Przegląd Statystyczny. Statistical Review, 2023, vol. 70, 4, 1–36 https://doi.org/10.59139/ps.2023.04.1
Received: 24.05.2024, revised: 6.07.2024, accepted: 9.07.2024

Modified Cramer-von Mises goodness-of-fit test for normality

Piotr Sulewski,[a] Damian Stoltmann[b]

Abstract. The first goal of the article is to apply the modified Cramer-von Mises (*CM*) goodness-of-fit test for normality to a practical problem. The modification of the test involves varying the formula for calculating the empirical distribution function (*EDF*). The critical values are obtained using the Monte Carlo method for sample sizes $n = 10{,}20$ and at a significance level of α=0.05. The second goal is to calculate the power of several tests for appropriately selected alternative distributions. The article shows that the values of constants α, β in the *EDF* formula affect the power of the *CM* test. The effectiveness of the new proposal is illustrated by the analysis of real data sets.

Keywords: empirical distribution function, goodness-of-fit test, Cramer-von Mises test, power of test
JEL: C02, C12, C46, G00

1. Introduction

Numerous goodness-of-fit tests (*GoFTs*) for normality have been considered and applied in many fields of science, including medicine, quality control and hydrology. *GoFTs* for normality are also very popular in economics and finance. They are used to analyse market behaviour (the distribution of rates of return, trading volume or asset prices), assess market efficiency, identify deviations from ideal market conditions and analyse stochastic processes (asset prices or changes in commodity prices). In econometrics, normality tests are used to check whether regression errors are normally distributed. This is important for the proper evaluation of regression models as the violation of the assumption of normality can lead to erroneous statistical conclusions. In demography, on the other hand, the fertility curve is almost normally distributed.

One of the most common normality testing procedures available in statistical software is the Cramer-von Mises (*CM*) test (Cramér, 1928; von Mises, 1931), which belongs to the group of empirical distribution function (*EDF*) tests. Other popular *EDF* tests include the Kolmogorov-Smirnov (*KS*) test (Kolmogorov, 1933; Smirnov, 1948), the Lilliefors (*LF*) test (Lilliefors, 1967), the Kuiper (*K*) test (Kuiper, 1960), the

[a] Pomeranian University, Institute of Exact and Technical Sciences, ul. Arciszewskiego 22a, 76-200 Słupsk, Poland, e-mail: piotr.sulewski@upsl.edu.pl, ORCID: https://orcid.org/0000-0002-0788-6567.
[b] Pomeranian University, Institute of Exact and Technical Sciences, ul. Arciszewskiego 22a, 76-200 Słupsk, Poland, e-mail: damian.stoltmann@upsl.edu.pl, ORCID: https://orcid.org/0000-0001-7053-2684.

Watson (*W*) test (Watson, 1962) and the Anderson-Darling (*AD*) test (Anderson & Darling, 1952).

Recently, many articles have been devoted to goodness-of-fit tests (*GoFTs*) for normality. Table 1 shows the authors of works created in the 21st century.

Table 1. Articles devoted to normal *GoFTs* created in the 21st century

Article	Sample sizes	Article	Sample sizes
Bonett and Seier (2002)	**10, 20, …, 50**, 100	Afeez et al. (2018)	**10, 30, 50**, 100, 300, 500, 1000
Aliaga et al. (2003)	X	Marange and Qin (2019)	**15, 30, 50**, 80, 100, 150, 200
Bontemps and Meddahi (2005)	100, 250, 500, 1000	Sulewski (2019)	**10, 12, …, 30, 40, 50**
Luceño (2006)	100	Tavakoli et al. (2019)	**5, 6, …, 15, 20, 25, 30, 40, 50,** …,100
Yazici and Yolacan (2007)	**20, 30, 40, 50**	Mishra et al. (2019)	**n<30, n>30**
Gel et al. (2007)	**20, 50,** 100	Kellner and Celisse (2019)	**50**, 75, 100, 200, 300, 400
Coin (2008)	**20, 50, 200**	Wijekularathna et al. (2020)	**5, 10, 20, 30, 50,** 75, 100, 200, 500, 1000, 2000
Brys et al. (2008)	100, 1000	Sulewski (2022)	**10, 14, 20**
Gel and Gastwirth (2008)	**30, 50**, 100	Hernandez (2021)	**5, 10, …, 30**
Romão et al. (2010)	**25, 50,** 100	Khatun (2021)	**10, 20, 25, 30, 40, 50,**100, 200, 300
Razali and Wah (2011)	20, 30, 50, 100, 200, …, 500, 1000, 2000	Arnastauskaitė et al. (2021)	**2^5**,2^6,…,2^10
Noughabi and Arghami (2011)	10, 20, 30, 50	Bayoud (2021)	**10, 20 ,…, 50**, 60, 80, 100
Yap and Sim (2011)	**10, 20, 30, 50,** 100, 300, 500, 1000, 2000	Uhm and Yi (2021)	**10, 20, 30,** 100, 200, 300
Chernobai et al. (2012)	X	Sulewski (2021)	**20, 50,** 100
Ahmad and Khan (2015)	**10, 20, …, 50,** 100, 200, 500	Desgagné et al. (2022)	**20, 50,** 100, 200
Mbah and Paothong (2015)	**10, 20, 30, 50,** 100, 200, …, 500, 1000, 2500, 5000	Uyanto (2022)	**10, 30, 50**, 70, 100
Torabi et al. (2016)	**10, 20, 50,** 100, 1000	Ma et al. (2024)	**10, 30, 50**
Feuerverger (2016)	200	Giles (2024)	**10, 25, 50,** 100, 250 ,500, 1000
Nosakhare and Bright (2017)	**5, 10, …, 50,** 100	Borrajo et al. (2024)	**50**, 100, 200, 500
Desgagné and Lafaye de Micheaux (2018)	**10, 12, …, 20, 50**, 100, 200	Terán-García and Pérez-Fernández (2024)	**25,** 900

Note. Sample sizes $n \leq 50$ are in bold.
Source: authors' work.

There are, of course, articles dedicated to the *CM* test. Durbin and Knott (1972) compared the *CM* test with *AD* and *W GoFTs*. Pettitt and Stephens (1976) used the *CM* test for a censored sample. Scott (2000) presented tables of unweighted *CM* statistics for one and two samples, and compared them to the limiting distribution.

Scott and Stewart (2011) presented tables for the Lilliefors distribution and the *CM* distribution, which are used to test for normality when the population mean and variance are unknown.

The small samples that dominate in Table 1 can be used in experimental economics where published papers can be found that describe samples of a dozen or so people in a group. In situations like these, strong testing proves very useful. In terms of the results, a hypothesis accepted in the original paper may be rejected if a more powerful test is applied.

Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be independent and identically distributed observations from unknown continuous cumulative distribution function (*CDF*) $F(x)$. We wish to determine whether $F(x)$ coincides with the *CDF* of normal distribution $\Phi(x)$. Then, we are interested in testing hypothesis $H_0\colon F(x) = \Phi(x)$ against hypothesis $H_1\colon F(x) \neq \Phi(x)$. The *EDF* is given by $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \theta(x - x_i)$, where $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$.

The δ-corrected *KS* test (Harter et al., 1984), investigated further by Khamis (1990, 1992, 1993) redefines the value of the *EDF* at the data points and compares the redefined *EDF* to the *CDF* at the data points. Let the *EDF* at the i-th data point be given by

$$F_\delta(x) = \frac{i-\delta}{n-2\delta+1}, 0 \leq \delta \leq 1 \tag{1}$$

Harter et al. (1984) selected $\delta = 0{,}0.5{,}1$ for their study.

Bloom (1958) proposed α, β transformation

$$F_{\alpha,\beta}\left(x_{(i)}\right) = \frac{i-\alpha}{n-\alpha-\beta+1}, \alpha, \beta \leq 1 \tag{2}$$

to decrease the MSE of certain statistics. Note that $F_{\delta,\delta}(x) = F_\delta(x)$. This transformation was used to create *GoFTs*.

Sulewski (2022) used the Bloom formula to create the one-component Lilliefors *GoFT* with statistic

$$LF_1 = \underbrace{max}_{i}\left\{\left|F_{\alpha,\beta}\left(x_{(i)}\right) - \Phi\left(x_{(i)}\right)\right|\right\}. \tag{3}$$

We know perfectly well that the greatest discrepancy between the theoretical and empirical distribution functions may occur at different positions in the series. The probability of this discrepancy occurring for a given positional statistic r becomes smaller the more extreme the r is. Hence the idea of a two-component test statistic described in Sulewski (2021). The first component is, as in the original *LF* test, the absolute value of the greatest discrepancy between sample and population distributions. The second component is the position in an ordered sample at which this discrepancy is located. The two-component Lilliefors statistic is given by

$$LF_2(r) = \underbrace{max}_{i} \{ |F_{\alpha,\beta}(x_{(i)}) - \Phi(x_{(i)})| \}. \quad (4)$$

Simulation studies for the one- and two-component Lilliefors tests were carried out for the following methods of calculating $F_{\alpha,\beta}(x_{(i)})$ $(\alpha, \beta \leq 1)$:

1. $F_{0,1}(x_{(i)}) = \frac{i}{n}$ – occurs in the *KS* statistic;
2. $F_{1,0}(x_{(i)}) = \frac{i-1}{n}$ – occurs in the *KS* statistic;
3. $F_{0.5,0.5}(x_{(i)}) = \frac{i-0.5}{n}$ – occurs in the *CM* statistic;
4. $F_{0,0}(x_{(i)}) = \frac{i}{n+1}$ – the mean value of *i*-th order statistics of the beta distribution;
5. $F_{0.3,0.3}(x_{(i)}) = \frac{i-0.3}{n+0.4}$ – the median of *i*-th order statistics of the beta distribution;
6. $F_{0.375,0.375}(x_{(i)}) = \frac{i-0.375}{n+0.25}$ – the mean value of *i*-th order statistics of the normal distribution;
7. $F_{0.3175,0.3175}(x_{(i)}) = \frac{i-0.3175}{n+0.365}$ – founded by Filliben (1975);
8. $F_{1,1}(x_{(i)}) = \frac{i-1}{n-1}$ – founded by Harter et al. (1984).

In six of the *EDF* definitions listed above (except $F_{0,1}$ and $F_{1,0}$), $\alpha = \beta$.

The first goal of this paper is to propose a modified *CM* test for normality. The second goal is to calculate the power of several tests for appropriately selected alternative distributions (alternatives).

The rest of this paper is structured as follows. In Section 2, we define a new version of the *CM* test. The similarity measure of the alternative to the normal distribution is described in Section 3. Section 4 presents the alternatives divided into nine groups according to their skewness and excess kurtosis. Simulation studies are presented in Section 5 and real data examples are provided in Section 6. Finally, the concluding remarks are presented in Section 7. Additional material can be found in the Appendix.

2. Modified Cramer-Von Mises test for normality

The *CM* statistic belongs to the class of quadratic *EDF* statistics with measure

$$n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \omega(x) dF(x), \tag{5}$$

where $\omega(x)$ is a weighting function. When the weighting function is $\omega(x) = 1$, the *CM* statistic is obtained:

$$n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 dF(x). \tag{6}$$

The *CM* test for normality is defined based on the following statistics:

$$CM = n \int_{-\infty}^{\infty} [F_n(x) - \Phi(x)]^2 \, \phi(x; \mu, \sigma) dx, \tag{7}$$

where $\phi(x), \Phi(x)$ are the *PDF* and *CDF* of the normal distribution, respectively. The simpler form of the *CM* statistic is

$$CM = \frac{1}{12n} + \sum_{i=1}^{n} \left[\Phi\left(x_{(i)}\right) - \frac{i - 0.5}{n} \right]^2. \tag{8}$$

The *CM* test is also presented in the second version, namely (Stephens, 1974)

$$CM_s = \left(1 + \frac{1}{2n}\right) CM. \tag{9}$$

We define the modified *CM* (*MCM*) statistic using the Bloom formula. The *MCM* statistic is given by

$$MCVM(\alpha, \beta) = \frac{1}{12n} + \sum_{i=1}^{n} \left[\Phi\left(x_{(i)}\right) - F_{\alpha,\beta}\left(x_{(i)}\right) \right]^2, \tag{10}$$

where $\alpha, \beta \in [0, 1]$. Note that $MCVM(0.5, 0.5) = CVM$. H_0 is rejected for large $MCVM$ statistic values.

3. Similarity measure

Let us assume that

$$\begin{gathered} \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \, , s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \, , z_{(i)} = \frac{(x_{(i)} - \bar{x})}{s}, \\ m_k = \frac{1}{n} \sum_{i=1}^{n} \left(x_{(i)} - \bar{x}\right)^k, \gamma_1 = \frac{m_3}{s^3}, \bar{\gamma}_2 = \frac{m_4}{s^4} - 3. \end{gathered} \tag{11}$$

Let us remember that the Malachov inequality is defined as $\bar{\gamma}_2 \geq {\gamma_1}^2 - 2$.

A review of recent statistical literature shows that the small skewness γ_1 and excess kurtosis $\bar{\gamma}_2$ values do not dominate in testing for normality. It is very interesting to see how the *GoFTs* respond to samples coming from alternatives close to the normal distribution.

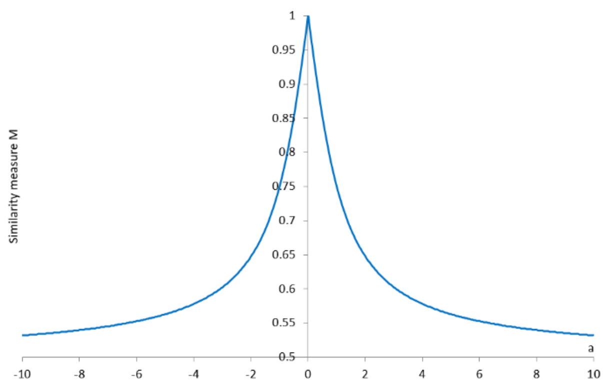
Let $f(x; \boldsymbol{\theta})$ be a *PDF* of the alternative with vector of parameters $\boldsymbol{\theta}$**.** Similarity measure M of alternative (A) to the normal distribution is defined as (Sulewski, 2022)

$$M_A(\boldsymbol{\theta}; \mu, \sigma) = \int_{-\infty}^{\infty} min[f(x; \boldsymbol{\theta}), \phi(x; \mu, \sigma)]dx \,, \tag{12}$$

where $\phi(x; \mu, \sigma)$ is the PDF of the normal distribution. $M_A(\boldsymbol{\theta}; \mu, \sigma)$ takes on the values of $[0, 1]$. $M_A(\boldsymbol{\theta}; \mu, \sigma) = 1$ when *PDFs* are identical.

Figure 1 shows the values of similarity measure (12) when an alternative is the skew normal (*SN*) distribution (Azzalini, 1985) with *PDF* $f_{SN}(x; a) = 2\phi(x; 0, 1)\Phi(ax; 0, 1)$ $(a \in R)$. Note that if $a \to \mp\infty$, then $M_{SN}(a; 0, 1) = 0.5$.

Figure 1. Similarity measure $M_{SN}(a; 0, 1)$ for the skew normal distribution



Source: authors' work.

4. Alternative distributions

As mentioned earlier, there are many articles devoted to testing for normality. In these articles, a lot of alternative distributions (alternatives) were used, including both asymmetric and symmetric ones. Symmetric distributions with undefined γ_1 and $\bar{\gamma}_2$ are Cauchy and slash distributions.

According to the statistical literature, alternatives can be divided into four groups, depending on the support and shape of their densities (see e.g. Esteban et al., 2001; Torabi et al., 2016). These groups include symmetric alternatives with support $(-\infty, \infty)$, asymmetric alternatives with support $(-\infty, \infty)$, alternatives with support

$(0, \infty)$ and alternatives with support $(0, 1)$. Gan and Koehler (1990), Krauczi (2009) and Torabi et al. (2016) divided alternatives into five groups: asymmetric short-tailed, asymmetric long-tailed, symmetric short-tailed, symmetric close to normal and symmetric long-tailed alternatives.

Our idea is to divide alternatives into nine groups according to their γ_1 and $\bar{\gamma}_2$ signs. Groups O–H are defined in Table 2.

Table 2. Groups of alternatives with signs of γ_1 and $\bar{\gamma}_2$

Group	γ_1	$\bar{\gamma}_2$
O	zero	zero
A	positive	positive
B	negative	positive
C	zero	positive
D	zero	negative
E	positive	negative
F	negative	negative
G	positive	zero
H	negative	zero

Source: authors' work.

The main criterion for selecting an alternative for the Monte Carlo simulation is that γ_1 and $\bar{\gamma}_2$ calculated for the alternative parameters belong to the O, A–H groups. This criterion is fulfilled by distributions defined in an infinite domain such as:

- the Edgeworth series (*ES*) with parameters γ_1 and $\bar{\gamma}_2$ as a monolithic distribution;
- the Pearson distribution (*P*) with parameters γ_1 and $\bar{\gamma}_2$ as a monolithic distribution;
- the normal mixture distribution (*NM*) with 5 parameters as a mixture of two normal distributions;
- the normal logistic mixture distribution (*NLM*) with 5 parameters as a mixture of normal and non-normal distributions;
- the normal distribution with a plasticising component (*NDPC*) with six parameters as a mixture of two various distributions;
- the plasticising component mixture (*PCM*) with seven parameters as a mixture of two identical non-normal distributions that characterise multimodality.

We chose values of alternative parameters to obtain desired similarity measure values of the alternative to the normal distribution. These analysed values, if possible, are $0.5, 0.75, 0.9$.

The appendix presents Tables 1A–6A with vectors of alternative parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M for the analysed alternatives. The *PDF* formulas and *PDF* curves (see Figures 1A–6A) for the alternative $\boldsymbol{\theta}$ values are also provided in the Appendix. As can be seen in Figure 1A, the *ES* distribution is not suitable for simulation studies as negative *PDF* values are observed even though the normalisation condition is met. Figure 2A indicates very interesting bimodal shapes. Figure 3A and Figure 5A present both unimodal and bimodal shapes. Figure 4A shows unimodal shapes, while very interesting multimodal shapes may be observed in Figure 6A.

5. Power comparisons

The new $MCM(\alpha,\beta)$ test, where $MCM(0.5,0.5)=CM$ was compared with the CM_S, AD, LF and the Shapiro-Wilk (SW) (Shapiro & Wilk, 1965) tests. To study the power of each of the discussed tests, critical values $cv_{0.05}$ (type I error equals $\alpha = 0.05$) were estimated using $m = 10^6$ order statistics. The power of tests (*PoTs*) was calculated based on $rep = 10^5$ test statistic values.

Table 3 shows critical values and test sizes of the analysed *GoFTs* for sample sizes $n = 10, 20$ and $\alpha = 0.05$. The TS values are close to 0.05, so the simulation procedures are correct.

Table 3. Critical values (*CV*) and test sizes (*TS*) of the analysed *GoFTs* for sample sizes $n = 10, 20$

No	*GoFT*	*CV*		*TS*	
		$n=10$	$n=20$	$n=10$	$n=20$
1	$MCM(0,1)$	0.15247	0.14008	0.051	0.051
2	$MCM(1,0)$	0.15232	0.13992	0.051	0.051
3	$MCM(0,0)$	0.11423	0.11992	0.052	0.051
4	$MCM(0.3,0.3)$	0.11487	0.12035	0.052	0.051
5	$MCM(1,1)$	0.14934	0.13820	0.052	0.052
6	$MCM(0.375,0.375)$	0.11612	0.12101	0.052	0.051
7	$MCM(0.3175,0.3175)$	0.11510	0.12048	0.052	0.051
8	$MCM(0.5,0.5)=CM$	0.11922	0.12285	0.051	0.051
9	CM_S	0.12518	0.12593	0.051	0.051
10	AD	0.68511	0.72118	0.052	0.051
11	LF	0.26186	0.19187	0.051	0.050
12	SW^*	0.84451	0.90441	0.051	0.050

Note. H_0 is rejected for small statistical values.
Source: authors' work.

Tables 7A–14A show how a selection of the *EDF* influences the power of the *MCM* test. The alternatives are indexed. The larger the index (ID), the more the distribution resembles a normal distribution, i.e. the PoTs should decrease as the index value increases. ID = 1 denotes similarity measure $M = 0.5$, while ID = 4 denotes similarity measure $M = 0.95$. The highest *PoTs* of the $MCM(\alpha,\beta)$ values are underlined. The highest *PoTs* for all the analysed tests are in bold.

The simulation results in Tables 7A–14A show that the *MCM* test with the analysed *EDFs* is the most powerful for all the considered similarity measures, alternatives and $n = 10, 20$ ($n = 10$) in 83.75% (94.38%.). The new proposal for group of alternatives A and $n = 10, 20$, ($n = 10$) is the most powerful in 82.5% (100%), for group B and $n = 10, 20$, ($n = 10$) is the most powerful in 85% (100%), for group C and $n =$

$10, 20$, $(n = 10)$ is the most powerful in 67.50% (90%), for group D and $n = 10, 20$, $(n = 10)$ is the most powerful in 77.5% (85%), for group E and $n = 10, 20$, $(n = 10)$ is the most powerful in 92.5% (95%), for group F and $n = 10, 20$, $(n = 10)$ is the most powerful in 90% (95%), for group G and $n = 10, 20$, $(n = 10)$ is the most powerful in 87.5% (95%) and for group H and $n = 10, 20$, $(n = 10)$ is the most powerful in 87.5% (95%). $MCM(0, 1)$ with $F_{0,1}(x_{(i)}) = \frac{i}{n}$ dominates for groups A, E and G. $MCM(1, 0)$ with $F_{1,0}(x_{(i)}) = \frac{i-1}{n}$ dominates for groups B, F and H. $MCM(0, 0)$ with $F_{1,1}(x_{(i)}) = \frac{i-1}{n+1}$ dominates for group C and $MCM(0, 0)$ with $F_{0,0}(x_{(i)}) = \frac{i}{n+1}$ dominates for group D. Powers of the *CM* and CM_S tests are the same. We assume that a *GoFT* detects abnormal samples if its power is at least 0.06. Thanks to similarity measures $M = 0.9, 0.95$, this situation occurs in 30%. For alternatives *P*, *NM*, *NLM*, *NDPC* and *PCM*, the test power is less than 0.6 in 14%, 45%, 4%, 44% and 43% of cases, respectively. For alternative groups A – H, the test power is less than 0.6 in 20%, 18%, 22%, 49%, 32%, 35%, 27% and 37% of cases, respectively.

6. Real data examples

In this section, we present an application of the *MCM* test in eight real data sets to illustrate its potentiality. Details related to examples I – VIII are presented in Table 4.

Table 4. Real data examples with sources, sample size, skewness and excess kurtosis values

Ex	Description	Source	n	γ_1	$\bar{\gamma}_2$
I	Strength measured in GPA for single carbon fibres and impregnated 1,000-carbon fibre tows (gauge lengths of 20 mm):1.312, 1.314, 1.479, 1.552, 1.7, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.14, 2.179, 2.224, 2.24, 2.253, 2.27, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.49, 2.511, 2.514, 2.535, 2.554, 2.566, 2.57, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.77, 2.773, 2.8, 2.809, 2.818, 2.821, 2.848, 2.88, 2.809, 2.818, 2.821, 2.848, 2.88, 2.954, 3.012, 3.067, 3.084, 3.09, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585.	Nofal et al., 2017	74	−0.154	−0.049
II	Macroeconomic data set with information on the number of members of the armed forces	R package longley[4]	16	−0.404	−0.949
III	Socio-economic data (percentage of males involved in agriculture as occupation) for 47 French-speaking provinces of Switzerland	R package swiss[2]	47	−0.331	−0.793

Table 4. Real data examples with sources, sample size, skewness and excess kurtosis values (cont.)

IV	Socio-economic data (percentage of draftees receiving the highest mark on army examination) for 47 French-speaking provinces of Switzerland	R package swiss[3]	47	0.461	−0.011
V	Average heights for American women aged 30–39	R package women[1]	15	0	−1.211
VI	Percent proportion of favourable responses to the question on promotion opportunities from a survey of the clerical employees of a large financial organisation; the data are aggregated from the questionnaires of approximately 35 employees for each of the 30 (randomly selected) departments.	R package attitude[6]	30	−0.911	0.388
VII	Percent proportion of favourable responses to the question on opportunities to learn from a survey of the clerical employees of a large financial organisation; the data are aggregated from the questionnaires of the approximately 35 employees for each of the 30 (randomly selected) departments.	R package attitude[3]	30	0.399	−0.229
VIII	Percent proportion of favourable responses to the question whether the organisation does not allow special privileges from a survey of the clerical employees of a large financial organisation; the data are aggregated from the questionnaires of the approximately 35 employees for each of the 30 (randomly selected) departments.	R package attitude[2]	30	−0.227	−0.514

Source: authors' work.

When fitting the normal distribution to the data, we calculate the p-values for the analysed *GoFTs* based on 10^5 statistic values (see Table 5).

Table 5. The *p*-values for the *GoFTs* related to examples I–VIII

GoFT	I	II	III	IV	V	VI	VII	VIII
$MCM(0,1)$	0.829	0.237	0.304	**<u>0.250</u>**	0.906	0.027	**<u>0.215</u>**	0.638
$MCM(1,0)$	**<u>0.660</u>**	**<u>0.090</u>**	**<u>0.149</u>**	0.493	0.907	**<u>0.005</u>**	0.455	**<u>0.516</u>**
$MCM(0,0)$	0.775	0.119	0.171	0.380	<u>0.797</u>	0.011	0.315	0.533
$MCM(0.3,0.3)$	0.763	0.127	0.192	0.362	0.897	0.010	0.312	0.563
$MCM(1,1)$	0.724	0.162	0.256	0.327	0.996	0.010	0.313	0.630
$MCM(0.375,0.375)$	0.759	0.129	0.198	0.358	0.918	0.010	0.312	0.571
$MCM(0.3175,0.3175)$	0.762	0.127	0.193	0.361	0.902	0.010	0.312	0.565
$MCM(0.5,0.5)=CM$	0.753	0.134	0.208	0.352	0.946	0.010	0.311	0.584
CM_S	0.753	0.134	0.208	0.352	0.946	0.010	0.311	0.584
AD	0.756	0.106	0.195	0.362	0.925	0.015	0.417	0.567
LF	0.826	0.094	0.231	0.287	0.996	0.028	0.529	0.568
SW	0.728	0.111	0.191	0.254	**0.729**	0.034	0.640	0.552

Note. The optimal *MCM(α,β)* test is underlined. The lowest *p*-value for all the analysed tests are in bold.
Source: authors' work.

The optimal $MCM(\alpha,\beta)$ test for examples I-III, VI and VIII is the $MCM(1,0)$ test. The obtained result is consistent with the simulation results showing that for groups B ($\gamma_1 < 0, \bar{\gamma}_2 > 0$) and F ($\gamma_1 < 0, \bar{\gamma}_2 < 0$), the most powerful is $MCM(1,0)$. Non-normality is the most pronounced by the $MCM(1,0)$ test.

The optimal $MCM(\alpha,\beta)$ test for examples IV and VII is the $MCM(0,1)$ test. The obtained result is consistent with the simulation results indicating that for groups E ($\gamma_1 > 0, \bar{\gamma}_2 < 0$), the most powerful is $MCM(0,1)$. The non-normality is the most pronounced by the $MCM(0,1)$ test.

The optimal $MCM(\alpha,\beta)$ test for example V is the $MCM(0,0)$ test. The obtained result is consistent with the simulation results stating that for groups D ($\gamma_1 = 0, \bar{\gamma}_2 < 0$), the most powerful is $MCM(0,0)$. The non-normality is the most pronounced by the SW test.

7. Real data examples

The obtained results show that the methods of calculating the *EDF* depend on the nature of the non-normal (alternative) distribution. There is a method of calculating the *EDF* $F_{\alpha,\beta}(x_i)$ for groups A – H which maximises the power of the $MCM(\alpha,\beta)$ test. $MCM(0,1)$ dominates for alternative groups A, E and G, $MCM(1,0)$ for groups B, F and H, $MCM(1,1)$ for group C and $MCM(0,0)$ for group D.

The new proposal is the most powerful in 95% of the cases for $n = 10$ and in 84% of the cases for $n = 20$.

The new *GoFT* is the best in 100% of cases in groups A and B, in 90% of cases in group C, in 85% of cases in group D and in 95% of cases in groups E – H.

For the normal logistic mixture and Pearson distributions, the analysed tests do not detect abnormal samples in only 4% and 14% of cases, respectively. With regards to the alternative groups, the best results are achieved by groups B, A and C (asymmetric alternatives with a positive excess kurtosis), while the worst by group D (symmetrical alternatives with a negative excess kurtosis).

The good performance of the *MCM* test against other most popular *GoFTs* is illustrated through the analysis of real data sets.

Appendix

Edgeworth series distribution

PDF of the Edgeworth series (*ES*) with parameters γ_1 and $\bar{\gamma}_2$ is given by

$$f_{ES}(x;\gamma_{1,},\bar{\gamma}_2) = \phi(x;0,1)\left(1+\frac{1}{3!}\gamma_{1,}(x^3-3x)+\frac{1}{4!}\bar{\gamma}_2(x^4-6x^2+3)\right)\ (x\in R),$$

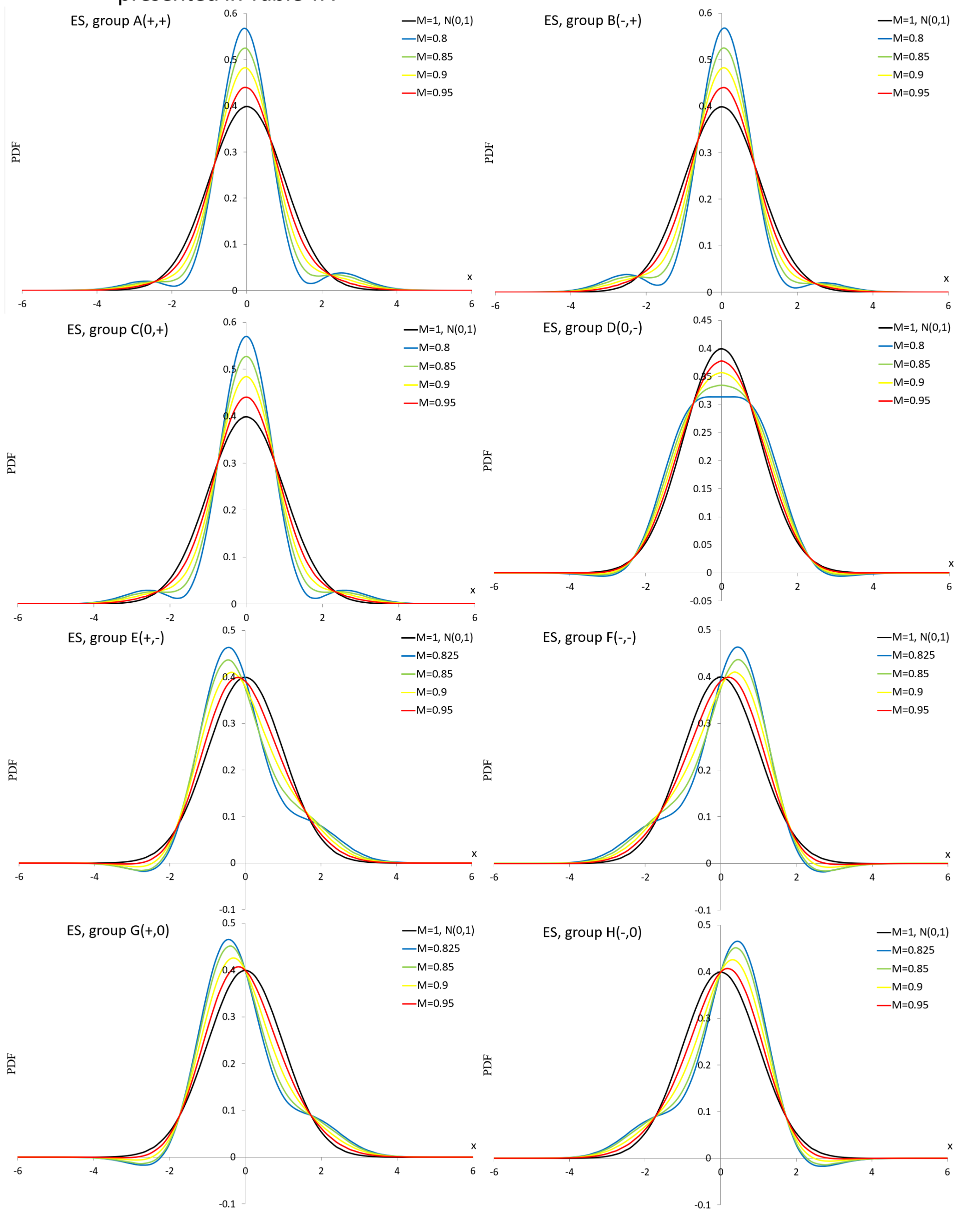
where $\gamma_{1,} \in R, \bar{\gamma}_2 \geq -2$.

Table 1A. Vectors of *ES* parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta} = (\gamma_1, \bar{\gamma}_2)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	$(0, 0)$	0	1	0	0	$M(\boldsymbol{\theta}; 0,1) = 1$
A	0.4, 3.33	0	1	0.4	3.33	$M(\boldsymbol{\theta}; 0,1) = 0.8$
	0.3, 2.499	0	1	0.3	2.499	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	0.2, 1.666	0	1	0.2	1.666	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.1, 0.833	0	1	0.1	0.833	$M(\boldsymbol{\theta}; 0,1) = 0.95$
B	-0.4, 3.33	0	1	-0.4	3.33	$M(\boldsymbol{\theta}; 0,1) = 0.8$
	-0.3, 2.499	0	1	-0.3	2.499	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	-0.2, 1.666	0	1	-0.2	1.666	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.1, 0.833	0	1	-0.1	0.833	$M(\boldsymbol{\theta}; 0,1) = 0.95$
C	0, 3.428	0	1	0	3.428	$M(\boldsymbol{\theta}; 0,1) = 0.8$
	0, 2.571	0	1	0	2.571	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	0, 1.71	0	1	0	1.71	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0, 0.85	0	1	0	0.85	$M(\boldsymbol{\theta}; 0,1) = 0.95$
D	0, -3.428	0	1	0	-3.428	$M(\boldsymbol{\theta}; 0,1) = 0.8$
	0, -2.571	0	1	0	-2.571	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	0, -1.71	0	1	0	-1.71	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0, -0.85	0	1	0	-0.85	$M(\boldsymbol{\theta}; 0,1) = 0.95$
E	1.39, -0.067	0	1	1.39	-0.067	$M(\boldsymbol{\theta}; 0,1) = 0.825$
	1.175, -0.46	0	1	1.175	-0.46	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	0.775, -0.408	0	1	0.775	-0.408	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.39, -0.15	0	1	0.39	-0.15	$M(\boldsymbol{\theta}; 0,1) = 0.95$
F	-1.39, -0.067	0	1	-1.39	-0.067	$M(\boldsymbol{\theta}; 0,1) = 0.825$
	-1.175, -0.46	0	1	-1.175	-0.46	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	-0.775, -0.408	0	1	-0.775	-0.408	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.39, -0.15	0	1	-0.39	-0.15	$M(\boldsymbol{\theta}; 0,1) = 0.95$
G	1.391, 0	0	1	1.391	0	$M(\boldsymbol{\theta}; 0,1) = 0.825$
	1.19, 0	0	1	1.19	0	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	0.795 ,0	0	1	0.795	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.4, 0	0	1	0.4	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$
H	-1.391, 0	0	1	-1.391	0	$M(\boldsymbol{\theta}; 0,1) = 0.825$
	-1.19, 0	0	1	-1.19	0	$M(\boldsymbol{\theta}; 0,1) = 0.85$
	-0.795, 0	0	1	-0.795	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.4, 0	0	1	-0.4	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$

Source: authors' work.

Figure 1A. *PDF* curves of the Edgeworth series distribution for parameter values presented in Table 1A



Source: authors' work.

Pearson distribution

Let $a = \frac{2\bar{\gamma}_2 - 3\gamma_1^2}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}$, $b = \frac{|\gamma_1|(\bar{\gamma}_2 + 6)}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}$, $c = \frac{4\bar{\gamma}_2 - 3\gamma_1^2 + 12}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}$, $\Delta = b^2 - 4ac$, then the *PDF* of the Pearson (P) distribution is given by

$$f_P(x; \gamma_{1,}, \bar{\gamma}_2) = \begin{cases} \dfrac{exp\left[\dfrac{2ab-b}{a(2ax+b)}\right]}{C_1(2ax+b)^{1/a}} & \Delta = 0 \\ \dfrac{exp\left[\dfrac{b-2ab}{a\sqrt{4ac-b^2}} tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)\right]}{C_2(ax^2+bx+c)^{1/(2a)}} & \Delta < 0 \\ \dfrac{\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right)^{\frac{b-2ab}{2a\sqrt{b^2-4ac}}}}{C_3(ax^2+bx+c)^{1/(2a)}} & \Delta > 0 \end{cases}$$

$$f_P(x; \gamma_{1,}, \bar{\gamma}_2) = \begin{cases} \dfrac{exp\left[\dfrac{2ab-b}{a(2ax+b)}\right]}{C_1(2ax+b)^{1/a}} & \Delta = 0 \\ \dfrac{exp\left[\dfrac{b-2ab}{a\sqrt{4ac-b^2}} tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)\right]}{C_2(ax^2+bx+c)^{1/(2a)}} & \Delta < 0, \\ \dfrac{\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right)^{\frac{b-2ab}{2a\sqrt{b^2-4ac}}}}{C_3(ax^2+bx+c)^{1/(2a)}} & \Delta > 0 \end{cases}$$

where $x \in R, \gamma_{1,} \in R, \bar{\gamma}_2 \geq -2$ and C_1, C_2, C_3 are normalising constants defined as

$$C_1 = \int_{-\infty}^{\infty} \frac{exp\left[\dfrac{2ab-b}{a(2ax+b)}\right]}{(2ax+b)^{1/a}} dx,$$

$$C_2 = \int_{-\infty}^{\infty} \frac{exp\left[\dfrac{b-2ab}{a\sqrt{4ac-b^2}} tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)\right]}{(ax^2+bx+c)^{1/(2a)}} dx,$$

$$C_3 = \int_{-\infty}^{\infty} \frac{\left(\dfrac{2ax+b-\sqrt{\Delta}}{2ax+b+\sqrt{\Delta}}\right)^{\frac{b-2ab}{2a\sqrt{\Delta}}}}{C_8(ax^2+bx+c)^{1/(2a)}} dx.$$

Table 2A. Vectors of the Pearson distribution parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta} = (\gamma_1, \bar{\gamma}_2)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	(0, 0)	0	1	0	0	$M(\boldsymbol{\theta}; 0{,}1) = 1$
A	(2.04, 4.1)	0	1	2.04	4.1	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(1.62, 3.845)	0	1	1.62	3.845	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(0.9, 2)	0	1	0.9	2	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(0.4, 0.94)	0	1	0.4	0.94	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
B	(-2.04, 4.1)	0	1	-2.04	4.1	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(-1.62, 3.845)	0	1	-1.62	3.845	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(-0.9, 2)	0	1	-0.9	2	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(-0.4, 0.94)	0	1	-0.4	0.94	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
C	(0, 11.2)	0	1	0	11.2	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(0, 3.65)	0	1	0	3.65	$M(\boldsymbol{\theta}; 0{,}1) = 0.925$
	(0, 1.521)	0	1	0	1.521	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
	(0, 0.55)	0	1	0	0.55	$M(\boldsymbol{\theta}; 0{,}1) = 0.975$
D	(0, -1.695)	0	1	0	-1.695	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(0, -1.315)	0	1	0	-1.315	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(0, -0.89)	0	1	0	-0.89	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(0, -0.588)	0	1	0	-0.588	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
E	(0.985, -0.5)	0	1	0.985	-0.5	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(0.715, -0.475)	0	1	0.715	-0.475	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(0.515, -0.2)	0	1	0.515	-0.2	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(0.315, -0.16)	0	1	0.315	-0.16	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
F	(-0.985, -0.5)	0	1	-0.985	-0.5	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(-0.715, -0.475)	0	1	-0.715	-0.475	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(-0.515, -0.2)	0	1	-0.515	-0.2	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(-0.315, -0.16)	0	1	-0.315	-0.16	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
G	(1.164, 0)	0	1	1.164	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(0.879, 0)	0	1	0.879	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(0.578, 0)	0	1	0.578	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(0.354, 0)	0	1	0.354	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
H	(-1.164, 0)	0	1	-1.164	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	(-0.879, 0)	0	1	-0.879	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	(-0.578, 0)	0	1	-0.578	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	(-0.354, 0)	0	1	-0.354	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$

Source: authors' work.

Figure 2A. *PDF* curves of the Pearson distribution for parameter values presented in Table 2A

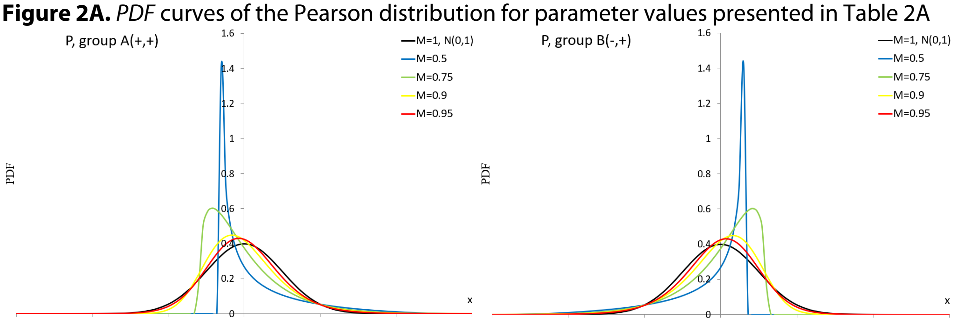
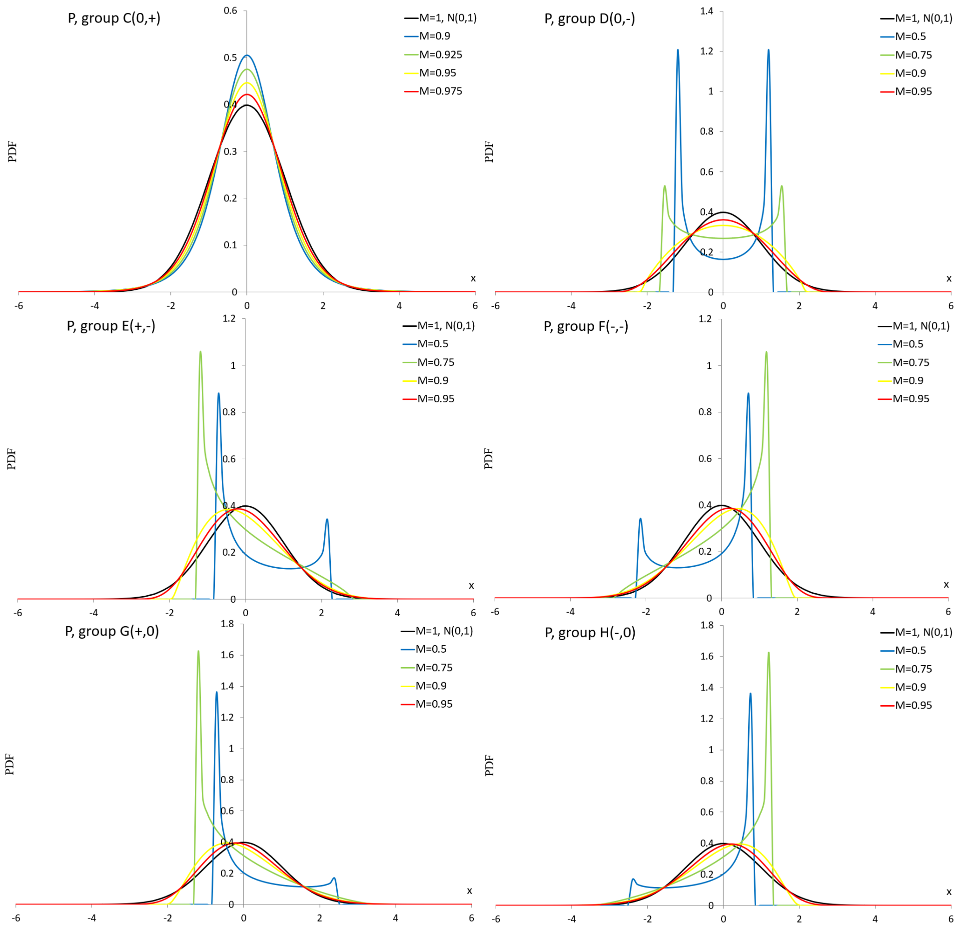


Figure 2A. *PDF* curves of the Pearson distribution for parameter values presented in Table 2A (cont.)



Source: authors' work.

Normal mixture distribution

PDF of the normal mixture (*NM*) distribution is given by

$$f_{NM}(x;\boldsymbol{\theta}) = \omega\phi(x;\mu_1,\sigma_1) + (1-\omega)\phi(x;\mu_2,\sigma_2)\ (x \in R),$$

where $\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,\omega)$ and $\mu_1,\mu_2 \in R, \sigma_1,\sigma_2 > 0, \omega \in [0,1]$. Special cases of the *NM* distribution are:

- normal $N(\mu_1,\sigma_1)$ for $\omega = 1$, $N(\mu_2,\sigma_2)$ for $\omega = 0$;
- location contaminated normal (*LCN*)

$f_{LCM}(x;\mu_1,\omega) = f_{NM}(x;\mu_1,1{,}0{,}1,\omega)$;

- scale contaminated normal (*SCN*)

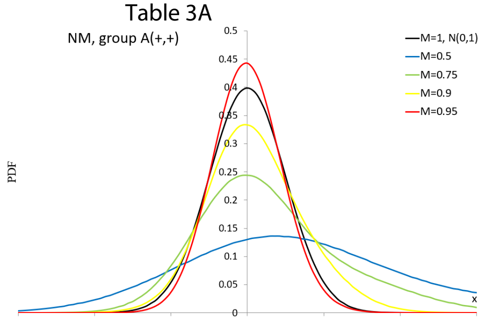
$f_{SCM}(x;\ \sigma_1,\omega) = f_{NM}(x;0,\sigma_1,0{,}1,\omega)$.

Table 3A. Vectors of the *NM* parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta}=(\mu_1,\sigma_1,\mu_2,\sigma_2,\omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta};\mu,\sigma)$
O	$(\mu_1,\sigma_1,\mu_2,\sigma_2,1)$	0	1	0	0	$M(\boldsymbol{\theta};\mu_1,\sigma_1)=1$
	$(\mu_1,\sigma_1,\mu_2,\sigma_2,0)$	0	1	0	0	$M(\boldsymbol{\theta};\mu_2,\sigma_2)=1$
A	0.572, 2.472, 5.614, 3.454, 0.787	1.646	3.408	0.685	0.755	$M(\boldsymbol{\theta};0,1)=0.5$
	-0.215, 1.254, 1.979, 1.99, 0.639	0.577	1.883	0.645	0.502	$M(\boldsymbol{\theta};0,1)=0.75$
	0.497, 1.376, -0.268, 0.884, 0.612	0.2	1.265	0.287	0.249	$M(\boldsymbol{\theta};0,1)=0.9$
	-0.098, 0.857, 0.31, 1.007, 0.767	-0.003	0.911	0.09	0.099	$M(\boldsymbol{\theta};0,1)=0.95$
B	0.502, 2.019, 1.708, 0.953, 0.36	1.274	1.544	-0.748	1.502	$M(\boldsymbol{\theta};0,1)=0.5$
	0.06, 1.437, 1.004, 0.609, 0.634	0.406	1.285	-0.5	0.499	$M(\boldsymbol{\theta};0,1)=0.75$
	0.709, 0.368, -0.072, 1.115, 0.193	0.079	1.06	-0.301	0.15	$M(\boldsymbol{\theta};0,1)=0.9$
	0.159, 0.955, -0.123, 1.158, 0.271	-0.047	1.114	-0.05	0.059	$M(\boldsymbol{\theta};0,1)=0.95$
C	0.519, 6.599, 0.519, 1.058, 0.665	0.519	5.416	0	1.398	$M(\boldsymbol{\theta};0,1)=0.5$
	0.137, 0.581, 0.137, 2.391, 0.294	0.137	2.034	0	1.054	$M(\boldsymbol{\theta};0,1)=0.75$
	0.225, 1.106, 0.225, 0.335, 0.89	0.225	1.049	0	0.299	$M(\boldsymbol{\theta};0,1)=0.9$
	-0.09, 1.029, -0.09, 1.37, 0.825	-0.09	1.096	0	0.201	$M(\boldsymbol{\theta};0,1)=0.95$
D	2.303, 0.51, 0.624, 0.481, 0.515	1.489	0.975	0	-1.099	$M(\boldsymbol{\theta};0,1)=0.5$
	2.707, 0.013, 0.017, 1.125, 0.238	0.657	1.509	0	-1.001	$M(\boldsymbol{\theta};0,1)=0.75$
	1.243, 0.621, -0.39, 0.811, 0.347	0.111	1.09	0	-0.63	$M(\boldsymbol{\theta};0,1)=0.9$
	-1.112, 0.794, 0.023, 0.974, 0.13	0	0.897	0	-0.329	$M(\boldsymbol{\theta};0,1)=0.95$
E	-0.475, 2.22, 5.318, 2.427, 0.721	1.141	3.457	0.5	-0.204	$M(\boldsymbol{\theta};0,1)=0.5$
	-0.019, 1.369, 2.979, 1.15, 0.829	0.494	1.748	0.339	-0.1	$M(\boldsymbol{\theta};0,1)=0.75$
	0.077, 0.844, 1.108, 0.779, 0.845	0.237	0.914	0.074	-0.035	$M(\boldsymbol{\theta};0,1)=0.9$
	1.091, 0.969, -0.111, 1.056, 0.1	0.009	1.108	0.05	-0.01	$M(\boldsymbol{\theta};0,1)=0.95$
F	-0.692, 0.705, 2.1, 0.679, 0.324	1.195	1.476	-0.542	-0.852	$M(\boldsymbol{\theta};0,1)=0.5$
	-0.055, 1.277, 1.781, 0.443, 0.775	0.358	1.377	-0.3	-0.5	$M(\boldsymbol{\theta};0,1)=0.75$
	-0.09, 1.08, -1.581, 0.92, 0.9	-0.239	1.155	-0.071	-0.042	$M(\boldsymbol{\theta};0,1)=0.9$
	0.386, 0.845, -0.145, 0.918, 0.1	-0.092	0.925	-0.01	-0.011	$M(\boldsymbol{\theta};0,1)=0.95$
G	2.686, 3.099, -0.964, 2.217, 0.471	0.755	3.232	0.4	0	$M(\boldsymbol{\theta};0,1)=0.5$
	-0.56, 1.465, 1.411, 1.45, 0.8	-0.166	1.661	0.151	0	$M(\boldsymbol{\theta};0,1)=0.75$
	-0.286, 1.114, 0.984, 1.105, 0.801	-0.033	1.222	0.101	0	$M(\boldsymbol{\theta};0,1)=0.9$
	-0.232, 0.938, 0.727, 0.897, 0.878	-0.115	0.984	0.051	0	$M(\boldsymbol{\theta};0,1)=0.95$
H	2.425, 1.101, 0.272, 1.693, 0.526	1.404	1.775	-0.499	0	$M(\boldsymbol{\theta};0,1)=0.5$
	0.864, 1.125, -1.339, 1.241, 0.735	0.28	1.511	-0.386	0	$M(\boldsymbol{\theta};0,1)=0.75$
	0.429, 1.078, -0.364, 1.228, 0.434	-0.02	1.23	-0.1	0	$M(\boldsymbol{\theta};0,1)=0.9$
	0.108, 1.088, -0.524, 1.073, 0.879	0.032	1.106	-0.01	0	$M(\boldsymbol{\theta};0,1)=0.95$

Source: authors' work.

Figure 3A. *PDF* curves of the normal mixture distribution for parameter values presented in Table 3A



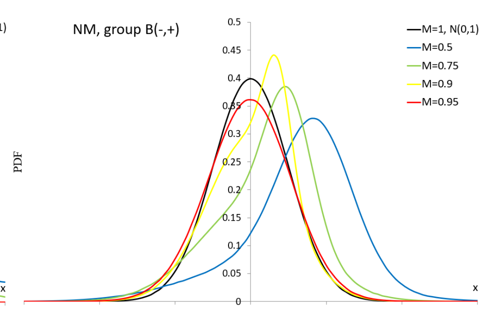
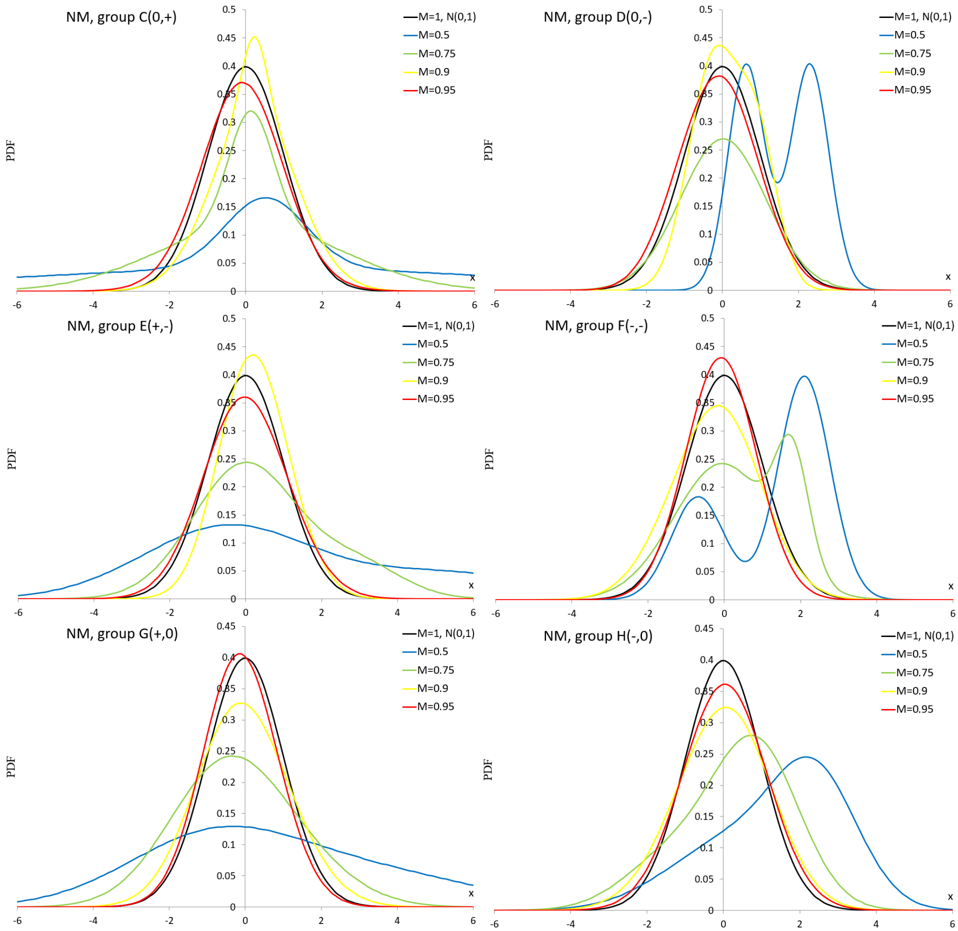


Figure 3A. *PDF* curves of the normal mixture distribution for parameter values presented in Table 3A (cont.)



Source: authors' work.

Normal logistic mixture distribution

PDF of the normal logistic mixture (*NLM*) distribution is given by

$$f_{NLM}(x;\boldsymbol{\theta}) = \omega\phi(x;\mu_1,\sigma_1) + (1-\omega)\frac{exp[-(x-\mu_2)/\sigma_2]}{\sigma_2\{1+xp[-(x-\mu_2)/\sigma_2]\}^2}\ (x \in R),$$

where $\boldsymbol{\theta} = (\mu_1, \sigma_1, \mu_2, \sigma_2, \omega)$ and $\mu_1, \mu_2 \in R, \sigma_1, \sigma_2 > 0, \omega \in [0,1]$. Special cases of the *NLM* distribution are:

- normal $N(\mu_1, \sigma_1)$ for $\omega = 1$;
- logistic (L) $f_L(x; \mu_2, \sigma_2)$ for $\omega = 0$.

Table 4A. Vectors of *NLM* parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1, \sigma_1, \mu_2, \sigma_2, \omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	$(\mu_1, \sigma_1, \mu_2, \sigma_2, 1)$	0	1	0	0	$M(\boldsymbol{\theta}; \mu_1, \sigma_1) = 1$
A	0.003, 0.223, 1.17, 0.701, 0.823	0.209	0.571	2.012	4.307	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	2.179, 2.001, -0.212, 0.917, 0.15	0.146	1.429	1.408	3.475	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	0.089, 1.119, 0.66, 1.78, 0.9	0.146	1.214	0.173	0.585	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.063, 1.019, 1.533, 0.962, 0.9	0.096	1.121	0.173	0.093	$M(\boldsymbol{\theta}; 0,1) = 0.95$
B	1.305, 0.572, 0.281, 1.747, 0.545	0.839	1.351	-0.851	1.731	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.449, 0.572, 0.281, 1.669, 0.479	0.361	1.271	-0.151	1.731	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	0.146, 0.843, 0.128, 1.669, 0.73	0.141	1.127	-0.015	1.578	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.034, 0.974, 0.038, 0.767, 0.535	0.035	0.884	-0.002	0.158	$M(\boldsymbol{\theta}; 0,1) = 0.95$
C	1.382, 0.88, 1.382, 1.769, 0.242	1.382	1.6	0	0.465	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.536, 1.412, 0.536, 1.855, 0.9	0.536	1.462	0	0.124	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	0.107, 0.595, 0.107, 0.528, 0.142	0.107	0.538	0	0.025	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.021, 0.875, 0.021, 0.756, 0.9	0.021	0.864	0	0.018	$M(\boldsymbol{\theta}; 0,1) = 0.95$
D	2.664, 1.103, 0.28, 1.333, 0.352	1.119	1.696	0	-0.458	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.772, 1.253, -1.236, 0.706, 0.899	0.569	1.353	0	-0.268	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	0.952, 0.489, 0.008, 0.682, 0.12	0.121	0.729	0	-0.176	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.321, 0.631, -0.209, 0.665, 0.268	-0.066	0.697	0	-0.035	$M(\boldsymbol{\theta}; 0,1) = 0.95$
E	2.917, 1.445, 0.207, 1.5, 0.292	0.999	1.929	0.201	-0.237	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.277, 0.569, 1.43, 0.334, 0.9	0.392	0.650	0.162	-0.205	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	0.425, 0.956, -0.714, 0.775, 0.673	0.053	1.047	0.106	-0.199	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.19, 0.909, 0.928, 0.889, 0.778	0.058	1.017	0.107	-0.034	$M(\boldsymbol{\theta}; 0,1) = 0.95$
F	0.757, 0.486, 1.444, 0.459, 0.45	1.134	0.582	-0.105	-0.24	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.263, 0.905, 1.505, 0.832, 0.571	0.795	1.069	-0.040	-0.232	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	-0.435, 0.71, 0.495, 0.765, 0.286	0.229	0.859	-0.038	-0.123	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.004, 1.011, -1.433, 0.77, 0.9	-0.139	1.079	-0.038	-0.115	$M(\boldsymbol{\theta}; 0,1) = 0.95$
G	0.131, 0.191, 2.38, 1.824, 0.393	1.497	1.8	0.807	0	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	1.419, 1.558, -0.332, 0.321, 0.649	0.804	1.520	0.688	0	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	-0.049, 0.982, 1.642, 0.994, 0.781	0.322	1.208	0.275	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	-0.104, 0.984, 0.585, 0.981, 0.802	0.032	1.021	0.028	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$
H	1.558, 1.02, -2.125, 0.982, 0.794	0.8	1.8	-0.8	0	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	0.769, 1.037, -1.115, 1.083, 0.762	0.320	1.320	-0.320	0	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	-0.071, 0.854, 0.669, 0.714, 0.496	0.302	0.869	-0.186	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	0.176, 0.841, -0.229, 0.858, 0.64	0.030	0.869	-0.019	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$

Source: authors' work.

Figure 4A. *PDF* curves of the normal logistic mixture distribution for parameter values presented in Table 4A

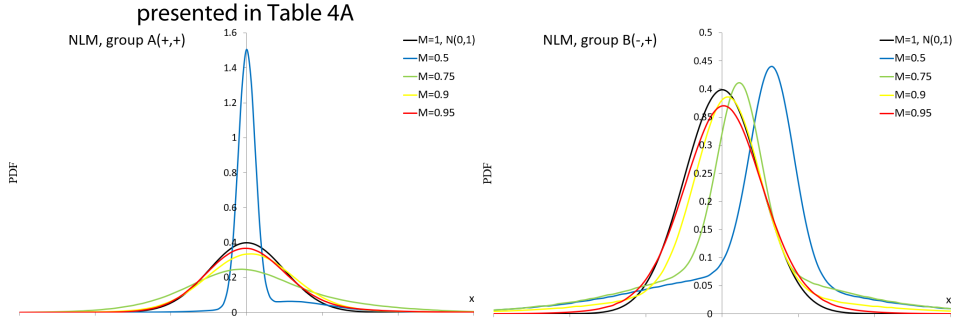
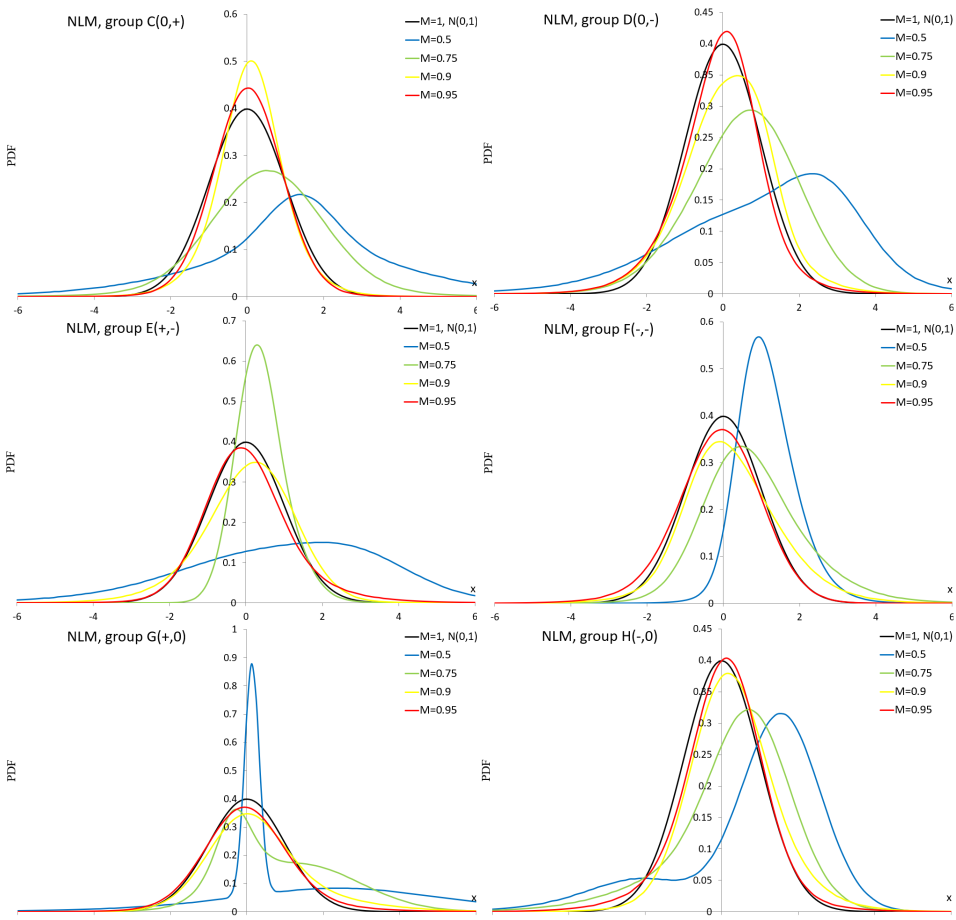


Figure 4A. *PDF* curves of the normal logistic mixture distribution for parameter values presented in Table 4A (cont.)



Source: authors' work.

Normal distribution with plasticising component

PDF of the normal distribution with plasticising component (*NDPC*) is given by

$$f_{NDPC}(x;\boldsymbol{\theta}) = \omega\phi(x;\mu_1,\sigma_1) + (1-\omega)\frac{c_2}{\sigma_2}\left|\frac{x-\mu_2}{\sigma_2}\right|^{c_2-1}\phi\left(\left|\frac{x-\mu_2}{\sigma_2}\right|^{c_2};0,1\right)\ (x \in R),$$

where $\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,c_2,\omega)$ and $\mu_1,\mu_2 \in R, \sigma_1,\sigma_2 > 0, c_2 \geq 1, \omega \in [0,1]$. Special cases of the *NDPC* distribution are:

- $N(\mu_1,\sigma_1)$ for $\omega = 1$ and $N(\mu_2,\sigma_2)$ for $c_2 = 1, \omega = 0$;
- plasticising component (*PC*) $f_{PC}(x;\mu_2,\sigma_2,c_2)$ for $\omega = 0$.

Table 5A. Vectors of *NDPC* parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1, \sigma_1, \mu_2, \sigma_2, c_2, \omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	$\mu_1, \sigma_1, \mu_2, \sigma_2, c_2, 1$	0	1	0	0	$M(\boldsymbol{\theta}; \mu_1, \sigma_1) = 1$
	$\mu_1, \sigma_1, \mu_2, \sigma_2, 1, 0$	0	1	0	0	$M(\boldsymbol{\theta}; \mu_2, \sigma_2) = 1$
A	1.948, 1.27, 1.143, 0.793, 1.482, 0.324	1.404	1.009	0.599	0.853	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.265, 0.415, 0.996, 1.541, 1.16, 0.313	0.767	1.288	0.426	0.152	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.173, 0.358, 0.289, 1.268, 1.132, 0.198	0.266	1.104	0.056	0.071	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.047, 1.02, -0.014, 0.872, 1, 0.214	-0.001	0.906	0.012	0.06	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
B	0.895, 0.421, -0.141, 1.9, 3.141, 0.872	0.762	0.804	-2	5.085	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.539, 0.632, -1.078, 2.061, 1.174, 0.741	0.12	1.34	-1.499	2.986	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	-0.966, 1.824, 0.259, 0.889, 1.1, 0.26	-0.059	1.305	-0.899	1.999	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	-0.099, 0.938, 0.399, 0.646, 1.204, 0.831	-0.015	0.911	-0.125	0.036	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
C	1.592, 1.867, 1.596, 1.215, 1.2, 0.249	1.595	1.365	-0.002	0.528	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.571, 1.023, 0.571, 1.962, 1.15, 0.505	0.571	1.508	0	0.325	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	-0.097, 1.332, -0.097, 1.058, 1.1, 0.614	-0.097	1.223	0	0.101	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.003, 1.135, 0.003, 0.95, 1.05, 0.874	0.003	1.112	0	0.026	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
D	-0.692, 2.203, -0.692, 2.544, 1.759, 0.25	-0.692	2.265	0	-1	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.323, 1.312, 0.605, 1.335, 1.2, 0.01	0.602	1.266	0	-0.587	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.179, 0.494, 0.179, 1.163, 1.426, 0.443	0.179	0.862	0	-0.202	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.195, 0.96, -0.719, 0.858, 1.109, 0.918	0.12	0.983	0	-0.05	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
E	0.675, 0.284, 2.122, 1.968, 2.104, 0.374	1.581	1.565	0.749	-0.849	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.423, 1.032, 1.058, 2.077, 1.815, 0.494	0.744	1.544	0.311	-0.667	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.159, 0.389, 0.296, 1.257, 1.649, 0.326	0.251	0.96	0.116	-0.597	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	1.081, 0.621, -0.216, 0.755, 1, 0.24	0.095	0.912	0.1	-0.298	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
F	1.609, 0.59, 0.322, 2.194, 1.609, 0.309	0.72	1.784	-0.491	-0.728	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.617, 0.737, 0.129, 1.752, 1.465, 0.332	0.291	1.395	-0.239	-0.526	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.189, 0.39, 0.007, 1.336, 1.739, 0.414	0.082	0.957	-0.195	-0.422	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.155, 0.882, 0.019, 1.184, 1.175, 0.581	0.098	0.995	-0.05	-0.188	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
G	1.877, 0.829, 0.573, 0.583, 2.562, 0.383	1.072	0.912	0.669	-0.001	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.362, 1.583, -1.112, 1.026, 1.283, 0.608	-0.216	1.55	0.405	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.055, 0.702, 0.474, 1.586, 1.328, 0.473	0.276	1.191	0.31	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.212, 1.443, -0.012, 1.057, 1.088, 0.1	0.01	1.079	0.05	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
H	1.064, 0.408, 0.687, 0.908, 2, 0.699	0.951	0.587	-0.601	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.186, 0.259, -0.072, 0.973, 1.697, 0.499	0.057	0.659	-0.473	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.116, 1.127, -0.443, 1.588, 1.225, 0.816	0.013	1.223	-0.143	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.029, 1.105, 0.161, 0.958, 1.036, 0.329	0.118	1.003	-0.028	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$

Source: authors' work.

Figure 5A. *PDF* curves of the *NDPC* for parameter values presented in Table 5A

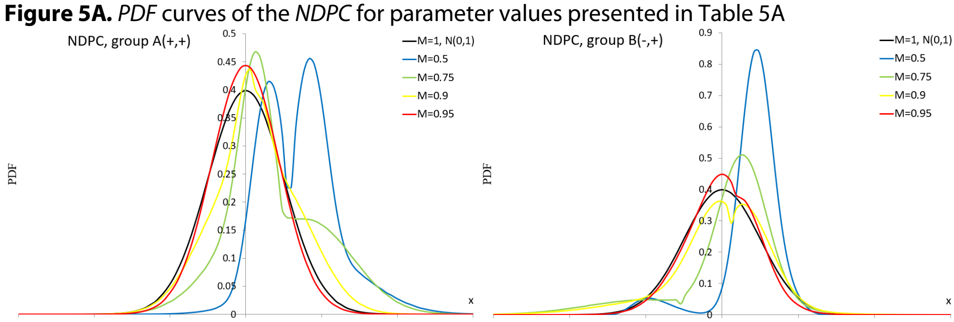
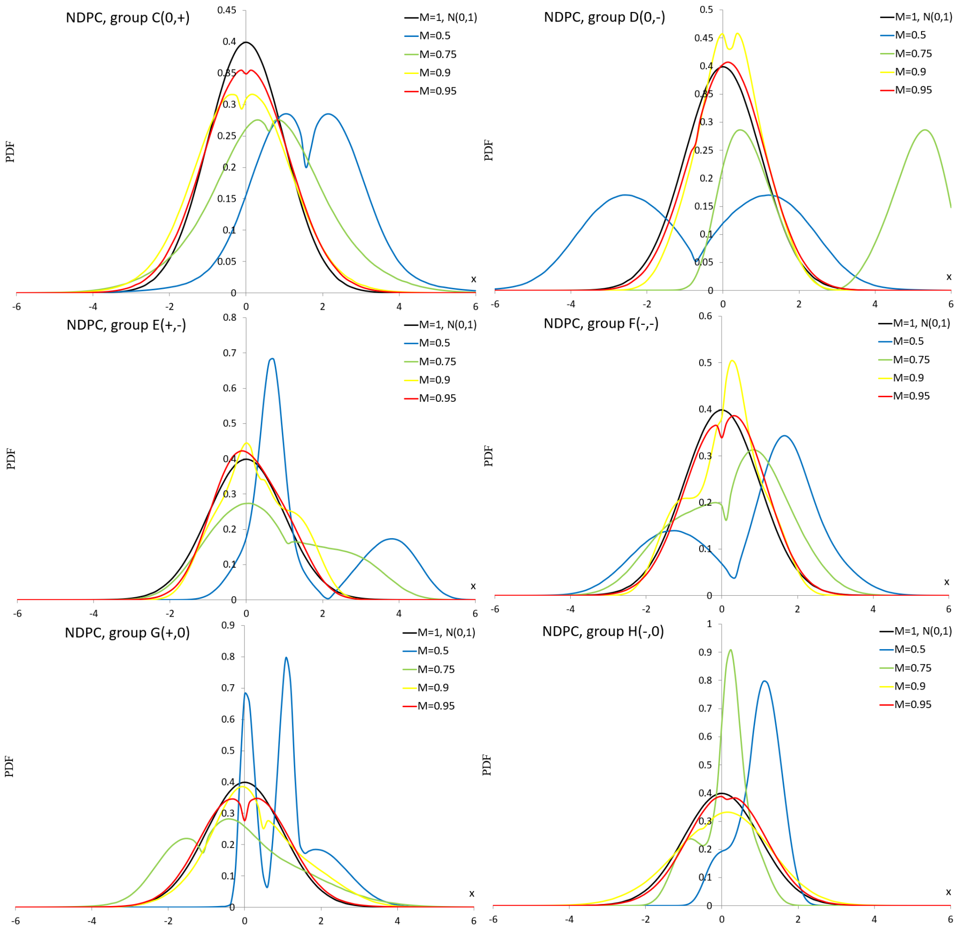


Figure 5A. *PDF* curves of the *NDPC* for parameter values presented in Table 5A (cont.)



Source: authors' work.

Plasticising component mixture distribution

PDF of the plasticising component mixture (*PCM*) distribution is given by

$$f_{PCM}(x;\boldsymbol{\theta}) = \omega f_{PC}(x;\mu_1,\sigma_1,c_1) + (1-\omega) f_{PC}(x;\mu_2,\sigma_2,c_2)\ (x \in R),$$

where $f_{PC}(x;\mu,\sigma,c) = \frac{c}{\sigma}\left|\frac{x-\mu}{\sigma}\right|^{c-1} \phi\left(\left|\frac{x-\mu}{\sigma}\right|^{c};0,1\right)$ $(x \in R)$ and $\boldsymbol{\theta} = (\mu_1,\sigma_1,c_1,\mu_2,\sigma_2,c_2,\omega)$, $\mu_1,\mu_2 \in R$, $\sigma_1,\sigma_2 > 0, c_1,c_2 \geq 1, \omega \in [0,1]$. Special cases of the *PCM* distribution are:

- $N(\mu_1,\sigma_1)$ for $c_1 = 1, \omega = 1$; $N(\mu_2,\sigma_2)$ for $c_2 = 1, \omega = 0$,
- plasticising component $PC(\mu_1,\sigma_1,c_1), PC(\mu_2,\sigma_2,c_2)$ for $\omega = 1, \omega = 0$, respectively.

Table 6A. Vectors of *PCM* parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure *M*. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1, \sigma_1, c_1, \mu_2, \sigma_2, c_2, \omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	$\mu_1, \sigma_1, 1, \mu_2, \sigma_2, c_2, 1$	0	1	0	0	$M(\boldsymbol{\theta}; \mu_1, \sigma_1) = 1$
	$\mu_1, \sigma_1, c_1, \mu_2, \sigma_2, 1{,}0$	0	1	0	0	$M(\boldsymbol{\theta}; \mu_2, \sigma_2) = 1$
A	1.415, 1.684, 2.194, 11.252, 5.474, 2.331, 0.9	2.399	3.622	2.647	7.663	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.444, 0.899, 1.602, 1.653, 2.506, 1.876, 0.64	0.879	1.604	0.913	0.412	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	-0.076, 1.056, 1.1, 0.701, 1.646, 1.095, 0.71	0.149	1.268	0.374	0.374	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.026, 1.078, 1.001, 0.701, 1.646, 1.174, 0.95	0.06	1.117	0.099	0.148	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
B	1.366, 0.572, 1.11, 0.502, 1.669, 1.253, 0.658	1.071	1.099	-0.978	1.565	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.67, 0.425, 1.576, -0.323, 1.696, 1.05, 0.349	0.024	1.444	-0.569	0.606	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	-0.204, 2.209, 1.205, 0.133, 1.139, 1.05, 0.076	0.107	1.224	-0.122	0.457	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.121, 0.936, 1.05, -0.17, 1.917, 1.411, 0.95	0.106	0.982	-0.1	0.204	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
C	1.597, 2.518, 1.263, 1.596, 0.856, 1.285, 0.526	1.597	1.797	0	0.601	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.012, 0.274, 1.256, 0.012, 2.046, 1.01, 0.183	0.012	1.846	0	0.598	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.127, 1.089, 1.01, 0.127, 0.183, 1.01, 0.863	0.127	1.01	0	0.401	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.075, 0.973, 1.01, 0.075, 1.964, 1.362, 0.867	0.075	1.119	0	0.387	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
D	1.631, 0.893, 1.05, 1.632, 2.104, 1.554, 0.498	1.632	1.488	0	-0.268	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.639, 1.576, 1.167, 0.64, 1.085, 1.199, 0.163	0.64	1.12	0	-0.251	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.666, 1.123, 4.041, 0.233, 1.069, 1.05, 0.01	0.237	1.052	0	-0.198	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.225, 1.087, 1.05, -0.067, 1.094, 1.05, 0.233	0.001	1.081	0	-0.18	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
E	1.472, 0.782, 1.11, 0.236, 0.291, 3.203, 0.692	1.091	0.861	0.38	-0.8	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	-0.196, 0.341, 1.064, 0.613, 0.758, 1.204, 0.153	0.489	0.734	0.201	-0.7	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.722, 0.703, 1.304, -0.57, 0.598, 1.05, 0.455	0.018	0.893	0.179	-0.617	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.584, 1.171, 9.804, -0.016, 1.024, 1.076, 0.05	0.014	1.013	0.028	-0.351	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
F	0.261, 1.419, 1.909, 3.099, 0.744, 1.567, 0.57	1.481	1.757	-0.3	-1.107	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	0.037, 1.295, 1.076, 1.316, 1.171, 1.654, 0.485	0.696	1.326	-0.204	-0.4	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.201, 0.121, 1.573, 0.184, 1.177, 1.161, 0.066	0.185	1.087	-0.003	-0.331	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.049, 1.063, 1.088, 1.392, 0.511, 1.05, 0.99	0.062	1.038	-0.008	-0.328	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
G	1.088, 0.894, 3.782, 1.969, 2.71, 1.792, 0.55	1.484	1.793	0.6	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	1.515, 2.553, 3.55, 0.07, 1.328, 1.619, 0.07	0.171	1.359	0.501	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	-0.034, 1.072, 1.159, 1.146, 1.51, 1.301, 0.756	0.254	1.238	0.401	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	0.825, 1.615, 1.868, 0.067, 0.934, 1.05, 0.141	0.174	1.044	0.336	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$
H	0.816, 1.867, 1.24, 1.787, 1.272, 1.05, 0.278	1.517	1.475	-0.302	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.5$
	-0.364, 1.889, 1.057, 0.29, 1.413, 1.05, 0.527	-0.055	1.682	-0.154	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.75$
	0.286, 0.405, 1.27, -0.263, 1.261, 1.05, 0.112	-0.202	1.188	-0.128	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.9$
	-0.153, 1.344, 1.349, -0.024, 0.539, 1.05, 0.565	-0.097	1	-0.12	0	$M(\boldsymbol{\theta}; 0{,}1) = 0.95$

Source: authors' work.

Figure 6A. *PDF* curves of the *PCM* distribution for parameter values presented in Table 6A

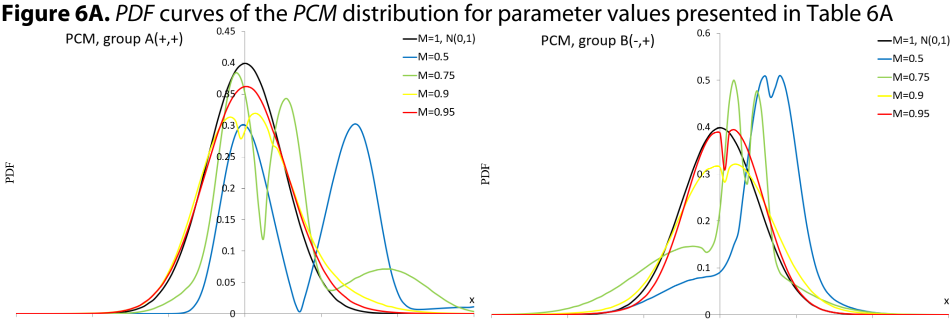
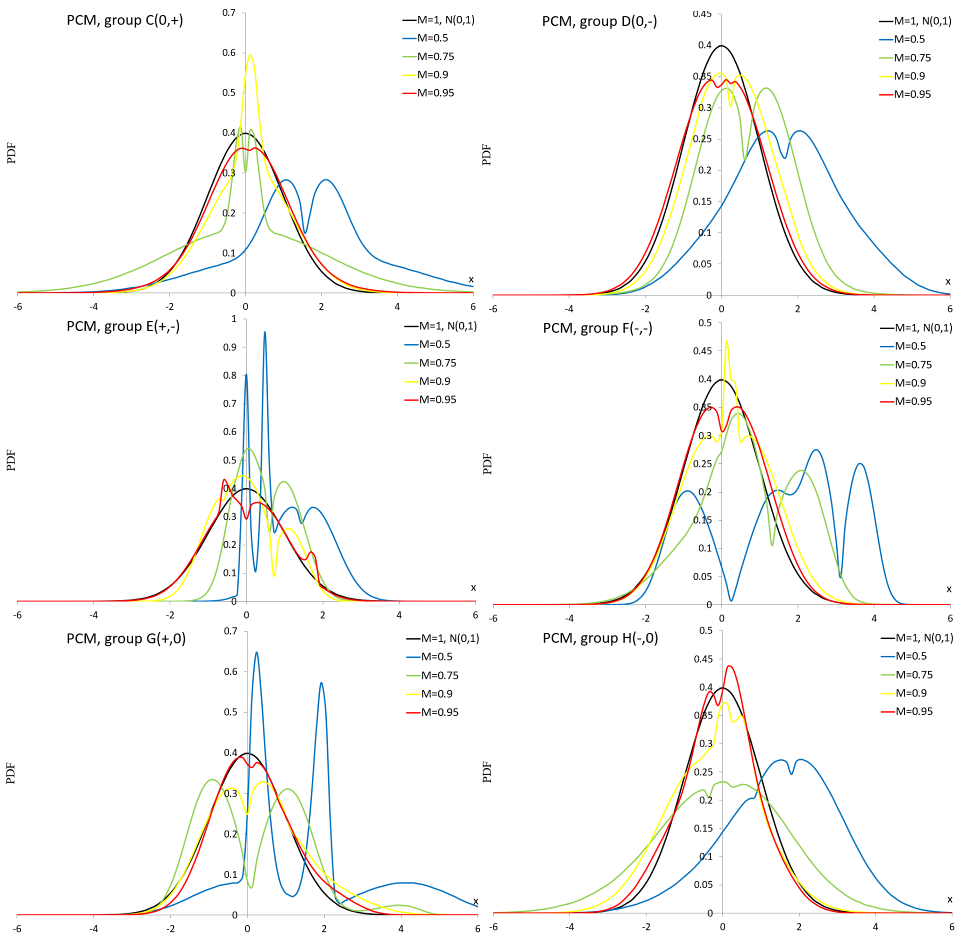


Figure 6A. *PDF* curves of the *PCM* distribution for parameter values presented in Table 6A (cont.)



Source: authors' work.

Table 7A. The power of *GoFTs* for group of alternatives A (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	**0.853**	0.614	0.789	0.792	0.764	0.791	0.792	0.789	0.789	0.817	0.658	0.847
	20	0.993	0.965	0.987	0.987	0.984	0.987	0.987	0.987	0.987	0.993	0.957	**0.997**
P_2	10	**0.374**	0.126	0.260	0.270	0.269	0.272	0.270	0.273	0.273	0.293	0.219	0.316
	20	0.650	0.380	0.533	0.544	0.541	0.545	0.544	0.546	0.546	0.599	0.421	**0.670**
P_2	10	**0.149**	0.049	0.093	0.099	0.107	0.101	0.100	0.103	0.103	0.108	0.092	0.111
	20	**0.227**	0.092	0.149	0.158	0.174	0.161	0.159	0.164	0.164	0.181	0.138	0.208
P_4	10	**0.092**	0.048	0.064	0.069	0.076	0.070	0.069	0.072	0.072	0.074	0.067	0.076
	20	**0.117**	0.060	0.080	0.086	0.100	0.088	0.087	0.091	0.091	0.098	0.081	0.108
NM_1	10	**0.146**	0.045	0.087	0.094	0.105	0.096	0.095	0.099	0.099	0.103	0.090	0.105
	20	**0.221**	0.087	0.142	0.153	0.170	0.156	0.153	0.159	0.159	0.171	0.136	0.185
NM_2	10	**0.143**	0.042	0.087	0.092	0.098	0.093	0.092	0.095	0.095	0.099	0.089	0.100
	20	**0.224**	0.084	0.147	0.155	0.165	0.157	0.156	0.159	0.159	0.169	0.138	0.179
NM_2	10	**0.084**	0.041	0.059	0.061	0.065	0.062	0.062	0.063	0.063	0.063	0.061	0.063
	20	**0.102**	0.048	0.071	0.073	0.079	0.075	0.074	0.075	0.075	0.078	0.071	0.081
NM_4	10	**0.057**	0.048	0.052	0.053	0.054	0.053	0.053	0.054	0.054	0.054	0.051	0.053
	20	**0.061**	0.047	0.053	0.053	0.055	0.054	0.054	0.054	0.054	0.055	0.053	0.056
NLM_1	10	**0.628**	0.469	0.552	0.577	0.604	0.581	0.578	0.588	0.588	0.596	0.541	0.591
	20	**0.881**	0.832	0.846	0.860	0.879	0.863	0.861	0.867	0.867	0.878	0.832	**0.885**
NLM_2	10	**0.112**	0.059	0.075	0.083	0.098	0.086	0.084	0.089	0.089	0.092	0.083	0.092
	20	**0.153**	0.084	0.102	0.115	0.139	0.118	0.116	0.123	0.123	0.131	0.106	0.137
NLM_2	10	0.138	0.116	0.114	0.127	**0.149**	0.130	0.128	0.135	0.135	0.142	0.121	0.145
	20	0.220	0.193	0.181	0.201	0.239	0.206	0.202	0.214	0.214	0.237	0.181	**0.267**
NLM_4	10	**0.113**	0.058	0.076	0.083	0.096	0.085	0.084	0.087	0.087	0.093	0.081	0.095
	20	**0.157**	0.087	0.106	0.118	0.141	0.121	0.119	0.126	0.126	0.138	0.107	**0.159**
$NDPC_1$	10	**0.117**	0.058	0.093	0.092	0.088	0.091	0.092	0.091	0.091	0.096	0.084	0.100
	20	0.170	0.082	0.127	0.128	0.128	0.129	0.128	0.129	0.129	0.150	0.115	**0.179**
$NDPC_2$	10	**0.183**	0.065	0.131	0.135	0.131	0.135	0.135	0.135	0.135	0.131	0.126	0.121
	20	**0.306**	0.157	0.243	0.246	0.242	0.247	0.246	0.246	0.246	0.231	0.224	0.188
$NDPC_2$	10	**0.063**	0.049	0.053	0.056	0.059	0.056	0.056	0.057	0.057	0.057	0.056	0.054
	20	**0.071**	0.052	0.056	0.060	0.067	0.061	0.060	0.062	0.062	0.060	0.062	0.053
$NDPC_4$	10	**0.052**	0.051	0.051	0.051	**0.052**	0.051	0.051	0.051	0.051	**0.052**	0.051	0.051
	20	**0.053**	0.051	0.051	0.051	**0.053**	0.051	0.051	0.052	0.052	0.052	0.051	**0.053**
PCM_1	10	0.594	0.476	**0.608**	0.595	0.540	0.590	0.594	0.581	0.581	0.593	0.527	0.594
	20	0.888	0.821	0.874	0.874	0.854	0.874	0.874	0.871	0.871	0.895	0.814	**0.896**
PCM_2	10	**0.296**	0.081	0.192	0.201	0.203	0.202	0.201	0.204	0.204	0.219	0.172	0.228
	20	**0.549**	0.267	0.416	0.431	0.439	0.433	0.431	0.437	0.437	0.481	0.342	0.494
PCM_2	10	**0.085**	0.041	0.061	0.063	0.065	0.063	0.063	0.064	0.064	0.066	0.062	0.067
	20	**0.104**	0.048	0.073	0.076	0.079	0.076	0.076	0.077	0.077	0.083	0.070	0.093
PCM_4	10	**0.058**	0.049	0.053	0.053	0.055	0.054	0.053	0.054	0.054	0.055	0.053	0.054
	20	**0.062**	0.049	0.053	0.054	0.057	0.055	0.054	0.055	0.055	0.056	0.054	0.058

Note. The highest *PoTs* of the *MCM(α,β)* values are underlined. The highest *PoTs* for all the analysed tests are in bold.
Source: authors' work.

Table 8A. The power of *GoFTs* for group of alternatives B (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.612	**0.853**	0.788	0.790	0.762	0.789	0.790	0.786	0.786	0.815	0.657	0.846
	20	0.964	**0.993**	0.986	0.987	0.984	0.987	0.987	0.986	0.986	0.993	0.957	**0.997**
P_2	10	0.125	**0.372**	0.260	0.268	0.267	0.269	0.269	0.271	0.271	0.292	0.217	0.315
	20	0.380	0.655	0.537	0.545	0.542	0.547	0.546	0.548	0.548	0.603	0.423	**0.673**
P_2	10	0.047	**0.149**	0.092	0.099	0.107	0.100	0.099	0.102	0.102	0.107	0.091	0.112
	20	0.093	**0.228**	0.150	0.160	0.175	0.163	0.161	0.166	0.166	0.181	0.140	0.207
P_4	10	0.048	**0.092**	0.065	0.069	0.076	0.070	0.069	0.072	0.072	0.074	0.067	0.074
	20	0.059	**0.119**	0.079	0.086	0.100	0.088	0.087	0.091	0.091	0.098	0.081	0.107
NM_1	10	0.068	**0.182**	0.112	0.125	0.144	0.128	0.126	0.132	0.132	0.138	0.116	0.139
	20	0.141	**0.293**	0.194	0.214	0.249	0.219	0.215	0.226	0.226	0.244	0.189	0.256
NM_2	10	0.049	**0.151**	0.097	0.102	0.107	0.103	0.102	0.105	0.105	0.107	0.096	0.106
	20	0.101	**0.237**	0.165	0.173	0.182	0.175	0.174	0.177	0.177	0.180	0.152	0.175
NM_2	10	0.042	**0.108**	0.077	0.078	0.076	0.078	0.078	0.078	0.078	0.078	0.073	0.077
	20	0.066	**0.153**	0.114	0.114	0.110	0.114	0.114	0.113	0.113	0.113	0.102	0.108
NM_4	10	0.048	**0.055**	0.051	0.051	0.052	0.051	0.051	0.051	0.051	0.052	0.052	0.051
	20	0.046	**0.056**	0.050	0.051	0.052	0.051	0.051	0.051	0.051	0.052	0.051	0.052
NLM_1	10	0.318	0.446	0.369	0.404	**0.451**	0.412	0.406	0.423	0.423	0.416	0.377	0.382
	20	0.683	0.758	0.695	0.729	**0.776**	0.736	0.730	0.746	0.746	0.735	0.666	0.659
NLM_2	10	0.301	0.322	0.289	0.326	**0.378**	0.333	0.328	0.346	0.346	0.338	0.310	0.305
	20	0.593	0.609	0.562	0.604	**0.671**	0.613	0.606	0.628	0.628	0.613	0.549	0.534
NLM_2	10	0.248	0.250	0.225	0.255	**0.302**	0.262	0.257	0.272	0.272	0.281	0.237	0.275
	20	0.458	0.460	0.414	0.455	**0.524**	0.464	0.458	0.478	0.478	0.504	0.402	0.508
NLM_4	10	0.075	0.075	0.067	0.073	**0.084**	0.075	0.074	0.077	0.077	0.080	0.072	0.081
	20	0.094	0.094	0.080	0.090	0.111	0.092	0.091	0.096	0.096	0.106	0.083	**0.120**
$NDPC_1$	10	0.265	**0.439**	0.348	0.378	0.414	0.384	0.380	0.393	0.393	0.410	0.343	0.415
	20	0.581	0.684	0.609	0.638	0.680	0.644	0.639	0.653	0.653	0.686	0.590	**0.714**
$NDPC_2$	10	0.197	**0.436**	0.310	0.338	0.370	0.344	0.340	0.351	0.351	0.364	0.304	0.365
	20	0.532	**0.722**	0.612	0.640	0.683	0.647	0.642	0.656	0.656	0.681	0.573	0.688
$NDPC_2$	10	0.067	**0.181**	0.111	0.124	0.143	0.127	0.125	0.131	0.131	0.140	0.116	0.146
	20	0.132	**0.284**	0.185	0.205	0.240	0.210	0.206	0.217	0.217	0.245	0.177	0.282
$NDPC_4$	10	0.044	**0.060**	0.052	0.052	0.053	0.052	0.052	0.052	0.052	0.053	0.053	0.052
	20	0.043	**0.062**	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.054	0.052	0.055
PCM_1	10	0.103	**0.290**	0.184	0.204	0.230	0.209	0.205	0.215	0.215	0.224	0.186	0.224
	20	0.289	**0.507**	0.380	0.408	0.450	0.414	0.410	0.423	0.423	0.444	0.351	0.441
PCM_2	10	0.095	**0.251**	0.179	0.187	0.189	0.189	0.188	0.190	0.190	0.188	0.174	0.177
	20	0.258	**0.442**	0.358	0.369	0.375	0.371	0.370	0.374	0.374	0.359	0.335	0.307
PCM_2	10	0.052	**0.062**	0.056	0.057	0.058	0.057	0.057	0.058	0.058	0.059	0.056	0.059
	20	0.055	0.070	0.061	0.063	0.066	0.063	0.063	0.064	0.064	0.069	0.059	**0.077**
PCM_4	10	0.048	**0.058**	0.054	0.053	0.053	0.053	0.053	0.053	0.053	0.054	0.052	0.055
	20	0.050	**0.062**	0.057	0.056	0.056	0.056	0.056	0.056	0.056	0.058	0.054	**0.063**

Note. As in Table 7A.
Source: authors' work.

Table 9A. The power of *GoFTs* for group of alternatives C (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.110	0.110	0.095	0.108	**0.130**	0.111	0.109	0.116	0.116	0.121	0.105	0.123
	20	0.170	0.170	0.143	0.163	0.202	0.169	0.165	0.176	0.176	0.191	0.148	**0.208**
P_2	10	0.092	0.092	0.079	0.090	**0.108**	0.093	0.091	0.097	0.097	0.101	0.088	0.101
	20	0.130	0.130	0.108	0.124	0.155	0.128	0.125	0.134	0.134	0.147	0.115	**0.162**
P_2	10	0.076	0.075	0.067	0.074	**0.085**	0.075	0.074	0.078	0.078	0.080	0.072	0.081
	20	0.092	0.094	0.079	0.089	0.110	0.092	0.089	0.096	0.096	0.104	0.084	**0.116**
P_4	10	0.061	0.061	0.057	0.061	**0.067**	0.062	0.061	0.063	0.063	0.064	0.060	0.063
	20	0.067	0.067	0.060	0.065	0.075	0.066	0.065	0.067	0.067	0.071	0.063	**0.078**
NM_1	10	0.194	0.196	0.179	0.204	**0.238**	0.209	0.205	0.217	0.217	0.205	0.203	0.174
	20	0.366	0.368	0.335	0.369	**0.427**	0.377	0.371	0.389	0.389	0.356	0.351	0.265
NM_2	10	0.122	0.121	0.105	0.122	**0.147**	0.125	0.122	0.131	0.131	0.128	0.121	0.115
	20	0.200	0.200	0.171	0.197	**0.244**	0.203	0.198	0.213	0.213	0.202	0.186	0.162
NM_2	10	0.061	0.061	0.056	0.060	**0.067**	0.061	0.060	0.063	0.063	0.062	0.061	0.060
	20	0.068	0.069	0.059	0.066	**0.079**	0.067	0.066	0.069	0.069	0.069	0.066	0.066
NM_4	10	0.053	0.054	0.051	0.053	**0.055**	0.054	0.053	0.054	0.054	0.055	0.052	0.055
	20	0.056	0.055	0.053	0.055	**0.059**	0.055	0.055	0.056	0.056	0.057	0.055	0.059
NLM_1	10	0.132	0.133	0.113	0.132	**0.163**	0.136	0.133	0.142	0.142	0.144	0.129	0.137
	20	0.223	0.224	0.188	0.218	**0.272**	0.225	0.220	0.236	0.236	0.239	0.198	0.224
NLM_2	10	0.094	0.095	0.086	0.095	**0.108**	0.096	0.095	0.099	0.099	0.105	0.090	0.107
	20	0.139	0.142	0.120	0.136	0.164	0.139	0.137	0.146	0.146	0.162	0.123	**0.186**
NLM_2	10	0.082	0.081	0.071	0.079	**0.093**	0.081	0.080	0.085	0.085	0.088	0.077	0.088
	20	0.105	0.105	0.086	0.099	0.126	0.103	0.100	0.108	0.108	0.117	0.093	**0.130**
NLM_4	10	0.060	0.062	0.058	0.061	**0.066**	0.062	0.061	0.063	0.063	0.065	0.060	0.064
	20	0.071	0.071	0.064	0.069	0.078	0.070	0.069	0.072	0.072	0.077	0.067	**0.086**
$NDPC_1$	10	0.061	0.061	0.065	0.063	0.061	0.063	0.063	0.062	0.062	0.065	0.061	**0.066**
	20	0.072	0.072	0.076	0.074	0.072	0.074	0.074	0.073	0.073	0.080	0.070	**0.092**
$NDPC_2$	10	0.059	0.060	0.054	0.057	**0.062**	0.057	0.057	0.059	0.059	0.060	0.057	0.060
	20	0.064	0.064	0.058	0.062	**0.071**	0.063	0.062	0.065	0.065	0.067	0.061	0.068
$NDPC_2$	10	0.053	0.052	0.053	0.053	0.052	0.053	0.053	0.053	0.053	**0.054**	0.052	0.053
	20	0.054	0.054	0.055	0.055	0.054	0.055	0.055	0.054	0.054	0.055	0.053	**0.057**
$NDPC_4$	10	0.051	0.051	0.050	0.050	**0.051**	0.050	0.050	0.050	0.050	0.051	0.051	0.050
	20	0.051	0.050	0.050	0.050	**0.051**	0.050	0.050	0.050	0.050	0.050	0.051	0.051
PCM_1	10	0.080	0.081	0.069	0.078	**0.092**	0.080	0.079	0.083	0.083	0.084	0.078	0.082
	20	0.103	0.103	0.086	0.099	**0.125**	0.103	0.100	0.108	0.108	0.110	0.093	0.101
PCM_2	10	0.101	0.101	0.091	0.102	**0.119**	0.104	0.102	0.108	0.108	0.102	0.107	0.089
	20	0.151	0.152	0.132	0.150	**0.181**	0.154	0.151	0.161	0.161	0.144	0.156	0.110
PCM_2	10	0.075	0.074	0.066	0.073	**0.085**	0.075	0.074	0.077	0.077	0.075	0.076	0.069
	20	0.096	0.096	0.082	0.093	**0.114**	0.096	0.094	0.100	0.100	0.094	0.096	0.079
PCM_4	10	0.060	0.059	0.056	0.058	**0.062**	0.059	0.058	0.060	0.060	0.061	0.058	0.061
	20	0.062	0.061	0.056	0.059	0.067	0.060	0.059	0.061	0.061	0.065	0.058	**0.071**

Note. As in Table 7A.
Source: authors' work.

Table 10A. The power of *GoFTs* for group of alternatives D (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.468	0.467	0.629	0.572	0.390	0.556	0.568	0.526	0.526	0.600	0.367	**0.667**
	20	0.871	0.871	0.932	0.910	0.815	0.903	0.909	0.890	0.890	0.950	0.715	**0.980**
P_2	10	0.090	0.093	**0.154**	0.125	0.064	0.118	0.123	0.105	0.105	0.118	0.082	0.127
	20	0.212	0.211	0.311	0.262	0.149	0.251	0.260	0.230	0.230	0.287	0.148	**0.350**
P_2	10	0.044	0.042	**0.061**	0.052	0.033	0.049	0.051	0.046	0.046	0.045	0.045	0.043
	20	0.057	0.056	**0.085**	0.070	0.039	0.066	0.069	0.059	0.059	0.063	0.053	0.059
P_4	10	0.040	0.041	**0.050**	0.045	0.035	0.044	0.045	0.041	0.041	0.041	0.043	0.039
	20	0.043	0.043	**0.057**	0.050	0.034	0.048	0.049	0.044	0.044	0.044	0.045	0.038
NM_1	10	0.098	0.099	**0.160**	0.132	0.073	0.125	0.131	0.113	0.113	0.112	0.101	0.104
	20	0.229	0.233	**0.330**	0.284	0.172	0.272	0.281	0.251	0.251	0.247	0.195	0.212
NM_2	10	0.157	0.128	0.208	0.184	0.116	0.177	0.183	0.167	0.167	0.201	0.142	**0.216**
	20	0.311	0.262	0.365	0.333	0.241	0.325	0.331	0.309	0.309	0.432	0.325	**0.477**
NM_2	10	0.051	0.043	**0.062**	0.054	0.039	0.052	0.054	0.049	0.049	0.048	0.049	0.046
	20	0.064	0.055	**0.082**	0.069	0.045	0.067	0.069	0.062	0.062	0.060	0.058	0.054
NM_4	10	0.047	0.048	**0.049**	0.048	0.047	0.048	0.048	0.048	0.048	0.048	0.048	0.047
	20	0.047	0.048	**0.050**	0.049	0.046	0.049	0.049	0.048	0.048	0.047	0.048	0.045
NLM_1	10	0.046	**0.130**	0.090	0.092	0.092	0.092	0.092	0.092	0.092	0.096	0.084	0.097
	20	0.082	**0.197**	0.139	0.143	0.146	0.144	0.144	0.145	0.145	0.156	0.123	0.170
NLM_2	10	0.044	**0.069**	0.056	0.057	0.058	0.057	0.057	0.057	0.057	0.058	0.056	0.058
	20	0.045	**0.080**	0.060	0.062	0.066	0.063	0.062	0.063	0.063	0.066	0.061	0.072
NLM_2	10	0.058	**0.089**	0.067	0.072	0.081	0.073	0.072	0.075	0.075	0.079	0.070	0.080
	20	0.076	0.116	0.085	0.093	0.110	0.096	0.094	0.098	0.098	0.108	0.086	**0.122**
NLM_4	10	0.069	**0.106**	0.078	0.087	0.101	0.088	0.087	0.091	0.091	0.095	0.083	0.096
	20	0.097	0.148	0.103	0.117	0.145	0.121	0.118	0.126	0.126	0.138	0.108	**0.150**
$NDPC_1$	10	0.097	0.096	**0.155**	0.128	0.072	0.121	0.126	0.109	0.109	0.109	0.098	0.101
	20	0.223	0.224	**0.321**	0.276	0.165	0.264	0.273	0.243	0.243	0.237	0.189	0.204
$NDPC_2$	10	0.052	0.051	**0.069**	0.060	0.042	0.058	0.060	0.055	0.055	0.054	0.055	0.051
	20	0.068	0.068	**0.095**	0.081	0.051	0.078	0.081	0.072	0.072	0.069	0.069	0.059
$NDPC_2$	10	0.047	0.046	**0.048**	0.047	0.045	0.046	0.046	0.046	0.046	0.046	0.047	0.045
	20	0.046	0.045	**0.048**	0.046	0.043	0.046	0.046	0.045	0.045	0.044	0.048	0.040
$NDPC_4$	10	0.050	0.050	**0.051**	0.050	0.049	0.051	0.051	0.051	0.051	0.050	0.051	0.049
	20	0.047	**0.050**	**0.049**	0.049	0.048	0.048	0.048	0.048	0.048	0.048	0.049	0.047
PCM_1	10	0.046	0.046	**0.050**	0.048	0.043	0.047	0.048	0.046	0.046	0.045	0.046	0.043
	20	0.045	0.044	**0.049**	0.046	0.041	0.046	0.046	0.045	0.045	0.043	0.047	0.039
PCM_2	10	0.054	0.054	**0.067**	0.061	0.047	0.059	0.060	0.056	0.056	0.055	0.056	0.054
	20	0.065	0.066	**0.084**	0.074	0.054	0.072	0.074	0.068	0.068	0.066	0.068	0.062
PCM_2	10	0.048	0.047	**0.053**	0.049	0.044	0.049	0.049	0.048	0.048	0.048	0.049	0.046
	20	0.049	0.047	**0.055**	0.051	0.043	0.050	0.051	0.048	0.048	0.047	0.050	0.045
PCM_4	10	0.046	0.047	**0.051**	0.048	0.044	0.047	0.048	0.046	0.046	0.046	0.047	0.046
	20	0.046	0.048	**0.053**	0.050	0.043	0.049	0.050	0.048	0.048	0.046	0.048	0.045

Note. As in Table 7A.
Source: authors' work.

Table 11A. The power of *GoFTs* for group of alternatives E (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.753	0.537	0.739	0.719	0.635	0.712	0.717	0.700	0.700	0.737	0.579	**0.775**
	20	0.972	0.927	0.972	0.967	0.942	0.965	0.966	0.961	0.961	0.979	0.901	**0.990**
P_2	10	**0.264**	0.081	0.212	0.200	0.162	0.196	0.199	0.189	0.189	0.206	0.150	0.224
	20	0.496	0.259	0.440	0.419	0.352	0.414	0.418	0.402	0.402	0.459	0.296	**0.530**
P_2	10	**0.106**	0.031	0.074	0.071	0.064	0.070	0.071	0.069	0.069	0.072	0.065	0.074
	20	**0.158**	0.052	0.116	0.110	0.096	0.109	0.110	0.106	0.106	0.115	0.096	0.127
P_4	10	**0.073**	0.033	0.057	0.054	0.050	0.054	0.054	0.054	0.054	0.054	0.053	0.054
	20	**0.086**	0.036	0.067	0.064	0.058	0.064	0.064	0.062	0.062	0.064	0.060	0.066
NM_1	10	**0.139**	0.036	0.094	0.094	0.086	0.093	0.094	0.092	0.092	0.093	0.086	0.092
	20	**0.227**	0.085	0.169	0.166	0.152	0.166	0.166	0.163	0.163	0.165	0.141	0.157
NM_2	10	**0.094**	0.035	0.065	0.065	0.065	0.065	0.065	0.066	0.066	0.066	0.063	0.065
	20	**0.124**	0.048	0.088	0.088	0.086	0.088	0.088	0.088	0.088	0.089	0.082	0.085
NM_2	10	**0.054**	0.045	0.050	0.050	0.049	0.050	0.050	0.049	0.049	0.049	0.049	0.048
	20	**0.056**	0.044	0.050	0.051	0.049	0.051	0.051	0.050	0.050	0.051	0.051	0.051
NM_4	10	**0.054**	0.047	0.052	0.051	0.051	0.051	0.051	0.051	0.051	0.052	0.050	0.051
	20	**0.054**	0.046	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
NLM_1	10	0.044	**0.094**	0.068	0.070	0.072	0.070	0.070	0.071	0.071	0.074	0.066	0.075
	20	0.057	**0.122**	0.086	0.089	0.095	0.090	0.089	0.091	0.091	0.099	0.082	0.114
NLM_2	10	**0.092**	0.043	0.063	0.068	0.073	0.068	0.068	0.070	0.070	0.071	0.066	0.072
	20	**0.119**	0.053	0.079	0.085	0.096	0.086	0.085	0.087	0.087	0.093	0.080	0.101
NLM_2	10	0.050	**0.104**	0.070	0.076	0.084	0.077	0.076	0.079	0.079	0.081	0.074	0.083
	20	0.068	**0.137**	0.090	0.099	0.116	0.102	0.100	0.105	0.105	0.115	0.091	0.129
NLM_4	10	**0.131**	0.062	0.086	0.096	0.111	0.099	0.097	0.102	0.102	0.106	0.092	0.109
	20	**0.194**	0.103	0.129	0.144	0.172	0.147	0.145	0.152	0.152	0.169	0.130	0.192
$NDPC_1$	10	0.703	0.463	**0.716**	0.689	0.567	0.679	0.687	0.660	0.660	0.648	0.597	0.603
	20	0.979	0.949	**0.983**	0.979	0.954	0.977	0.978	0.974	0.974	0.969	0.940	0.940
$NDPC_2$	10	**0.105**	0.033	0.088	0.080	0.061	0.077	0.079	0.074	0.074	0.074	0.071	0.072
	20	**0.172**	0.072	0.154	0.139	0.104	0.136	0.138	0.129	0.129	0.128	0.111	0.115
$NDPC_2$	10	**0.060**	0.033	0.057	0.051	0.041	0.050	0.051	0.048	0.048	0.047	0.048	0.044
	20	**0.073**	0.038	0.071	0.063	0.046	0.061	0.063	0.058	0.058	0.056	0.056	0.048
$NDPC_4$	10	**0.057**	0.038	0.053	0.049	0.044	0.048	0.049	0.047	0.047	0.048	0.048	0.046
	20	**0.062**	0.038	0.057	0.053	0.045	0.052	0.053	0.051	0.051	0.050	0.052	0.047
PCM_1	10	**0.146**	0.058	0.143	0.125	0.084	0.120	0.124	0.113	0.113	0.116	0.105	0.117
	20	0.272	0.151	**0.280**	0.248	0.172	0.241	0.247	0.226	0.226	0.243	0.208	0.255
PCM_2	10	0.085	0.043	**0.089**	0.077	0.053	0.074	0.077	0.069	0.069	0.068	0.067	0.065
	20	0.137	0.077	**0.148**	0.128	0.083	0.123	0.127	0.114	0.114	0.111	0.102	0.097
PCM_2	10	**0.071**	0.031	0.062	0.057	0.045	0.055	0.056	0.053	0.053	0.052	0.052	0.050
	20	**0.091**	0.040	0.082	0.073	0.054	0.072	0.073	0.068	0.068	0.070	0.061	0.064
PCM_4	10	0.054	0.041	**0.055**	0.052	0.043	0.050	0.051	0.049	0.049	0.049	0.049	0.046
	20	0.059	0.042	**0.061**	0.055	0.043	0.054	0.055	0.052	0.052	0.049	0.053	0.044

Note. As in Table 7A.
Source: authors' work.

Table 12A. The power of *GoFTs* for group of alternatives F (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.538	0.755	0.741	0.721	0.636	0.714	0.720	0.701	0.701	0.740	0.582	**0.777**
	20	0.926	0.971	0.972	0.966	0.942	0.964	0.965	0.961	0.961	0.979	0.900	**0.990**
P_2	10	0.081	0.268	0.214	0.203	0.164	0.199	0.202	0.192	0.192	0.209	0.152	0.227
	20	0.261	0.498	0.441	0.420	0.356	0.415	0.419	0.404	0.404	0.461	0.298	**0.532**
P_2	10	0.030	**0.106**	0.075	0.072	0.064	0.071	0.071	0.070	0.070	0.072	0.066	0.073
	20	0.052	**0.156**	0.115	0.110	0.095	0.108	0.110	0.106	0.106	0.114	0.095	0.127
P_4	10	0.032	**0.074**	0.056	0.054	0.051	0.054	0.054	0.053	0.053	0.054	0.052	0.053
	20	0.036	**0.086**	0.065	0.062	0.056	0.062	0.062	0.061	0.061	0.063	0.059	0.064
NM_1	10	0.120	**0.326**	0.297	0.274	0.204	0.267	0.272	0.255	0.255	0.254	0.216	0.240
	20	0.447	**0.656**	0.642	0.614	0.517	0.606	0.612	0.591	0.591	0.584	0.476	0.530
NM_2	10	0.041	**0.087**	0.086	0.075	0.053	0.072	0.074	0.068	0.068	0.069	0.064	0.069
	20	0.067	**0.129**	0.129	0.114	0.077	0.110	0.113	0.103	0.103	0.109	0.091	0.110
NM_2	10	0.045	**0.055**	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
	20	0.044	**0.057**	0.051	0.051	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.048
NM_4	10	0.049	0.049	**0.050**	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049
	20	0.049	**0.051**	**0.051**	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.050	0.050
NLM_1	10	**0.112**	0.047	0.074	0.079	0.087	0.080	0.080	0.082	0.082	0.086	0.077	0.087
	20	**0.162**	0.074	0.108	0.116	0.130	0.118	0.117	0.121	0.121	0.131	0.105	0.146
NLM_2	10	**0.125**	0.050	0.080	0.087	0.097	0.089	0.087	0.091	0.091	0.094	0.083	0.096
	20	**0.181**	0.082	0.118	0.129	0.148	0.131	0.130	0.135	0.135	0.148	0.116	0.163
NLM_2	10	**0.108**	0.057	0.075	0.081	0.091	0.082	0.081	0.085	0.085	0.088	0.078	0.090
	20	**0.149**	0.080	0.101	0.111	0.130	0.114	0.112	0.117	0.117	0.128	0.101	0.142
NLM_4	10	0.050	**0.086**	0.062	0.067	0.073	0.068	0.067	0.069	0.069	0.071	0.065	0.072
	20	0.059	**0.103**	0.073	0.078	0.089	0.080	0.079	0.082	0.082	0.088	0.074	0.100
$NDPC_1$	10	0.101	**0.281**	0.251	0.234	0.181	0.228	0.233	0.219	0.219	0.211	0.202	0.192
	20	0.365	**0.567**	0.549	0.523	0.436	0.516	0.522	0.502	0.502	0.473	0.439	0.394
$NDPC_2$	10	0.034	**0.091**	0.076	0.070	0.056	0.068	0.070	0.066	0.066	0.065	0.064	0.062
	20	0.057	**0.133**	0.119	0.108	0.083	0.106	0.108	0.101	0.101	0.097	0.095	0.082
$NDPC_2$	10	0.034	**0.079**	0.061	0.060	0.056	0.060	0.060	0.059	0.059	0.057	0.058	0.052
	20	0.044	**0.104**	0.081	0.079	0.072	0.078	0.079	0.077	0.077	0.073	0.075	0.058
$NDPC_4$	10	0.045	0.052	**0.053**	0.050	0.046	0.049	0.050	0.048	0.048	0.048	0.049	0.047
	20	0.043	**0.055**	0.054	0.051	0.045	0.050	0.051	0.049	0.049	0.048	0.049	0.046
PCM_1	10	0.065	0.150	**0.158**	0.135	0.084	0.129	0.134	0.119	0.119	0.128	0.101	0.130
	20	0.183	0.321	0.336	0.298	0.201	0.288	0.296	0.271	0.271	0.322	0.195	**0.340**
PCM_2	10	0.051	0.055	**0.065**	0.060	0.047	0.058	0.059	0.056	0.056	0.055	0.055	0.052
	20	0.062	0.068	**0.082**	0.075	0.056	0.073	0.074	0.069	0.069	0.070	0.066	0.066
PCM_2	10	0.045	0.046	**0.051**	0.048	0.042	0.047	0.047	0.046	0.046	0.045	0.047	0.043
	20	0.043	0.045	**0.050**	0.047	0.039	0.046	0.046	0.044	0.044	0.043	0.046	0.040
PCM_4	10	0.047	0.047	**0.054**	0.050	0.042	0.049	0.050	0.048	0.048	0.047	0.048	0.046
	20	0.049	0.051	**0.060**	0.055	0.042	0.053	0.054	0.051	0.051	0.049	0.052	0.046

Note. As in Table 7A.
Source: authors' work.

Table 13A. The power of *GoFTs* for group of alternatives G (ALTs)

ALT	N	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.790	0.565	0.760	0.747	0.678	0.742	0.746	0.733	0.733	0.767	0.612	**0.800**
	20	0.981	0.940	0.977	0.974	0.959	0.973	0.973	0.971	0.971	0.984	0.922	**0.992**
P_2	10	**0.298**	0.089	0.224	0.217	0.189	0.215	0.217	0.210	0.210	0.229	0.167	0.250
	20	0.551	0.288	0.467	0.456	0.410	0.453	0.455	0.445	0.445	0.505	0.330	**0.584**
P_2	10	**0.114**	0.031	0.077	0.075	0.070	0.075	0.075	0.074	0.074	0.078	0.069	0.080
	20	**0.172**	0.057	0.121	0.119	0.109	0.118	0.119	0.117	0.117	0.127	0.102	0.143
P_4	10	**0.079**	0.034	0.058	0.057	0.056	0.057	0.057	0.056	0.056	0.058	0.055	0.057
	20	**0.095**	0.038	0.068	0.068	0.065	0.067	0.068	0.067	0.067	0.070	0.064	0.074
NM_1	10	**0.096**	0.033	0.065	0.065	0.064	0.065	0.065	0.065	0.065	0.066	0.064	0.066
	20	**0.128**	0.047	0.088	0.089	0.086	0.089	0.089	0.089	0.089	0.091	0.081	0.092
NM_2	10	**0.061**	0.043	0.053	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
	20	**0.068**	0.042	0.055	0.054	0.054	0.055	0.054	0.055	0.055	0.055	0.054	0.055
NM_2	10	**0.057**	0.045	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.050
	20	**0.059**	0.043	0.050	0.051	0.051	0.051	0.051	0.051	0.051	0.052	0.050	0.052
NM_4	10	**0.054**	0.048	0.051	0.051	0.050	0.051	0.051	0.051	0.051	0.051	0.050	0.049
	20	**0.054**	0.047	0.052	0.051	0.052	0.052	0.052	0.051	0.051	0.052	0.051	0.052
NLM_1	10	**0.526**	0.351	0.472	0.482	0.468	0.482	0.482	0.481	0.481	0.473	0.454	0.431
	20	**0.803**	0.709	0.767	0.777	0.779	0.778	0.777	0.780	0.780	0.766	0.753	0.672
NLM_2	10	**0.189**	0.056	0.143	0.137	0.118	0.136	0.137	0.133	0.133	0.134	0.119	0.133
	20	**0.332**	0.156	0.277	0.268	0.233	0.265	0.268	0.260	0.260	0.263	0.222	0.254
NLM_2	10	**0.170**	0.066	0.105	0.117	0.135	0.119	0.118	0.123	0.123	0.130	0.109	0.134
	20	**0.265**	0.129	0.176	0.195	0.226	0.199	0.196	0.205	0.205	0.224	0.173	0.251
NLM_4	10	**0.106**	0.068	0.076	0.084	0.099	0.086	0.084	0.089	0.089	0.094	0.081	0.097
	20	0.148	0.099	0.105	0.119	0.145	0.122	0.120	0.127	0.127	0.141	0.109	**0.161**
$NDPC_1$	10	**0.180**	0.071	0.144	0.141	0.124	0.139	0.140	0.137	0.137	0.144	0.133	0.145
	20	0.333	0.140	0.257	0.256	0.234	0.255	0.256	0.251	0.251	0.301	0.236	**0.336**
$NDPC_2$	10	**0.087**	0.032	0.060	0.060	0.059	0.060	0.060	0.060	0.060	0.061	0.059	0.061
	20	**0.113**	0.042	0.078	0.078	0.075	0.078	0.078	0.078	0.078	0.083	0.071	0.092
$NDPC_2$	10	**0.097**	0.037	0.067	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.067	0.067
	20	**0.130**	0.053	0.092	0.093	0.093	0.094	0.094	0.094	0.094	0.093	0.088	0.087
$NDPC_4$	10	0.050	0.047	**0.052**	0.050	0.045	0.049	0.050	0.048	0.048	0.049	0.049	0.049
	20	0.056	0.051	**0.060**	0.057	0.051	0.056	0.057	0.055	0.055	0.056	0.054	0.058
PCM_1	10	**0.260**	0.106	0.203	0.204	0.189	0.203	0.204	0.202	0.202	0.206	0.198	0.194
	20	**0.484**	0.265	0.400	0.405	0.391	0.406	0.405	0.405	0.405	0.409	0.402	0.339
PCM_2	10	0.143	0.088	**0.165**	0.144	0.097	0.139	0.143	0.129	0.129	0.131	0.118	0.127
	20	0.278	0.194	**0.302**	0.275	0.197	0.268	0.273	0.253	0.253	0.261	0.210	0.260
PCM_2	10	**0.084**	0.034	0.061	0.060	0.058	0.060	0.060	0.060	0.060	0.061	0.058	0.061
	20	**0.106**	0.041	0.074	0.074	0.071	0.074	0.074	0.074	0.074	0.078	0.070	0.086
PCM_4	10	**0.083**	0.036	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.061	0.059	0.061
	20	**0.102**	0.041	0.071	0.072	0.071	0.072	0.072	0.072	0.072	0.075	0.069	0.077

Note. As in Table 7A.
Source: authors' work.

Table 14A. The power of *GoFTs* for group of alternatives H (ALTs)

ALT	n	Numbered *GoFT* (see Table 3)											
		1	2	3	4	5	6	7	8	9	10	11	12
P_1	10	0.565	0.792	0.761	0.748	0.680	0.743	0.747	0.733	0.733	0.767	0.612	**0.801**
	20	0.942	0.982	0.978	0.975	0.960	0.974	0.975	0.972	0.972	0.985	0.924	**0.993**
P_2	10	0.086	**0.299**	0.222	0.217	0.190	0.215	0.217	0.210	0.210	0.229	0.164	0.250
	20	0.284	0.552	0.467	0.456	0.408	0.452	0.455	0.445	0.445	0.504	0.329	**0.583**
P_2	10	0.032	**0.116**	0.080	0.078	0.072	0.077	0.077	0.076	0.076	0.079	0.072	0.081
	20	0.057	**0.169**	0.120	0.118	0.108	0.117	0.118	0.116	0.116	0.125	0.101	0.142
P_4	10	0.034	**0.078**	0.058	0.057	0.055	0.056	0.057	0.056	0.056	0.058	0.055	0.058
	20	0.039	**0.096**	0.069	0.069	0.066	0.069	0.069	0.068	0.068	0.070	0.065	0.075
NM_1	10	0.033	**0.119**	0.080	0.079	0.076	0.079	0.079	0.079	0.079	0.080	0.074	0.080
	20	0.065	**0.182**	0.130	0.129	0.121	0.128	0.129	0.128	0.128	0.132	0.113	0.134
NM_2	10	0.034	**0.097**	0.066	0.066	0.066	0.066	0.066	0.067	0.067	0.067	0.064	0.067
	20	0.047	**0.129**	0.088	0.089	0.088	0.089	0.089	0.089	0.089	0.091	0.082	0.092
NM_2	10	0.045	**0.058**	0.052	0.052	0.051	0.052	0.052	0.051	0.051	0.052	0.050	0.050
	20	0.044	**0.060**	0.051	0.051	0.051	0.051	0.051	0.052	0.052	0.052	0.051	0.051
NM_4	10	0.049	**0.051**	0.050	0.049	0.050	0.049	0.049	0.049	0.049	0.050	0.049	0.049
	20	0.049	**0.051**	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
NLM_1	10	0.119	**0.360**	0.241	0.257	0.269	0.260	0.258	0.264	0.264	0.273	0.229	0.274
	20	0.378	**0.638**	0.510	0.529	0.548	0.533	0.530	0.538	0.538	0.559	0.456	0.567
NLM_2	10	0.067	**0.184**	0.115	0.127	0.145	0.130	0.128	0.135	0.135	0.141	0.118	0.144
	20	0.143	**0.297**	0.199	0.218	0.251	0.223	0.219	0.230	0.230	0.250	0.192	0.276
NLM_2	10	**0.094**	0.058	0.068	0.074	0.086	0.075	0.074	0.078	0.078	0.082	0.072	0.084
	20	0.127	0.074	0.087	0.096	0.116	0.099	0.097	0.102	0.102	0.113	0.090	**0.128**
NLM_4	10	0.082	**0.115**	0.085	0.097	0.115	0.099	0.098	0.103	0.103	0.109	0.093	0.111
	20	0.122	0.167	0.121	0.138	0.173	0.142	0.139	0.149	0.149	0.165	0.127	**0.186**
$NDPC_1$	10	0.041	**0.157**	0.098	0.101	0.103	0.102	0.101	0.103	0.103	0.106	0.094	0.104
	20	0.100	**0.261**	0.181	0.186	0.187	0.187	0.186	0.188	0.188	0.196	0.160	0.193
$NDPC_2$	10	0.066	**0.188**	0.130	0.136	0.140	0.137	0.136	0.139	0.139	0.135	0.129	0.121
	20	0.166	**0.328**	0.250	0.260	0.265	0.261	0.260	0.263	0.263	0.247	0.237	0.194
$NDPC_2$	10	0.042	**0.064**	0.053	0.052	0.053	0.052	0.052	0.053	0.053	0.053	0.052	0.052
	20	0.042	**0.068**	0.056	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055
$NDPC_4$	10	0.049	**0.052**	0.051	0.051	0.050	0.050	0.051	0.050	0.050	0.051	0.050	0.050
	20	0.048	**0.052**	0.051	0.051	0.050	0.051	0.051	0.051	0.051	0.051	0.050	0.051
PCM_1	10	0.036	**0.077**	0.057	0.057	0.056	0.057	0.057	0.057	0.057	0.058	0.055	0.057
	20	0.039	**0.094**	0.067	0.067	0.065	0.067	0.067	0.067	0.067	0.068	0.064	0.070
PCM_2	10	0.042	**0.063**	0.053	0.053	0.052	0.053	0.053	0.053	0.053	0.054	0.051	0.053
	20	0.041	**0.067**	0.055	0.055	0.053	0.055	0.055	0.054	0.054	0.055	0.054	0.056
PCM_2	10	0.042	**0.073**	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.058	0.057	0.056
	20	0.046	**0.087**	0.069	0.068	0.066	0.068	0.068	0.068	0.068	0.066	0.066	0.063
PCM_4	10	0.041	**0.067**	0.052	0.054	0.056	0.054	0.054	0.055	0.055	0.055	0.054	0.054
	20	0.044	**0.074**	0.057	0.059	0.062	0.059	0.059	0.060	0.060	0.059	0.059	0.054

Note. As in Table 7A.
Source: authors' work.

References

Afeez, B. M., Maxwell, O., Otekunrin, O. A., & Happiness, O. I. (2018). Selection and Validation of Comparative Study of Normality Test. *American Journal of Mathematics and Statistics*, *8*(6), 190–201. https://doi.org/10.5923/j.ajms.20180806.05.

Ahmad, F., & Khan, R. A. (2015). Power Comparison of Various Normality Tests. Pakistan *Journal of Statistics and Operation Research*, *11*(3), 331–345. https://doi.org/10.18187/pjsor.v11i3.845.

Aliaga, A. M., & Martínez-González, E., & Cayón, L., & Argüeso, F., & Sanz, J. L., & Barreiro, R .B. (2003). Goodness-of-fit tests of Gaussianity: constraints on the cumulants of the MAXIMA data. *New Astronomy Reviews*, *47*(8–10), 821–826. https://doi.org/10.1016/j.newar.2003.07.010.

Anderson, T. W., & Darling, D. A. (1952). Asymptotic theory of certain "goodness of fit" criteria based on stochastic processes. *Annals of Mathematical Statistics*, *23*(2), 193–212. https://doi.org /10.1214/aoms/1177729437.

Arnastauskaitė, J., Ruzgas, T., & Bražėnas, M. (2021). An Exhaustive Power Comparison of Normality Tests. *Mathematics*, *9*(7), 1–20. https://doi.org/10.3390/math9070788.

Azzalini, A. (1985). A Class of Distributions which Includes the Normal Ones. *Scandinavian Journal of Statistics*, *12*(2), 171–178.

Bayoud, H. A. (2021). Tests of normality: new test and comparative study. *Communications in Statistics – Simulation and Computation*, *50*(12), 4442–4463. https://doi.org/10.1080/03610918.2019.1643883.

Bloom, G. (1958). *Statistical estimates and transformed beta-variables*. John Wiley & Sons.

Bonett, D. G., & Seier, E. (2002). A test of normality with high uniform power. *Computational Statistics and Data Analysis*, *40*(3), 435–445. https://doi.org/10.1016/S0167-9473(02)00074-9.

Bontemps, C., & Meddahi, N. (2005). Testing normality: a GMM approach. *Journal of Econometrics*, *124*(1), 149–186. https://doi.org/10.1016/j.jeconom.2004.02.014.

Borrajo, M. I., González-Manteiga, W., & Martínez-Miranda, M. D. (2024). Goodness-of-fit test for point processes first-order intensity. *Computational Statistics* & *Data Analysis*, *194*, 1–16. https://doi.org/10.1016/j.csda.2024.107929.

Brys, G., Hubert, M., & Struyf, A. (2008). Goodness-of-fit tests based on a robust measure of skewness. *Computational Statistics*, *23*(3), 429–442. https://doi.org/10.1007/s00180-007-0083-7.

Chernobai, A., Rachev, S. T., & Fabozzi, F. J. (2012). Composite Goodness-of-Fit Tests for Left-Truncated Loss Samples. In C. F. Lee (Ed.), *Handbook of Financial Econometrics and Statistics* (pp. 575–596). Springer.

Coin, D. (2008). A goodness-of-fit test for normality based on polynomial regression. *Computational Statistics and Data Analysis*, *52*(4), 2185–2198. https://doi.org/10.1016/j.csda.2007.07.012.

Cramér, H. (1928). On the Composition of Elementary Errors. First paper: Mathematical deductions. *Scandinavian Actuarial Journal*, (1), 13–74. https://doi.org/10.1080/03461238.1928.10416862.

Desgagné, A., & Lafaye de Micheaux, P. (2018). A powerful and interpretable alternative to the Jarque-Bera test of normality based on 2nd-power Skewness and Kurtosis, using the Rao's Score Test on the APD family. *Journal of Applied Statistics*, *45*(13), 2307–2327. https://doi.org/10.1080 /02664763.2017.1415311.

Desgagné, A., Lafaye de Micheaux, P., & Ouimet, F. (2022). Goodness-of-fit tests for Laplace, Gaussian and exponential power distributions based on λ-th power skewness and kurtosis. Statistics. *A Journal of Theoretical and Applied Statistics*, *57*(1), 94–122. https://doi.org/10.1080/02331888.2022.2144859.

Durbin, J., & Knott, M. (1972). Components of Cramér-von Mises statistics. I. *Journal of the Royal Statistical Society: Series B (Methodological)*, *34*(2), 290–307.

Esteban, M. D., Castellanos, M. E., Morales, D., & Vajda, I. (2001). Monte Carlo comparison of four normality tests using different entropy estimates. *Communications in Statistics – Simulation and Computation*, *30*(4), 761–785.

Feuerverger, A. (2016). On Goodness of Fit for Operational Risk. *International Statistical Review*, *84*(3), 434–455.

Filliben, J. J. (1975). The Probability Plot Correlation Coefficient Test for Normality. *Technometrics*, *17*(1), 111–117. https://doi.org/10.1080/00401706.1975.10489279.

Gan, F. F., & Koehler, K. J. (1990). Goodness-of-Fit Tests Based on P-P Probability Plots. *Technometrics*, *32*(3), 289–303. https://doi.org/10.2307/1269106.

Gel, Y. R., & Gastwirth, J. L. (2008). A robust modification of the Jarque-Bera test of normality. *Economics Letters*, *99*(1), 30–32. https://doi.org/10.1016/j.econlet.2007.05.022.

Gel, Y. R., Miao, W., & Gastwirth, J. L. (2007). Robust directed tests of normality against heavy-tailed alternatives. *Computational Statistics & Data Analysis*, *51*(5), 2734–2746. https://doi.org/10.1016/j.csda.2006.08.022.

Giles, D. E. (2024). New Goodness-of-Fit Tests for the Kumaraswamy Distribution. *Stats*, *7*(2), 373–389. https://doi.org/10.3390/stats7020023.

Harter, H. L., Khamis, H. J., & Lamb, R. E. (1984). Modified Kolmogorov-Smirnov tests of goodness of fit. *Communications in Statistics – Simulation and Computation*, *13*(3), 293–323. https://doi.org/10.1080/03610918408812378.

Hernandez, H. (2021). *Testing for Normality: What is the Best Method?* (ForsChem Research Reports, vol. 6, 2021-05). https://doi.org/10.13140/RG.2.2.13926.14406.

Kellner, J., & Celisse, A. (2019). A one-sample test for normality with kernel methods. *Bernoulli*, *25*(3), 1816–1837. https://doi.org/10.3150/18-BEJ1037.

Khamis, H. J. (1990). The δ-corrected Kolmogorov-Smirnov test for goodness-of-fit. *Journal of Statistical Planning and Inference*, *24*(3), 317–335. https://doi.org/10.1016/0378-3758(90)90051-U.

Khamis, H. J. (1992). The Δ-corrected Kolmogorov-Smirnov test with estimated parameters. *Journal of Nonparametric Statistics*, *2*(1), 17–27. https://doi.org/10.1080/10485259208832539.

Khamis, H. J. (1993). A comparative study of the &-corrected Kolmogorov-Smirnov test. *Journal of Applied Statistics*, *20*(3), 401–421. https://doi.org/10.1080/02664769300000040.

Khatun, N. (2021). Applications of Normality Test in Statistical Analysis. *Open Journal of Statistics*, *11*(1), 113–122. https://doi.org/10.4236/ojs.2021.111006.

Kolmogorov, A. (1933). Sulla determinazione empirica di unalegge di distributione. *Giornale dell' Istituto Italiano degli Attuari*, (4), 83–91.

Krauczi, É. (2009). A study of the quantile correlation test for normality. TEST. *An Official Journal of the Spanish Society of Statistics and Operations Research*, *18*(1), 156–165. https://doi.org/10.1007/s11749-007-0074-6.

Kuiper, N. H. (1960). Tests concerning random points on a circle. *Indagationes Mathematicae (Proceedings)*, *63*, 38–47. https://doi.org/10.1016/S1385-7258(60)50006-0.

Lilliefors, H. W. (1967). On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown. *Journal of the American Statistical Association*, *62*(318), 399–402. https://doi.org/10.1080/01621459.1967.10482916.

Luceño, A. (2006). Fitting the generalized Pareto distribution to data using maximum goodness-of-fit estimators. *Computational Statistics* & *Data Analysis*, *51*(2), 904–917. https://doi.org/10.1016/j.csda.2005.09.011.

Ma, Y., Kitani, M., & Murakami, H. (2024). On modified Anderson-Darling test statistics with asymptotic properties. *Communications in Statistics – Theory and Methods*, *53*(4), 1420–1439. https://doi.org/10.1080/03610926.2022.2101121.

Marange, C. S., & Qin, Y. (2019). An Empirical Likelihood Ratio Based Comparative Study on Tests for Normality of Residuals in Linear Models. *Metodoloski zvezki*, *16*(1), 1–16. https://doi.org/10.51936/ramh7128.

Mbah, A. K., & Paothong, A. (2015). Shapiro-Francia test compared to other normality test using expected *p*-value. *Journal of Statistical Computation and Simulation*, *85*(15), 3002–3016. https://doi.org/10.1080/00949655.2014.947986.

von Mises, R. (1931). Über einige Abschätzungen von Erwartungswerten. *Journal für die reine und angewandte Mathematik*, *1931*(165), 184–193. https://doi.org/10.1515/crll.1931.165.184.

Mishra, P., Pandey, C. M., Singh, U., Gupta, A., Sahu, C., & Keshri, A. (2019). Descriptive Statistics and Normality Tests for Statistical Data. *Annals of Cardiac Anaesthesia*, *22*(1), 67–72. https://doi.org/10.4103/aca.ACA_157_18.

Nofal, Z. M., Afify, A. Z., Yousof, H. M., & Cordeiro, G. M. (2017). The generalized transmuted-G family of distributions. *Communications in Statistics – Theory and Methods*, *46*(8), 4119–4136. https://doi.org/10.1080/03610926.2015.1078478.

Nosakhare, U. H., & Bright, A. F. (2017). Evaluation of Techniques for Univariate Normality Test Using Monte Carlo Simulation. *American Journal of Theoretical and Applied Statistics*, *6*(5–1), 51–61. https://doi.org/10.11648/j.ajtas.s.2017060501.18.

Noughabi, H. A., & Arghami, N. R. (2011). Monte Carlo comparison of seven normality tests. *Journal of Statistical Computation and Simulation*, *81*(8), 965–972. https://doi.org/10.1080/00949650903580047.

Pettitt, A. N., & Stephens, M. A. (1976). Modified Cramér-von Mises statistics for censored data. *Biometrika*, *63*(2), 291–298. https://doi.org/10.1093/biomet/63.2.291.

Razali, N. M., & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *Journal of Statistical Modeling and Analytics*, *2*(1), 21–33.

Romão, X., Delgado, R., & Costa, A. (2010). An empirical power comparison of univariate goodness-of-fit tests for normality. *Journal of Statistical Computation and Simulation*, *80*(5), 545–591. https://doi.org/10.1080/00949650902740824.

Scott, W. F. (2000). Tables of the Cramér-von Mises distributions. *Communications in Statistics – Theory and Methods*, *29*(1), 227–235. https://doi.org/10.1080/03610920008832479.

Scott, W. F., & Stewart, B. (2011). Tables for the Lilliefors and Modified Cramer-von Mises Tests of Normality. *Communications in Statistics – Theory and Methods*, *40*(4), 726–730. https://doi.org/10.1080/03610920903453467.

Shapiro, S. S., & Wilk, M. B. (1965). An analysis of variance test for normality (complete samples). *Biometrika*, *52*(3–4), 591–611. https://doi.org/10.1093/biomet/52.3-4.591.

Smirnov, N. (1948). Table for estimating the goodness of fit of empirical distributions. *Annals of Mathematical Statistics*, *19*(2), 279–281. https://doi.org/10.1214/aoms/1177730256.

Stephens, M. A. (1974). EDF Statistics for Goodness of Fit and Some Comparisons. *Journal of the American Statistical Association, 69*(347), 730–737. https://doi.org/10.1080/01621459.1974.10480196.

Sulewski, P. (2019). Modification of Anderson-Darling goodness-of-fit test for normality. *Afinidad. Journal of Chemical Engineering Theoretical and Applied Chemistry*, *76*(588), 270–277. https://www.raco.cat/index.php/afinidad/article/view/361876.

Sulewski, P. (2021). Two component modified Lilliefors test for normality. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, *16*(2), 429–455. https://doi.org/10.24136/eq.2021.016.

Sulewski, P. (2022). Modified Lilliefors goodness-of-fit test for normality. *Communications in Statistics – Simulation and Computation*, *51*(3), 1199–1219. https://doi.org/10.1080/03610918.2019.1664580.

Tavakoli, M., Arghami, N., & Abbasnejad, M. (2019). A Goodness of Fit Test For Normality Based on Balakrishnan-Sanghvi Information. *Journal of The Iranian Statistical Society*, *18*(1), 177–190. https://doi.org/10.29252/jirss.18.1.177.

Terán-García, E., & Pérez-Fernández, R. (2024). A robust alternative to the Lilliefors test of normality. *Journal of Statistical Computation and Simulation*, *94*(7), 1494–1512. https://doi.org/10.1080/00949655.2023.2291138.

Torabi, H., Montazeri, N. H., & Grané, A. (2016). A test for normality based on the empirical distribution function. *SORT*, *40*(1), 55–88. https://doi.org/10.2436/20.8080.02.35.

Uhm, T., & Yi, S. (2021). A comparison of normality testing methods by empirical power and distribution of *P*-values. *Communications in Statistics – Simulation and Computation*, *52*(9), 4445–4458. https://doi.org/10.1080/03610918.2021.1963450.

Uyanto, S. S. (2022). An Extensive Comparisons of 50 Univariate Goodness-of-fit Tests for Normality. *Austrian Journal of Statistics*, *51*(3), 45–97. https://doi.org/10.17713/ajs.v51i3.1279.

Watson, G. S. (1962). Goodness-of-fit tests on a circle. II. *Biometrika*, *49*(1–2), 57–63. https://doi.org/10.1093/biomet/49.1-2.57.

Wijekularathna, D. K., Manage, A. B. W., & Scariano, S. M. (2020). Power analysis of several normality tests: A Monte Carlo simulation study. *Communications in Statistics – Simulation and Computation*, *51*(3), 757–773. https://doi.org/10.1080/03610918.2019.1658780.

Yap, B. W., & Sim, C. H. (2011). Comparisons of various types of normality tests. *Journal of Statistical Computation and Simulation*, *81*(12), 2141–2155. https://doi.org/10.1080/00949655.2010.520163.

Yazici, B., & Yolacan, S. A. (2007). A comparison of various tests of normality. *Journal of Statistical Computation and Simulation*, *77*(2), 175–183. https://doi.org/10.1080/10629360600678310.