zero for Bitcoin and Ether and close to one for Tether. In the case of the first two assets, the optimal portfolios do not differ significantly from the index, whereas in the case of the third one, Tether dominates in the portfolio. Such an assumption gives us a nearly one-component portfolio and prevents us from comparing the relative benefits from hedging. By using an equal-weighted portfolio, we are able to verify the potential of the considered assets for extreme risk reduction in comparison to extreme profit sacrifice.

Figure 5 and Figures A6–A10 show the tails of the return distributions for the indices and equal-weighted portfolios. In the same way as in the previous figures, we present the index right tail area of 5% and the other tails against its background. The distribution tails for portfolios consisting of particular indices and Tether are the

thinnest; thus, Tether outperforms other assets in terms of extrema reduction. The asymmetry between risk reduction and profit sacrifice (for gold and Tether) is interpreted as the inequality of differences between the grey lines and the black lines, respectively. More precisely, the difference shows how much the hedging instrument added to a portfolio makes the return distribution tail thinner than the tail for the index. It is more profitable for investors to reduce the lower tail (difference in grey lines) more intensively than the upper tail (difference in black lines), which is interpreted as a higher risk reduction over profit sacrifice. The beneficial asymmetry is the most visible for the S&P 500–gold and FTM–gold pairs, whereas adverse asymmetry is for the NKX–Tether pair. Bitcoin and Ether increase substantially the tail risk of a base investment. Distribution tails of hedge portfolios begin at much higher return levels and decay at a slower rate than the tails for indices. For instance, for the NKX–Ether portfolio, the upper tail begins in a place where the upper tail for the index disappears. In terms of the relative benefits of investment in pair-wise portfolios with Bitcoin or Ether, portfolios with Bitcoin indicate much greater differences between the black lines than between the grey lines for almost all indices. The results show that Bitcoin added to the index increases potential profits much more than the downside risk. It is evident that along with the inclusion of Bitcoin in the portfolio, investors benefit from higher probability of extreme profits in relation to the probability of extreme losses. We observed a similar effect to the Bitcoin case in the Ether portfolio with S&P 500 or TSX. For SHC, CAC and FTM, the same relation holds, however, the differences are not as distinct as for the former indices. The result is ambiguous only for NKX.

As in the previous section, we have done computations of CVaR which are presented in Table 6. The results based on the tail measure do not coincide with those based on the distribution tails. Even when gold and Tether decrease the CVaR more in the lower tail than in an upper tail, the differences are not as clear as shown in the tail plots (e.g. FTM–gold pair). The highest discrepancy between the results from the used methods is noticeable for Bitcoin and Ether. CVaR yields the opposite results relative to the tail analysis for SPX, NKX, CAC and FTM indices in pair with both Bitcoin or Ether, which suggests a higher risk increase of extreme losses compared to the potential extreme profits. These exceptions indicate the need for caution when interpreting results based on a single risk measure. CVaR provides an adequate picture of the risks reflected in the most extreme values in the tail, but it fails to properly capture most of the data from the tail. An analysis based on the entire tail of the return distribution is more general and reliable.

Figure 5. Distribution tails (pdf) of the SPX returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio



Source: authors' work.

Table 6. Reduction in 5% CVaR (in pp.) by equal-weighted portfolios relative to indices

Portfolio	CVaR Lower tail	CVaR Upper tail	ΔCVaR Lower tail	ΔCVaR Upper tail
SPX	3.47	3.04		
SPX + Gold	2.08	1.99	1.39	1.05
SPX + Tether	1.69	1.44	1.78	1.60
SPX + Bitcoin	6.17	5.55	-2.70	-2.51
SPX + Ether	7.77	6.86	-4.30	-3.82
TSX	3.09	2.33		
TSX + Gold	2.01	1.73	1.08	0.60
TSX + Tether	1.46	1.14	1.63	1.19
TSX + Bitcoin	5.99	5.55	-2.90	-3.22
TSX + Ether	7.56	6.81	-4.47	-4.48
SHC	2.62	2.35		
SHC + Gold	1.83	1.69	0.79	0.66
SHC + Tether	1.41	1.22	1.22	1.13
SHC + Bitcoin	5.56	5.47	-2.94	-3.12
SHC + Ether	7.17	6.98	-4.55	-4.63

Table 6. Reduction in 5% CVaR (in pp.) by equal-weighted portfolios relative to indices (cont.)

Portfolio	CVaR Lower tail	CVaR Upper tail	ΔCVaR Lower tail	ΔCVaR Upper tail
NKX	2.77	2.82		
NKX + Gold	1.85	1.85	0.92	0.97
NKX + Tether	1.43	1.40	1.34	1.42
NKX + Bitcoin	5.97	5.43	-3.20	-2.61
NKX + Ether	7.86	6.86	-5.09	-4.03
CAC	3.41	2.94		
CAC + Gold	2.09	1.80	1.31	1.14
CAC + Tether	1.71	1.50	1.70	1.43
CAC + Bitcoin	5.98	5.43	-2.57	-2.49
CAC + Ether	7.48	6.74	-4.07	-3.80
FTM	3.03	2.82		
FTM + Gold	1.93	1.81	1.09	1.01
FTM + Tether	1.55	1.42	1.48	1.40
FTM + Bitcoin	5.91	5.34	-2.88	-2.52
FTM + Ether	7.45	6.62	-4.43	-3.80

Source: authors' work.

4. Conclusions

Hedging strategies against the risk of six global stock markets are considered in this paper. We compare the effectiveness of two hedging strategies, i.e. optimal hedging and equal-weighted portfolio hedging using gold and cryptocurrencies in the research period covering the COVID-19 pandemic and the Russia-Ukraine war. The empirical study provides an examination of the relative risk reduction in the lower and upper tails of the return distribution through the analysis of the portfolio tail thickness. We are able to verify how much the extreme risk is reduced with the hedging instrument relative to the profit sacrifice. We find several results that shed new light on the benefits of hedging with cryptocurrencies and gold, and thus provide findings relevant for individual and institutional investors.

The optimal hedge strategy is appropriate only when gold is applied as a hedge or a safe-haven instrument. The optimal weight for Tether is close to one; thus, it almost replaces equities from the portfolio built on variance minimisation. On the other hand, optimal weights for Bitcoin and Ether are close to zero. The high volatility of both assets does not allow for an effective risk reduction. Meanwhile, gold provides a good shelter for stock markets, since it reduces volatility, downside risk and provides the highest profit/risk ratio. Moreover, gold is able to reduce the probability of extreme losses more intensively than the probability of extreme profits.

In the equal-weighted portfolio strategy, Tether can still reduce the risk more effectively than gold. However, Tether is an asset which demonstrates the lowest profit/risk ratio among the considered hedges. Moreover, along with the reduction

of volatility both extreme losses and extreme profits are reduced at the same rate. Identifying which one is a better shelter for the global stock markets is ambiguous and highly depends on investor preferences. Bitcoin and Ether fail to act as effective hedges or safe-haven assets since they substantially increase volatility and the downside risk. However, we provide convincing arguments that the latter instruments added to stock market indices increase the extreme potential profits on investments more intensively than extreme losses for most of the returns from the tail area of 5%.

Our empirical findings have significant implications for the financial market participants. We address the key question: Is it possible to reduce extreme losses and save extreme profits in a hedging strategy? The answer is negative, safe-haven assets added to the base portfolio always reduce the potential extreme profits along with the unwanted huge losses. Profit sacrifice is an alternative cost of downside risk reduction. However, the relationship between losses and profits depends on the type of the hedging strategy and the hedging assets. The empirical results presented in this study reveal which popular safe-haven candidates offer a beneficial profit/loss relationship. Using the findings, investors can improve their asset allocation and hedging effectiveness by taking into account the asymmetry between profits and losses according to individual expectations and risk tolerance.

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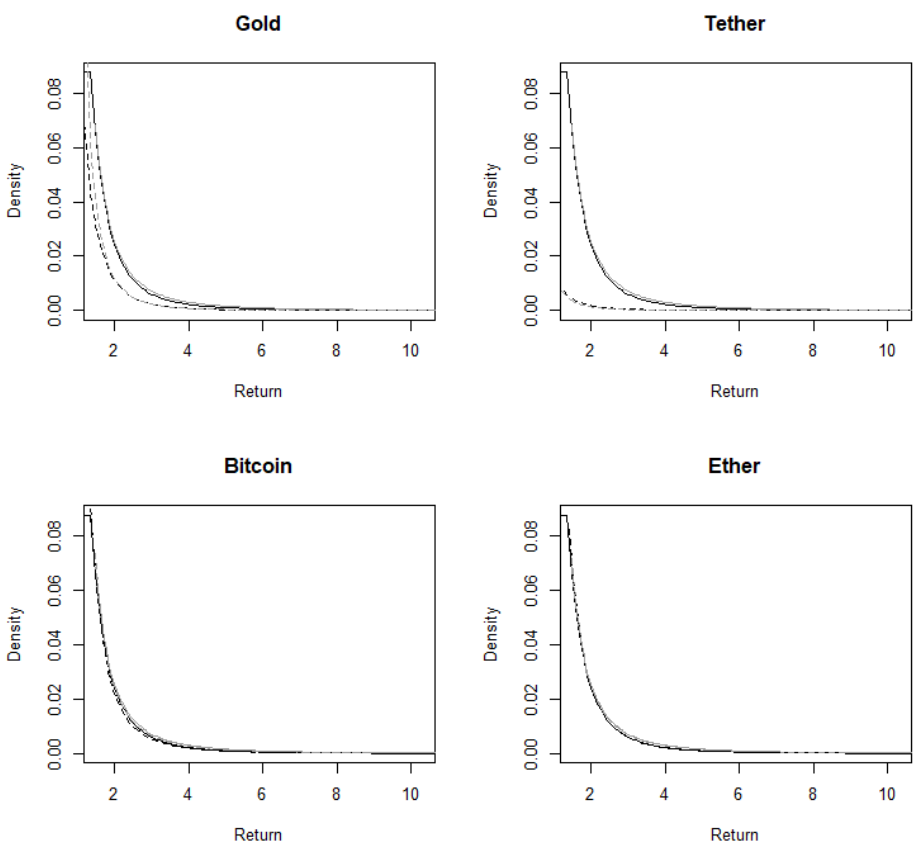
Appendix

Figure A1. Distribution tails (pdf) of the TSX returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio



Source: authors' work.

Figure A2. Distribution tails (pdf) of the SHC returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio

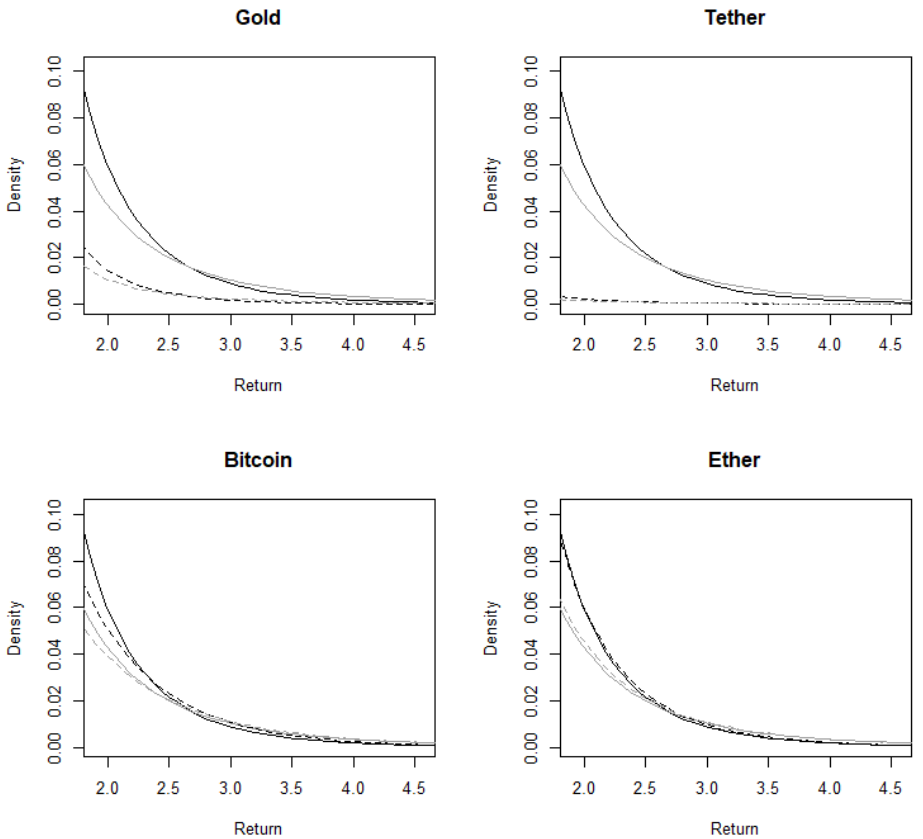


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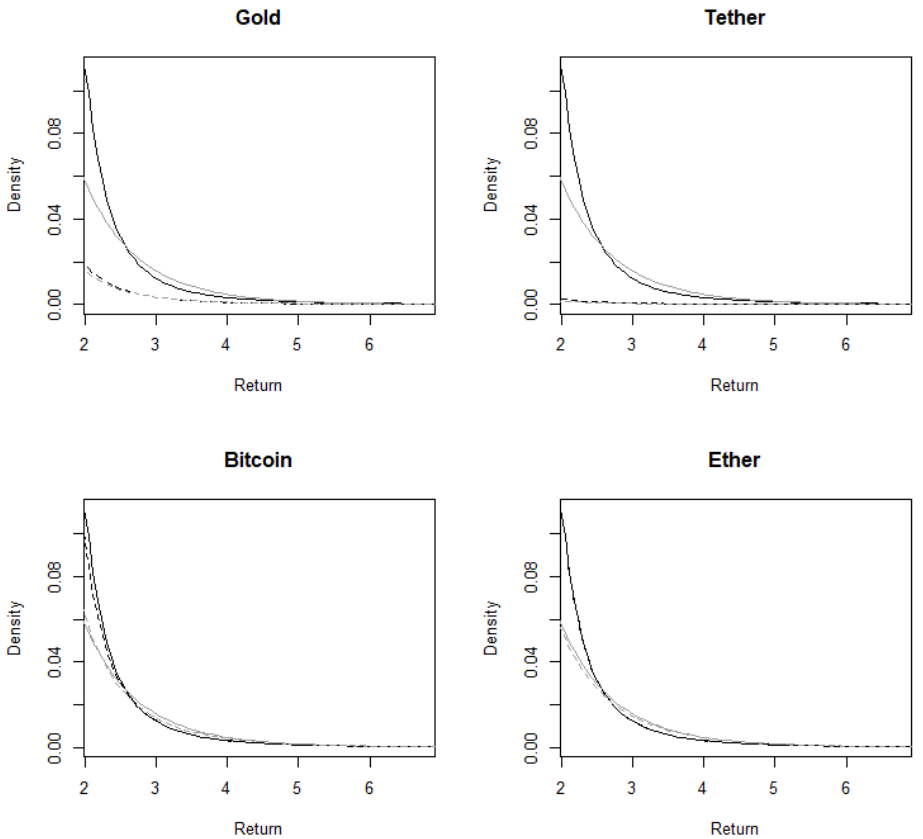
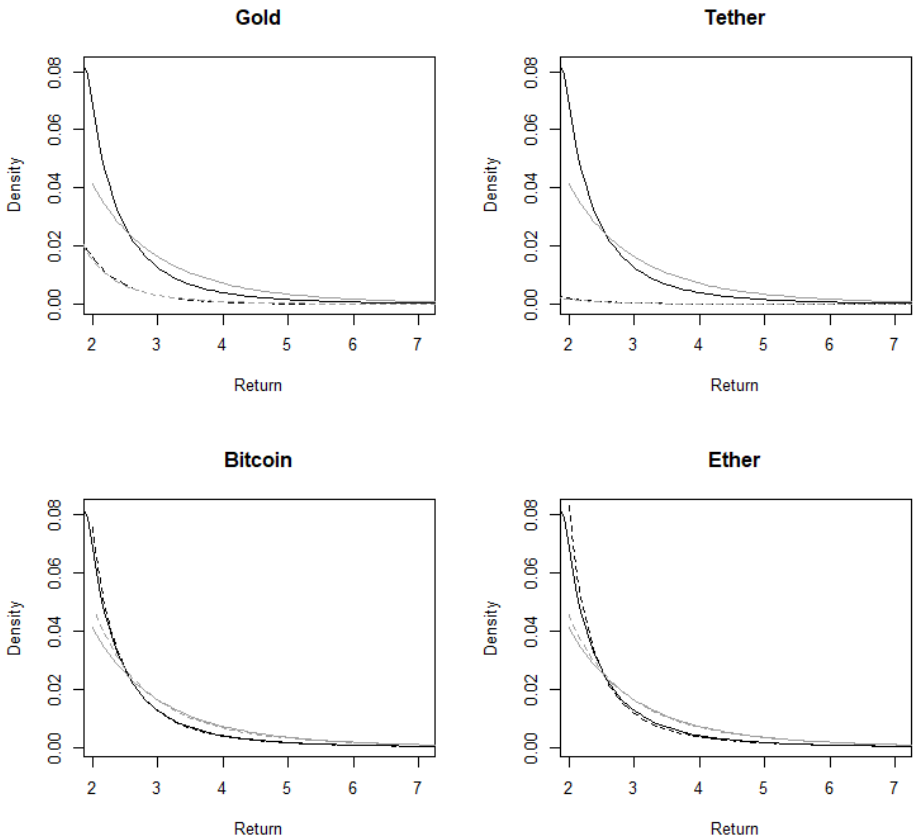
Figure A3. Distribution tails (pdf) of the NKX returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio



Source: authors' work.

Figure A4. Distribution tails (pdf) of the CAC returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio



Source: authors' work.

Figure A5. Distribution tails (pdf) of the FTM returns and an optimal portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – optimal portfolio

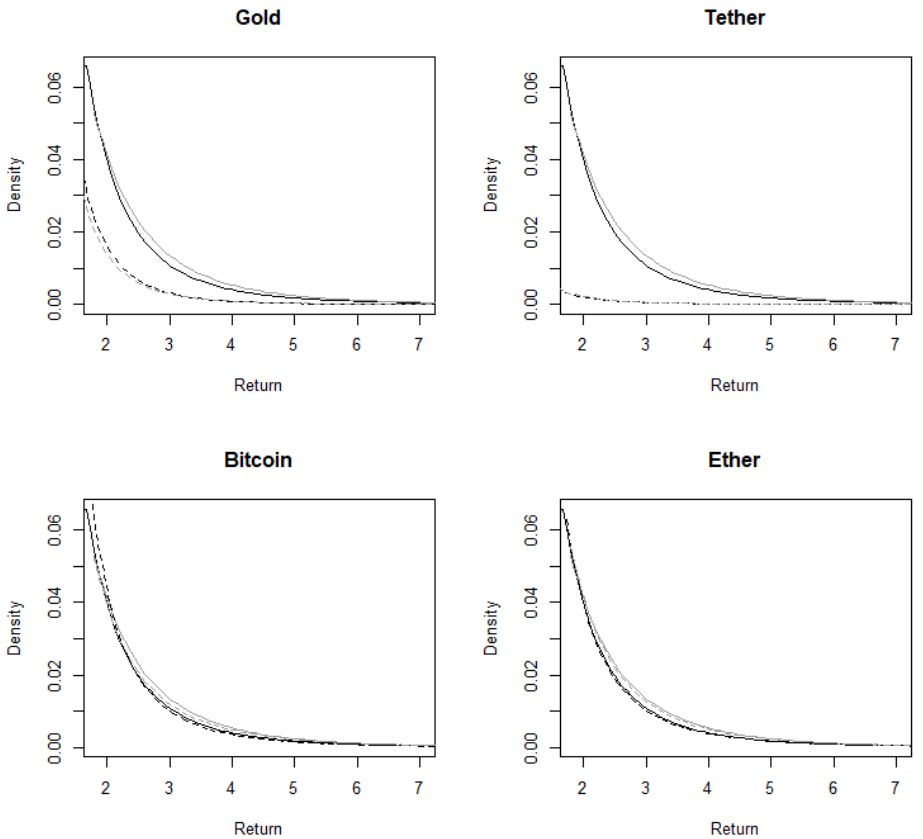


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Figure A6. Distribution tails (pdf) of the TSX returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio

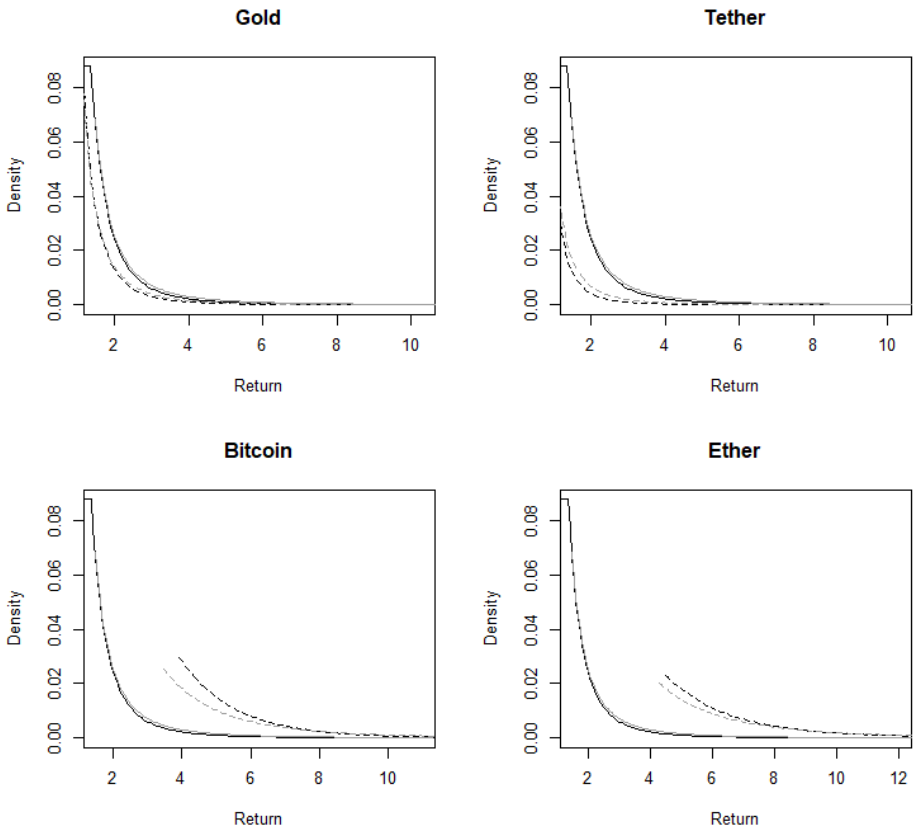

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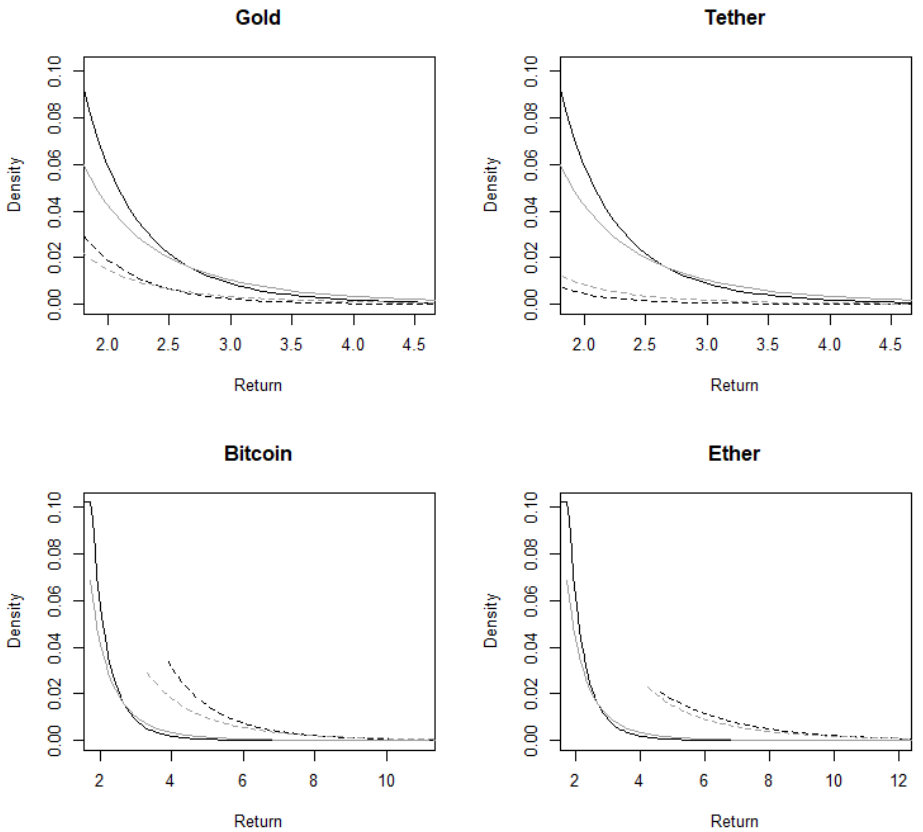
Figure A7. Distribution tails (pdf) of the SHC returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio



Source: authors' work.

Figure A8. Distribution tails (pdf) of the NKX returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio

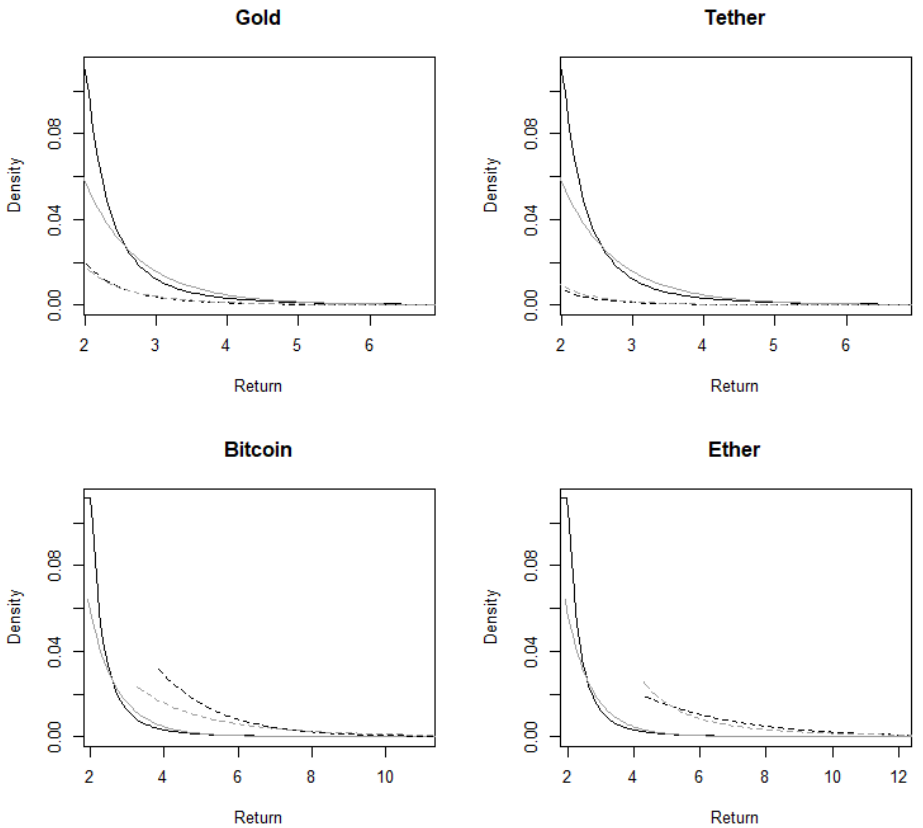


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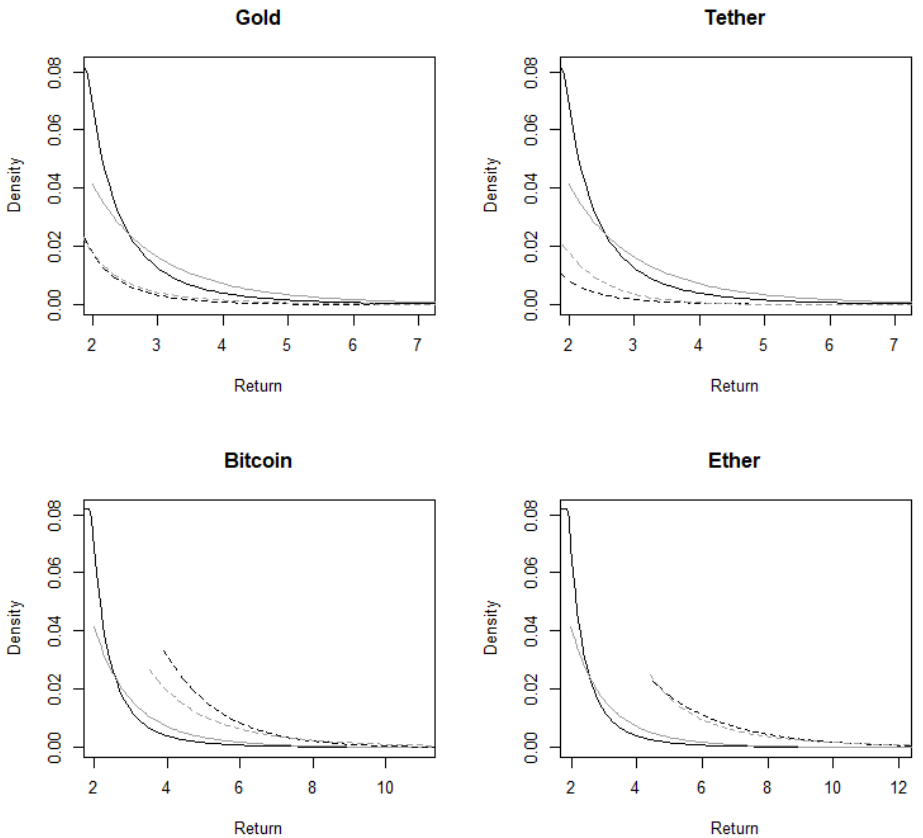
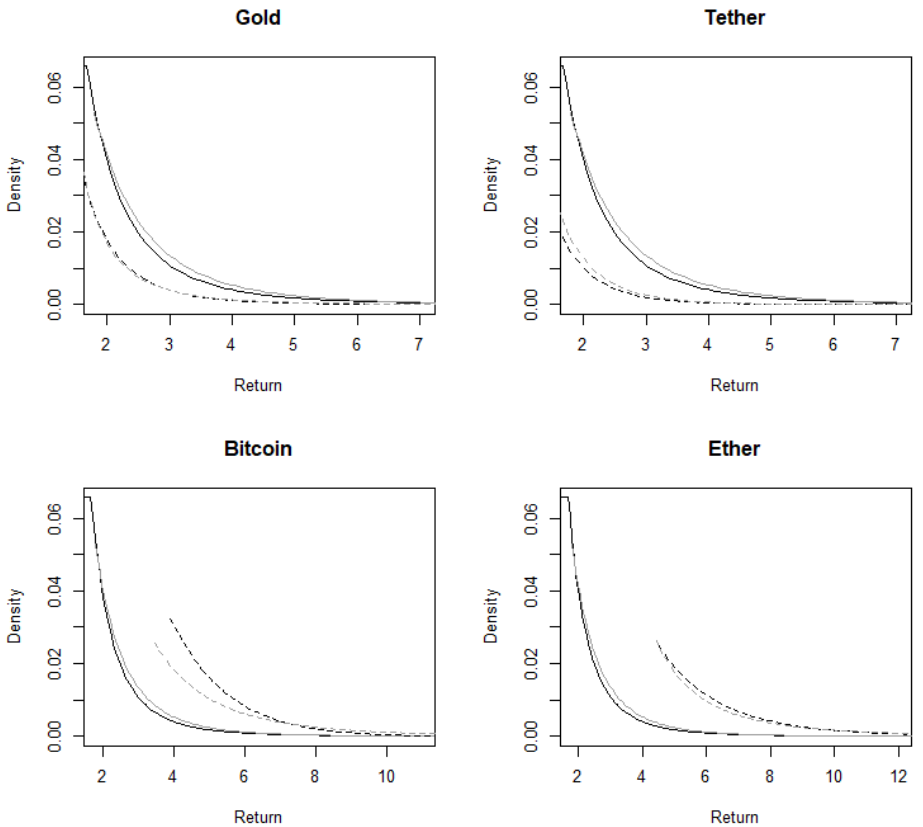
Figure A9. Distribution tails (pdf) of the CAC returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio



Source: authors' work.

Fig. A10. Distribution tails (pdf) of the FTM returns and an equal-weighted portfolio consisting of an index and hedge asset. Black line – 5% upper tail, grey line – lower tail (symmetrical image), solid line – index, dashed line – equal-weighted portfolio



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