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LETTER FROM THE EDITOR

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Do strict environmental policies in European countries reduce CO_2 emissions?

Dawid Jan Bonar[a]

Abstract. This article uses fixed-effects and random-effects panel data models to examine the effectiveness of environmental policies, and additional determinants on carbon dioxide (CO_2) emissions in 21 selected European OECD countries from 1990 to 2020. Specifically, the analysis investigates the impact of individual subgroups constituting the total Environmental Policy Stringency (EPS) index, namely market-based instruments, non-market-based instruments and technological support. Furthermore, the impact of these instruments is examined considering two types of CO_2 measurements: production-based (PBA) and consumption-based (CBA). The obtained results demonstrate that the impact of each subgroup varies and the strength of their influence depends on the method of CO_2 measurement. Finally, the study examines whether the 2008 changes to the Emissions Trading System (ETS) influenced the effectiveness of the instruments within the EPS. The results indicate that these changes significantly improved policy effectiveness when CO_2 is measured using the PBA. In contrast, the post-2008 changes had a minimal effect on reducing CO_2 emissions measured using the CBA, which may be related to the phenomenon of outsourcing.

Keywords: EPS, carbon dioxide, environmental policies, emissions
JEL: Q50, Q54, Q56

1. Introduction

Society in the 21st century is facing one of its most serious challenges – global warming. Greenhouse gases, with carbon dioxide (CO_2) at the forefront, are the main contributors to this phenomenon. This chemical is emitted into the atmosphere mainly as a result of human activities, such as the burning of fossil fuels and massive deforestation. Scientists emphasize that greenhouse gases, especially CO_2, have been identified as the most significant factor influencing climate change (Lv & Xu, 2019). However, it is worth noting that CO_2 emissions are not only caused by human activities, but also by natural processes such as volcanic eruptions.

Countries, especially the more developed ones, are trying to slow down the warming process by reducing carbon emissions. To this end, legislative bodies are formulating various policy programmes to mitigate the negative impact of economic entities on the natural environment.

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The European Union (EU) plays a major role in environmental protection across Europe. Currently, the climate policy of this organisation encompasses 142 directives. The first significant document is Directive 2003/87/EC, concerning the greenhouse gas emissions trading scheme. The objective of this policy is to reduce the production of atmospheric pollutants by 62% compared to the levels in 2005. However, according to the European Council & Council of the European Union (n.d. a, n.d. b), by 2023, emissions had decreased by 41%. The system aims to ensure that entities producing pollutants contribute financially to the green transformation within the EU. A cap is set for the total amount of greenhouse gases that can be produced by facilities covered by the programme, including factories and power plants.

However, the year 2008 is more important from the perspective of research on the effectiveness of the Emissions Trading System (ETS) (ETS Phase II), when the EU significantly expanded the scope of the system, thus initiating its practical implementation. Another important EU project aimed at environmental protection is 'Fit for 55'. Introduced in 2021, this package of climate regulations aims to reduce greenhouse gas emissions by 55% by 2030. It includes a reformed EU ETS that will encompass emissions from maritime transport and increase the stringency of the policy by gradually phasing out free allowances.

The Environmental Policy Stringency (EPS) index, developed by Botta and Koźluk (2014), was created for comparative research on environmental policies. It consists of three subgroups of environmental policy instruments: market-based, non-market-based and technology support. In the countries of the EU, the most important instrument is the ETS, which sets a cap on CO_2 emissions. Entities participating in this market can buy and sell allowances depending on their CO_2 emission levels. It is worth noting that a higher price in the ETS is associated with a more stringent policy aimed at reducing greenhouse gas emissions.

The literature distinguishes between two main approaches to measuring CO_2 emissions. The first is production-based accounting (PBA), which focuses on accounting only for gases produced within the territory of a given country or geographic area. The primary criticism of this measurement method is the phenomenon of outsourcing, whereby activities with a significant environmental impact are relocated to countries with less stringent climate regulations. In response to this criticism, a second approach was developed: consumption-based accounting (CBA). CBA takes into account CO_2 emissions based on both domestic activities and imports. This is particularly relevant when a country imports a significant amount of goods whose production processes emit large quantities of greenhouse gases. As noted by Papież et al. (2021), EU countries tend to show a greater reduction in emissions when measured by PBA than by CBA. This trend may be related to the issue of outsourcing.

Studies on the impact of EPS instruments on CO_2 emission has been conducted in BRICS countries (Wang et al., 2022), in both BRICS and G7 countries (Sezgin et al., 2021), in China, the USA, India, Russia and Japan (Yirong, 2022), and in Czechia, Greece, Hungary, Korea, South Africa, and Turkey (Wolde-Rufael & Mulat-Weldemeskel, 2021). The most extensive study (covering the largest number of OECD countries) was conducted by Albulescu et al. (2022) and Frohm et al. (2023). The impact of EPS in the most polluted Asian countries was examined by Liu et al. (2023).

In previous studies, variables such as GDP *per capita* (Ahmed & Ahmed, 2018; Albulescu et al., 2022; Frohm et al., 2023; Liu et al., 2023; Wang et al., 2022; Wolde-Rufael & Mulat-Weldemeskel, 2021; Yirong, 2022), the Human Development Index (HDI; Sezgin et al., 2021), the share of renewable energy sources (RES) (Albulescu et al., 2022; Khan & Imran, 2023; Liu et al., 2023; Morales-Lage et al., 2016; Wang et al., 2022; Wolde-Rufael & Mulat-Weldemeskel, 2021), industrial value added (Wang et al., 2022), the inflow of foreign direct investment (FDI) (Albulescu et al., 2022; Aller et al., 2021), the impact of environmental and energy taxes (Wolde-Rufael & Mulat-Weldemeskel, 2021), and globalisation (Sabir & Gorus, 2019) have been used to model CO_2 emissions.

In most studies, the authors considered only production-based (i.e. PBA) CO_2 emissions (Ahmed & Ahmed, 2018; Albulescu et al., 2022; Frohm et al., 2023; Liu et al., 2023; Wang et al., 2022). However, the use of the consumption-based (i.e. CBA) CO_2 emissions by Wolde-Rufael and Mulat-Weldemeskel (2021) and the measurement of CO_2 as the sum of production and consumption activities by Sezgin et al. (2021) are worth highlighting.

In all the mentioned studies, the EPS index was treated as a whole, customarily not considering its subgroups separately. In the current literature, the separate impact of the instrument subgroups was examined by Guo et al. (2021). Furthermore, due to the analysis focusing mainly on countries outside the non-EU or non-European countries, none of the above works takes into account the impact of EU policies, including the ETS.

The objective of this study is to examine the impact of strict environmental policies on the production of CO_2 *per capita* in 21 selected European countries that are members of the Organisation for Economic Co-operation and Development (OECD) from 1995 to 2020. The study is limited to selected European OECD countries due to the availability and quality of EPS index data.

Panel data estimation methods, such as the fixed effects estimator and random effects estimator, were used in the study to determine their relationships. The main hypothesis posited in the study is: environmental policy instruments included in the EPS index significantly impact the reduction of CO_2 emissions *per capita*.

The following hypotheses are also considered in detail in this paper:

- the choice of the CO_2 measurement method, whether based on the place of CO_2 production (PBA) or consumption (CBA), influences the effectiveness of environmental policies in reducing CO_2 emissions *per capita*;
- the introduction of changes to the EU ETS in 2008 influences the effectiveness of environmental policies in reducing CO_2 emissions *per capita* in the European OECD countries;
- the different subgroups of the EPS index, i.e. market-based, non-market-based and technology support vary in terms of their impact on CO_2 production.

This paper presents three novelties. The first novelty of this article is to examine whether the introduction of the ETS system in 2008 has a significant impact on the effectiveness of environmental policies in reducing CO_2 emissions. Most of the countries selected for analysis are members of the EU, and it can therefore be hypothesised that the currently most important ETS has influenced the level of CO_2 production.

The second novelty is the fact that two groups of models are considered: one using the PBA method as the dependent variable and the other using the CBA method. In the aforementioned studies, most researchers rely on either one of these two approaches (usually PBA). There is a lack of research in the current literature on the impact of the EPS index on CO_2 production measured using both approaches, which would allow for an assessment of whether CO_2 reduction in Europe results from internal European actions to limit CO_2 and is a consequence of stringent policies, or merely from the relocation of production to countries with less stringent environmental regulations.

The third novelty of the work involves the examination of whether the different subgroups of the EPS indicator, i.e. market-based, non-market-based and technology support differ in their impact on CO_2 production. The existing literature lacks such studies, as most authors consider the simultaneous impact of all subgroups in their models based on the calculation of an arithmetic mean. This approach does not allow for a deeper understanding of how each subgroup individually affects the reduction of CO_2 emissions.

This paper is organised as follows. Section 2 reviews the relevant literature concerning the analysis of CO_2 emissions in European OECD countries. Section 3 is devoted to presenting the area under study, while the used methodology is sketched in Section 4, and Section 5 presents the main outcome of this work. Finally, Section 6 shows the conclusions and policy implications.

2. Literature review

The most extensive study on the impact of the EPS index on CO_2 emission reductions was conducted by Albulescu et al. (2022). In their work, they used data from 30 countries, either OECD members or developing countries, concerning the overall EPS index, GDP *per capita*, the inflow of FDI, the share of RES and CO_2 production. The relationship between the EPS index, additional determinants and CO_2 production was examined using panel data models based on quantile regression with fixed effects. Their results show that the greatest impact on reducing CO_2 emissions through increased environmental stringency occurs in countries with low levels of emissions. Furthermore, the impact of the EPS index is greater in EU countries due to the 20-20-20 targets for greenhouse gas emissions.

The second most extensive study in terms of the number of countries is the analysis conducted by Frohm et al. (2023). They include data on the total EPS index, GDP *per capita* and the share of fossil fuels in energy consumption from 30 selected OECD countries. Panel data models were used for the analysis. They find that the impact of policies is significant but varies across economic sectors. This variation may result from the differing intensity of fossil fuel usage in the particular sectors of the economy. In order to achieve net-zero emissions by 2050, it is necessary for the current policies to be rapidly tightened.

In Asian countries, the role of EPS instruments in reducing CO_2 emissions has been analysed by Liu et al. (2023). Using the autoregressive distributed lag stationarity (ARDL) and nonlinear autoregressive distributed lag (NARDL) models, they conclude that the positive impact of environmental policies is the greatest in the most polluted countries. Due to stringent regulations, enterprises are compelled to implement changes in production technologies toward more environmentally friendly solutions. This, in turn, encourages companies to seek innovations in zero-emission technologies.

Ahmed and Ahmed (2018) analyse the impact of environmental policy stringency instruments in China based on PBA emissions and the overall EPS index and GDP *per capita* in US dollars using the corrected grey model with convolution (CGMC). They find that the EPS index positively impacts CO_2 production, but its strength is weaker compared to the negative impact of GDP *per capita*.

A broader analysis of the impact of the EPS index and the HDI on CO_2 production was conducted by Sezgin et al. (2021). The study utilised data from the BRICS and G7 countries. They measure CO_2 production as the sum of production and consumption activities. Using cointegration tests and Granger causality analysis, they found that the EPS index positively influences the reduction of CO_2 emissions in developed countries and long-term increases in a country's development lead to a decrease in CO_2 emissions.

Wang et al. (2022) examined the potential impact of the overall EPS index, the share of RES, GDP *per capita*, and industrial value added on reducing CO_2 emissions exclusively in BRICS countries. They used a single approach to measuring CO_2 emissions. Based on cross-sectional autoregressive distributed lag (CS-ARDL) models, the researchers confirmed the positive impact of the EPS index on CO_2 emissions in the long term. Furthermore, the combined impact of the EPS index and the share of RES is greater than their individual effects.

The impact of the EPS index and additional variables, such as environmental tax, energy tax and the share of renewable energy sources on CO_2 emissions in developing countries was studied by Wolde-Rufael and Mulat-Weldemeskel (2021). They utilised consumption-based accounting to analyse CO_2 emissions, which is not common. Using panel data models, they demonstrated that the effectiveness of environmental policy stringency requires time. The authors also found causality between the increase in the EPS index and the decrease in CO_2 emissions.

Yirong (2022) extensively examined the impact of environmental policy stringency on CO_2 emissions mainly in Asian countries and the USA. To estimate the impact of the overall EPS index and additional determinants such as GDP *per capita*, technological innovations and population on CO_2 production, nonlinear ARDL panel models were used. The main conclusion of the study is that increasing environmental policy stringency leads to a reduction in CO_2 emissions in the long term.

The impact of environmental policy stringency instruments on greenhouse gas emissions in Western and Central European countries was examined by Dmytrenko et al. (2024). The study utilised panel data models that considered the separate effects of market-based and non-market-based instruments. Based on these models, the authors concluded that the policies implemented in Europe play a crucial role only in Western countries. The most significant factor contributing to the reduction of greenhouse gases in both groups was R&D expenditure.

Based on the existing literature, it is evident that there is a lack of analyses focusing exclusively on European countries. Moreover, few studies analyse the impact of the EPS index on carbon dioxide emissions measured using both approaches (PBA, CBA). Additionally, a novel aspect of this article is the examination of the impact of the introduction of the ETS on the effectiveness of the EPS index. Finally, the separate impact of EPS index subgroups on CO_2 emissions was also considered, which is rare in the existing literature.

3. Data

The aim of this study is to investigate whether environmental policy instruments and additional determinants influence the reduction of CO_2 emissions, measured using PBA and CBA. The analysis encompasses annual data from 1995 to 2020 for 21 European countries that are members of the OECD, namely: Austria, Belgium, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, the Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, and the United Kingdom. These countries were selected for analysis due to data availability.

The variables selected for the analysis are shown in Table 1, which provides information regarding the abbreviation used, the full name of the variable, the unit of measurement and the data source. Table 1 is divided into two parts: the first one presents the dependent variables and the second the explanatory variables.

Table 1. Variables selected for research

Symbol	Variable Name	Unit	Data Source
Dependent variables			
PBA	CO_2 production measured by PBA	Tonnes *per capita*	World Bank
CBA	CO_2 production measured by CBA	Tonnes *per capita*	Our World in Data
Explanatory variables			
GDP	Gross domestic product *per capita* in nominal prices	US Dollar	World Bank
FDI	FDI	% of GDP	World Bank
RES	Share of energy consumption from RES	Percentage of total energy consumption	World Bank
KOFGI	KOF Globalization Index	Percentage of globalisation (0–100%)	ETH Zurich University
EPS	EPS Index	Points, ranging from 0 to 6	OECD
TECH	Technology support of EPS	Points, ranging from 0 to 6	OECD
MARKET	Market instruments of EPS	Points, ranging from 0 to 6	OECD
NON-MARKET	Non-market instruments of EPS	Points, ranging from 0 to 6	OECD

Source: author's work.

Table 1 lists the variables used for modelling CO_2 production. The dependent variables are CO_2 production, measured using both CBA and PBA, expressed in tonnes *per capita*. Data on CO_2 production measured by CBA were obtained from Our World in Data, while data on production measured by PBA from the official World Bank website. To describe the impact of the environmental policy stringency on CO_2 emissions, the main EPS Index and its three subgroups were utilised:

technology support EPS, market-based EPS instruments and non-market-based EPS instruments. Additional explanatory variables used include GDP *per capita* in nominal prices expressed in US dollars, FDI as a percentage of GDP and the consumption of electricity from RES as a percentage of its total consumption. These data were also sourced from the official World Bank website. Furthermore, to account for globalisation, the KOF Globalization Index, developed by the Federal Institute of Technology (ETH) Zurich was used. This index takes percentage values from 1 to 100, reflecting the degree of globalisation in a given country.

The EPS index was created for international comparative research on policies aimed at reducing environmental pollution. It consists of three subgroups of environmental policy instruments: market-based, non-market-based and technology support. The EPS index ranges from 0 to 6, with data obtained from the OECD.

The components of market-based instruments (Kruse et al., 2022) are: ETS, Renewable Energy Exchange Instruments, CO_2 tax, nitrate tax and sulphur oxide. However, the non-market-based instruments include (Kruse et al., 2022): nitrate emissions, sulphur oxide emissions, particulate matter emissions and sulphur content in diesel.

The latest update, which introduced the technology sub-index, added two new categories. The first category is 'upstream', which includes public expenditures on R&D and the discovery of zero-emission technologies that may be currently unprofitable. The second category, 'downstream', encompasses support for the existing RES in the form of subsidies. This subgroup aims to facilitate the operation of the existing technologies. The motivation for creating the third technology subgroup was the distinct nature of these instruments compared to the market-based and non-market-based ones. According to the International Energy Agency data (2021), it is estimated that half of the technologies that will contribute to zero emissions by 2050 are currently in the prototype phase.

One of the greatest challenges regarding CO_2 emissions is the method of measurement. Currently, the two most popular methods are the previously mentioned PBA and CBA. Kozul-Wright and Fortunato (2012) indicate that the reduction in CO_2 emissions in developed countries often results from outsourcing, i.e. relocating environmentally harmful activities to countries with less stringent regulations. In such cases, the PBA does not account for production outside the country, leading to an apparent reduction in emissions. Research conducted in the UK by Barrett et al. (2013) shows the need to apply CBA in territorial CO_2 emission measurements, especially in highly developed countries. Unfortunately, data on emissions measured using CBA are often inconsistent or lack reliability. Therefore, Peters (2008) suggests combining both approaches to diversify research and demonstrate the effectiveness of climate policies.

In most studies, only one method of measuring CO_2 is considered, most commonly PBA (Drastichová, 2018; Vavrek & Chovancova, 2016), and less frequently CBA (Sanyé-Mengual et al., 2019; Wolde-Rufael & Mulat-Weldemeskel, 2021). Studies that incorporate both approaches are the least common (Franzen & Mader, 2018).

4. Methodology

Panel data contain information about multiple units (cross-sectional data) over different time periods (time series). Unlike cross-sectional or time series data, models estimated using panel data allow for the relaxation of assumptions that are implicitly made in cross-sectional data analysis (Maddala, 2006, p. 643). In cross-sectional data analysis, it is often assumed that unobserved factors either do not affect the dependent variable or remain constant. However, with panel data, these unobserved factors can be modelled using fixed or random effects, which account for variations among units over time. Panel data also allow the model to be estimated on an incremental basis, allowing for the avoidance of estimator bias that arises from omitting time-invariant explanatory variables. These variables form part of the unit-specific effect and are removed when calculating first differences (Dańska-Borsiak, 2011).

The first model to be analysed is the fixed effects model. The form of this model is as follows:

$$y_{it} = \eta_i + x_{it}\beta + \nu_{it}, \tag{1}$$

where:
y_{it} – dependent variable for the i-th unit in the t-th period,
x_{it} – vector of explanatory variables for the i-th unit in the t-th period,
η_i – captures specific factors for the i-th unit that are constant over time,
β – the vector of parameters,
ν_{it} – the random component with a normal distribution.

Moreover, the model has a key assumption that allows for the identification of parameters:

$$\mathrm{E}[\nu_i | x_i, \alpha_i] = 0. \tag{2}$$

This means that random component ν_i is not correlated with explanatory variables x_i and fixed effects α_i. The model, in contrast to the classical linear regression model, has 'i' specific intercept terms that account for the effects for each unit. It is also important to note that this model is consistent even when the heterogeneous specific component is correlated with one or more explanatory variables.

The second model, which assumes that individual effects α_i are random variables rather than fixed, is the random effects model. The random effects model can be described by the following equation:

$$y_{it} = x_{it}\beta + (\eta_i + v_{it}) = x_{it}\beta + u_{it}, where\ \eta_i + v_{it} = u_{it}. \tag{3}$$

The model also assumes that both random components are uncorrelated with the observed explanatory variables:

$$E[u_{it}|x_i] = 0. \tag{4}$$

This assumption excludes the estimation of the model through the ordinary least squares (OLS) method.

The Hausman test applied to panel data, is used to chose between fixed effects and random effects models. The random effects model is based on the assumption that group effects are uncorrelated with exogenous variables (Greene, 2000, pp. 301–303). In other words, this test assists in selecting the model specification, particularly in deciding between random effects or fixed effects models. The hypothesis framework for this test is as follows:

$$H_0: E[\epsilon_i|x_{it}] = 0\ vs\ H_1: E[\epsilon_i|x_{it}] \neq 0. \tag{5}$$

The null hypothesis supports the use of the random effects model; rejecting it in favour of the alternative hypothesis, on the other hand, suggests the application of the fixed effects model.

To ensure the normality of the dependent variable's distribution and to facilitate the conduct of statistical tests, a logarithmic transformation was applied to the variables, namely CO_2 production using CBA and PBA approaches, GDP *per capita* in US dollars (lnGDP), share of renewable energy consumption (lnRES), and values of the KOF Globalization Index (lnKOF). Two general model formulas were considered in this study: one using the overall EPS index (1) and the other based on its three subgroups (2). Additionally, these two types of models were considered for two measures of CO_2 – PBA and CBA.

The formulas for the models used are as follows:

$$lnCO_2 = \beta_0 + \beta_1 lnPKB + \beta_2 lnREC + \beta_3 lnKOF + \beta_7 EPS + \epsilon,$$
$$\text{where } lnCO_2 \text{ is } lnPBA \text{ or } lnCBA, \tag{6}$$

$$lnCO_2 = \beta_0 + \beta_1 lnPKB + \beta_2 lnREC + \beta_3 lnKOF + \beta_4 TECH$$
$$+ \beta_5 MARKET + \beta_6 NONMARKET + \epsilon, \tag{7}$$
$$\text{where } lnCO_2 \text{ is } lnPBA \text{ or } lnCBA.$$

In order to investigate whether the introduction of climate policies increased the impact of EPS on CO_2 production, models 1 and 2 were estimated on data subsets covering the following periods:

A) years 1995–2020;

B) years 1995–2008, i.e. the period before the introduction of the ETS;

C) years 2008–2020, i.e. directly after the introduction of the ETS.

For example, the PBA_A_1 model is interpreted as the model with the overall EPS index, estimated based on data from 1995–2020, where the dependent variable is CO_2 production *per capita* measured using the PBA approach.

To verify whether the group effect in the random effects models is statistically significant, the Breusch-Pagan test (1980) can be applied. The null hypothesis supports the use of the classical OLS estimator, as the variance of the individual effect is equal to zero, while the alternative hypothesis suggests the significance of individual effects.

5. Empirical results

The choice between the fixed effects estimator or the random effects estimator was based on the results of the Hausman test for pairs of models estimated on the same datasets and identical sets of explanatory variables. The significance level for the test is set at 0.05. The test results are presented in Table 2.

Due to the long time series in the utilised dataset, the possibility of autocorrelation was tested. A suitable test was conducted and models robust to autocorrelation were estimated (Appendix 1), demonstrating that both the sign and significance of the parameters are nearly identical.

Table 2. Hausman test results

Model	p-value	Conclusion
PBA_A_1	0.558	Random effect
PBA_B_1	0.860	Random effect
PBA_C_1	0.674	Random effect
PBA_A_2	0.781	Random effect
PBA_B_2	0.951	Random effect
PBA_C_2	0.826	Random effect
CBA_A_1	0.436	Random effect
CBA_B_1	0.669	Random effect
CBA_C_1	0.519	Random effect
CBA_A_2	0.637	Random effect
CBA_B_2	0.772	Random effect
CBA_C_2	0.811	Random effect

Source: author's work.

Table 2 provides a detailed description of model pairs created based on the previously mentioned criteria, such as the dataset, the dependent variable and the inclusion of either the overall EPS or its subgroups. Based on Table 2, we can conclude that at a significance level of 0.05, in all cases, there is no basis for rejecting the null hypothesis stating that the random effects estimator is a more appropriate model. Based on the conducted Hausman test, the Balestra-Nerlove estimator can be used to estimate the parameters of the 12 random effects models.

To verify the appropriateness of using the random effects estimator over the classical OLS method, the Breusch-Pagan test was applied. The conclusion along with the p-value is presented in Table 3.

Table 3. Breusch-Pagan test results

Model	p-value	Conclusion
PBA_A_1	0	Significant individual effects
PBA_A_2	0	Significant individual effects
CBA_A_1	0	Significant individual effects
CBA_A_2	0	Significant individual effects
PBA_B_1	~0	Significant individual effects
PBA_B_2	~0	Significant individual effects
CBA_B_1	~0	Significant individual effects
CBA_B_2	~0	Significant individual effects
PBA_C_1	~0	Significant individual effects
PBA_C_2	~0	Significant individual effects
CBA_C_1	~0	Significant individual effects
CBA_C_2	~0	Significant individual effects

Source: author's work.

For all models in which the Breusch-Pagan test was conducted, the p-value was close to zero. This indicates that the individual effect was significant in all cases.

The values of the individual parameters for the models estimated on the dataset covering the years 1995–2020, along with their statistical significance and the coefficient of determination are presented in Table 4.

Table 4. Models estimated on a dataset from the years 1995–2020

Variable	PBA_A_1	PBA_A_2	CBA_A_1	CBA_A_2
lnGDP	0.122***	0.128***	0.196***	0.201***
lnRES	−0.217***	−0.209***	−0.205***	−0.198***
lnKOFGI	0.269**	0.188	−0.472**	−0.479**
EPS	−0.079***	–	−0.027*	–
TECH	–	−0.012**	–	0.004
MARKET	–	−0.088***	–	−0.045***
NONMARKET	–	−0.019***	–	−0.009
Constant	0.378	0.683	2.976***	2.965***
Coefficient of determination	0.631	0.658	0.336	0.345

Note. *, ** and *** – the statistical significance of parameters at the significance levels of 1%, 5%, and 10%, respectively.

Source: author's work.

The aggregated impact of all three subgroups is represented by the EPS variable, which shows statistical significance in all examined models. In the model estimated on data covering the years 1995–2020 (PBA_A_1), the value of this parameter was –0.079, indicating that an increase in the stringency of policies included in the EPS index leads to a decrease in *per capita* CO_2 production (PBA). In the model estimated on data covering the years 1995–2020 (CBA_A_1), the parameter value was –0.027, suggesting that an increase in the stringency of policies included in the EPS index results in a reduction of *per capita* CO_2 production (CBA).

The parameter for the EPS technological support variable (TECH) in the model estimated on data for the years 1995–2020 (PBA_A_2) is –0.012 and is statistically significant. This indicates that increased support for renewable energy sources and expenditures on R&D translates into a reduction in *per capita* CO_2 emissions. The parameter for the EPS technology support variable (TECH) in the model estimated on data for the years 1995–2020 (CBA_A_2) is 0.004, but statistically insignificant.

Another analysed variable is MARKET, representing the effect of market-based EPS instruments. The parameter for the variable describing the effect of market-based EPS instruments (MARKET) in the model estimated on data for the years 1995–2020 (PBA_A_2) is –0.088 and is statistically significant. This means that increasing the stringency of instruments such as the ETS, raising the CO_2 or nitrate tax rate, leads to a decrease in *per capita* CO_2 production. The parameter for this variable describing the effect of market-based EPS instruments (MARKET) in the model estimated on data for the years 1995–2020 (CBA_A_2) is –0.045 and statistically significant.

The last subgroup of the EPS index encompasses the non-market instruments (NONMARKET). Based on Table 4, it can be stated that the variable in the model estimated from data for the years 1995–2020 (model PBA_A_2) is statistically significant with a value of –0.019. This indicates that increasing the stringency of instruments such as limits on nitrate, sulfur oxide or suspended particulate emissions leads to a decrease in *per capita* CO_2 emissions. The variable in the model estimated from data for the years 1995–2020 (model CBA_A_2) is –0.009 but statistically insignificant.

The impact of the logarithmic gross domestic product *per capita*, expressed in nominal prices in US dollars (lnGDP), is statistically significant, ranging from 0.122 to 0.201. This means that as the gross domestic product *per capita* increases, the production of CO_2 *per capita* also increases.

Another statistically significant variable in all models is lnRES, which ranged from –0.217 to –0.198. The parameter values are negative, indicating that the share of RES positively affects the reduction of CO_2 production *per capita*.

The last variable present in each model is lnKOF. In two models estimated on data for the years 1995–2020 (CBA_A_1, CBA_A_2), this parameter is statistically significant and positive. This indicates that as globalisation increases in European countries, the production of CO_2 *per capita* rises. In one model (PBA_A_1), the parameter is statistically significant but has a negative value. This suggests that as globalisation increases in European countries, the production of CO_2 *per capita* decreases.

Table 5 presents the values of individual parameters for models, where the dependent variable is CO_2 production measured using production-based accounting. Their statistical significance and coefficient of determination is also provided. The values of the parameters were estimated on datasets for the years 1995–2008 and 2008–2020.

Table 5. PBA models estimated on datasets from the years 1995–2008 and 2008–2020

Variable	PBA_B_1	PBA_C_1	PBA_B_2	PBA_C_2
lnGDP	0.089***	0.123***	0.099***	0.129***
lnRES	−0.133***	−0.351***	−0.134***	−0.318***
lnKOFGI	0.189**	−1.350***	0.172*	−0.449
EPS	−0.023**	−0.051***	–	–
TECH	–	–	−0.017**	−0.004
MARKET	–	–	−0.036**	−0.049***
NONMARKET	–	–	−0.002	−0.054***
Constant	0.799**	7.826***	0.807**	3.878*
Coefficient of determination	0.231	0.632	0.251	0.654

Note. As in Table 4.
Source: author's work.

The aggregated impact of all three subgroups is represented by the EPS variable. In the models before the introduction of the ETS, in the years 1995–2008 (model PBA_B_1), the parameter value was −0.023, while after its introduction, in the years 2008–2020 (model PBA_C_1), it was −0.051. This indicates that the policy related to the ETS was effective, as the overall stringency resulted in a greater reduction in *per capita* CO_2 production than before 2008, the year in which the scope of the ETS was significantly expanded.

In the models, before the introduction of the ETS, in the years 1995–2008 (model PBA_B_2), the value of the TECH parameter was −0.017, and after its introduction, in the years 2008–2020 (model PBA_C_2), it was −0.004, but not statistically significant. This may suggest that the impact of technology support policies on reducing *per capita* CO_2 production weakened.

The next variable analysed is MARKET. In the models before the introduction of the ETS from 1995–2008 (PBA_B_2), the value of the MARKET parameter was −0.036, and after its introduction, in the years 2008–2020 (model PBA_C_2), it was −0.049. In both models, the parameter was statistically significant, suggesting that this subgroup of environmental policies influenced the reduction of *per capita* CO_2 emissions after the policy tightening associated with the ETS expansions post-2008. Moving to the last subgroup of the EPS index, the non-market instruments (NONMARKET), in the models before the introduction of the ETS, in the years 1995–2008 (PBA_B_2), the value of the NONMARKET variable was −0.002 but statistically insignificant, whereas after its introduction, in the years 2008–2020 (model PBA_C_2), it was −0.054. This indicates that the subgroup of environmental policies based on non-market instruments began to have an effect after the increased stringency of the ETS in 2008.

Table 6 shows the values of the individual parameters for the models, where the dependent variable is CO_2 production measured using CBA, along with their statistical significance and coefficient of determination. The estimations concerned datasets covering the years 1995–2008 and 2008–2020.

Table 6. CBA models estimated on datasets from the years 1995–2008 and 2008–2020

Variable	CBA_B_1	CBA_C_1	CBA_B_2	CBA_C_2
lnGDP	0.097***	0.435***	0.107***	0.409***
lnRES	−0.096***	−0.349***	−0.095***	−0.304***
lnKOFGI	−0.221	−1.872***	−0.228	−0.564
EPS	0.054**	−0.003	–	–
TECH	–	–	0.011	0.012
MARKET	–	–	−0.021	−0.008
NONMARKET	–	–	0.024***	−0.076***
Constant	2.469***	6.973***	2.477***	1.7
Coefficient of determination	0.225	0.567	0.235	0.604

Note. As in Table 4.
Source: author's work.

The aggregated impact of all three subgroups is represented by the EPS variable. In the models before the introduction of the ETS, in the years 1995–2008 (model CBA_B_1), the parameter value was 0.054, and after its introduction, in the years 2008–2020 (model CBA_C_1), it was 0.003, but it was not statistically significant. This indicates that the policy negatively impacted the reduction of CO_2 before the increased stringency of the ETS system introduced in 2008. After 2008, it had no impact on production.

In the models before the introduction of the ETS, in the years 1995–2008 (model CBA_B_2) and after its introduction, in the years 2008–2020 (model CBA_C_2), the TECH parameter was not statistically significant. This suggests that EPS technology support policies did not affect CO_2 production *per capita* measured using CBA in any of the studied periods.

Interestingly, in the models before the introduction of the ETS, in the years 1995–2008 (CBA_B_2), the value of the MARKET variable was −0.021, and after its introduction, in the years 2008–2020 (model CBA_C_2), it was −0.008; however, in both models, the variable was statistically insignificant. This suggests that the changes in the ETS did not significantly impact the reduction of CO_2 emissions measured using CBA.

In the models before the introduction of the ETS, in the years 1995–2008 (CBA_B_2), the value of the NONMARKET parameter was 0.024, and after its introduction, in the years 2008–2020 (model CBA_C_2), it was −0.076. Both parameters were statistically significant, suggesting that non-market instruments began positively impacting the reduction of CO_2 emissions after the changes introduced to the ETS system in 2008.

In all the analysed models, the impact of the logarithm of GDP *per capita*, expressed in nominal US dollars (lnGDP), was statistically significant. The models estimated based on data from the years 1995–2020 (PBA_B_1, PBA_B_2, CBA_B_1, CBA_B_2) had lower parameter values than those from 2008–2020 (PBA_C_1, PBA_C_2, CBA_C_1, CBA_C_2). This suggests that after the changes to the ETS system in 2008, the influence of GDP on CO_2 production *per capita* was greater.

Another statistically significant variable in all models is the logarithm of the share of RES in the total energy consumption. Models estimated based on data before the introduction of the ETS (PBA_B_1, PBA_B_2, CBA_B_1, CBA_B_2) had higher parameter values than those after the introduction of the ETS (PBA_C_1, PBA_C_2, CBA_C_1, CBA_C_2). This indicates that following 2008, the impact of the share of RES on CO_2 production *per capita* was greater.

The last variable present in each model is lnKOF. It is worth noting that the sign of the parameter in the years 1995–2008 (PBA_B_1, PBA_B_2) is opposite to that in the years 2008–2020 (PBA_C_1). This difference suggests that after 2008, globalisation began to positively influence *per capita* CO_2 emissions. Similarly, models estimated based on data before the introduction of the ETS in 2008 (CBA_B_1, CBA_B_2) had higher values than those after the introduction of the ETS (CBA_C_1, CBA_C_2), but they were not statistically significant. This implies that after the changes to the ETS system in 2008, globalisation began to positively influence the reduction of *per capita* CO_2 emissions.

5.1. Robustness check

To verify the robustness of the parameters obtained using the random effects estimator, an alternative estimator for panel data that accounts for lags was conducted.[1] The estimation results are presented in the Table 7.

Table 7. Random effects with autocorrelation correction

Variable	PBA_A_1	PBA_A_2	CBA_A_1	CBA_A_2
lnGDP	0.0621**	0.0712***	0.161***	0.163***
lnREC	−0.259***	−0.253***	−0.228***	−0.227***
lnKOFGI	0.181	0.148	−0.001	−0.0526
TECH		−0.00177		−0.00888
MARKET		−0.042***		−0.0157
NONMARKET		−0.010**		−0.00268
EPS	−0.032***		−0.021*	
Constant	1.344**	1.406**	1.269	1.469*

Note. As in Table 4.
Source: author's work.

[1] In STATA, models were estimated with a correction for residual autocorrelation.

In all models incorporating autocorrelation correction, parameters lnGDP and lnREC retain the same sign and exhibit very similar levels of statistical significance. The lnKOFGI parameter also demonstrates the same direction of change as in the random effects models. The most significant finding from these new models pertains to the results for parameters associated with environmental stringency (TECH, MARKET, NONMARKET and EPS). In models where the EPS index is included in its entirety, the parameter associated with this variable displays the same direction of change and a comparable level of statistical significance. In the remaining models, where the impact of environmental policies is captured separately, the parameters for these variables also exhibit a similar direction of change and statistical significance. Only in model CBA_A_2 does the parameter associated with the MARKET variable lack clear statistical significance.

6. Conclusions

The objective of this study was to examine the impact of strict environmental policies on the production of CO_2 *per capita*, measured using both PBA and CBA. Furthermore, the study examined whether EU policies affect the reduction of CO_2 production. Based on the above, several hypotheses were formulated and empirically tested with fixed and random effects using panel data models. The main hypothesis posited that the environmental policy instruments described in the EPS index significantly influence the reduction of *per capita* CO_2 emissions. However, the study inconclusively confirmed this thesis, despite the significance of the parameters corresponding to these variables in many models.

The second hypothesis was that the choice of the CO_2 measurement method depending on the place of production (PBA) or consumption (CBA), influences the effectiveness of environmental policies in reducing *per capita* CO_2 emissions. This study demonstrated that the choice of the CO_2 measurement method is significant, as in the vast majority of PBA models, the impact of the implemented policies was greater than in the CBA models.

Next, the study attempted to determine whether the changes made to the ETS system in 2008 in the EU influenced the effectiveness of environmental policies in reducing *per capita* CO_2 emissions in European OECD countries. For this purpose, models were constructed for two periods: one covering the years 1995–2008 and the other 2008–2020. In the models with the PBA dependent variable, it was found that the changes introduced to the ETS system increased the effectiveness of the policies, with the exception of the technology subgroup. In the models with the CBA dependent variable, the effectiveness was much lower, as the hypothesis was only proven for the non-market instruments subgroup.

The final hypothesis posited that the various subgroups of the EPS index, namely market-based, non-market-based and technology support, have differing impacts on CO_2 production. The breakdown of the overall EPS index proved accurate and the influence of each subgroup varied depending on the model formula and the analysed time period. In the CBA models, the non-market instruments subgroup had the greatest impact on reducing CO_2 emissions, while in the PBA models the market-based instruments subgroup had the most significant impact. It is also crucial to note that the technology subgroup was statistically insignificant in the PBA models and showed no significance in the CBA models. This may be due to the fact that this is a relatively new subgroup of policy instruments, with effects that are expected to contribute to zero emissions only by 2050.

The final hypothesis was that the different subgroups of the EPS indicator, i.e. market, non-market and technology support, affect carbon production in a different way. The breakdown of the overall EPS indicator proved to be accurate, with the impact of the individual subgroups depending on the model formula and the time period analysed. In the CBA models, the non-market instrument subgroup had the greatest impact on CO_2 reduction, while in the PBA models, it was the impact of the market instrument subgroup. It is also key to note that the technology subgroup was found to be insignificant in the PBA models and showed no significance in the CBA models. This may be due to the fact that it is a relatively new subgroup of policy instruments and is based on technologies that will only contribute to zero-carbon in 2050. As demonstrated by the analyses conducted on the obtained models, the choice of the dependent variable is a key factor in determining the strength and direction of the influence of climate policies and other determinants. Evaluating the impact of policies reveals a significant difference in the parameter results between the CBA and PBA models. Climate policy instruments, particularly those included in the EPS, appear to better explain CO_2 emissions measured using PBA. Additionally, the choice of the dependent variable seems to affect the significance of the parameters. Models measuring CO_2 emissions based on PBA generally show statistical significance more frequently compared to those using CBA. Considering the obtained results, it can be assumed that policies may not fully achieve their intended role and the reduction in CO_2 production may be the result of outsourcing, which involves relocating environmentally harmful activities to countries with less stringent regulatory frameworks.

The models estimated using data from the years 1995–2008 and 2008–2020 allow for the assessment of the impact of important changes introduced to the ETS system. Although the system was established in 2005, the changes introduced in 2008 significantly increased the stringency of this policy. As a result of these changes, there was a substantial reduction in CO_2 emissions by entities covered by the system.

It is important to note that the method of measuring CO_2 played a crucial role, as policies within the market-based and non-market-based subgroups had a much greater impact on reducing CO_2 emissions measured using PBA after 2008. In the CBA analysis, most results regarding the impact of specific EPS index subgroups on CO_2 emissions were statistically insignificant.

The obtained results may serve as a warning to legislators, prompting deeper reflections on the necessary changes. One of the most significant current challenges is enforcing responsibility for goods consumed in Europe, the production of which contributes to environmental pollution in countries such as India and China. One idea that could help address this problem is the introduction of additional border fees in Europe that would compensate for the harmful effects a product caused during its production process. An interesting tool currently being developed is the Carbon Border Adjustment Mechanism (CBAM). Its role may become significant in the near future due to its impact on enforcing the consequences of outsourcing.

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Peaks over Threshold Approach with a time-varying scale parameter and range-based volatility estimator for Value-at-Risk and Expected Shortfall estimation

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Abstract. Exploiting daily high-low range has become increasingly popular among volatility models due to valuable information about volatility dynamics. It has been shown in the literature that range-based volatility estimators can improve volatility and covariance forecasts, and thus models that use high and low prices can outperform standard volatility models based on closing prices solely. This paper incorporates a range-based volatility estimator in an extreme value theory framework to provide better estimates of the tails of daily asset returns. We introduce the Peaks over Threshold model with a range-based volatility estimator depicting the volatility of extreme returns that can contribute to more accurate tail risk estimation. We evaluate the proposed model based on the Monte Carlo simulation and long-period sample of the empirical financial time series by forecasting the Value-at-Risk and Expected Shortfall. We provide evidence that the proposed model can lead to better risk measure forecasts, especially for high tail probabilities.

Keywords: GARCH, Value-at-Risk, Expected Shortfall, Peaks over Threshold, Extreme Value Theory
JEL: C51, C53, C58

1. Introduction

Volatility plays an important role in many areas of economics and finance, where there are countless models and methods of estimating volatility. This topic still attracts many researchers who want to find new ways of describing volatility to better understand its behaviour and to be able to leverage that in practice. The GARCH model is the most popular time-varying volatility model introduced by Engle (1982) and Bollerslev (1986). The GARCH models are formulated solely on closing prices, whereas more accurate estimates of variance can be constructed from daily low and high prices (Parkinson, 1980). The use of high and low prices and volatility estimators constructed on the basis of the range of a maximum and minimum prices provided more accurate volatility models (see, e.g., Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fiszeder & Perczak, 2016; Fiszeder et al., 2023a, 2023b; Molnár, 2016; Xie, 2019). Daily low and high prices are almost always commonly available with closing prices for financial time series, therefore their

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application in volatility models is important from the practical point of view, and in most cases is relatively easy to implement. The application of such prices has also economic consequences (see Chou & Liu, 2010; Wu & Liang, 2011). All in all, the literature showing that range-based volatility models outperform models based on closing prices has recently been gaining popularity and expanding (see the reviews in Chou et al., 2015; Fałdziński et al., 2024; Petropoulos et al., 2022).

Extreme quantile estimation has been one of the main focuses of risk management for researchers and financial institutions, especially in the aftermath of the 2008 financial crisis. Effective risk forecasting plays a role of immense importance, not only in meeting regulatory requirements, but also to providing optimal capital allocation and investment decisions. For this purpose, several risk measures have been introduced that require extreme quantiles estimation, specifically in the left tail of the return distribution. It turns out that Value-at-risk (VaR) and Expected Shortfall (ES) are two of the most widely used risk measures in quantitative risk management. Many different VaR and ES forecasting models and methods have been proposed in the literature. They can be divided into four main groups: parametric, non-parametric, semi-parametric and hybrid (see overviews for VaR in Abad et al., 2014; Nieto & Ruiz, 2016). Standard parametric methods that use an entire dataset for the estimation of the returns distribution are not the best choice for high-quantile estimation. In such cases, a model is fitted to the data better where most of the data points reside, and not surprisingly, it is the mid-regions of the distribution. On the other hand, for risk measures, we focus specifically on the extreme quantiles where there are few observations, so we need more specialised approaches.

The extreme value theory (EVT) is a probabilistic theory with the principal role of describing extreme observations and providing models and methods built specifically for such extraordinary observations and their dynamics. This theory focuses on the tails of the distribution by taking advantage of the limiting laws of extremes. The EVT has been applied to many areas in finance (see an overview in Candia & Herrera, 2024; Echaust & Just, 2020a, 2020b; Herrera & Clements, 2020; Herrera & Schipp, 2013; Rocco, 2014), but its prevailing purpose is extreme quantiles estimation, as it is well suited to estimating and predicting the tails of the distribution, thus being a natural candidate for VaR and ES estimation.

Fisher and Tippett (1928) and Gnedenko (1943) proved that the distribution of the extreme values that are i.i.d.[1] for an unknown cumulative distribution function F converges to a Generalised Extreme Value (GEV) distribution that comprises three distributions. Interestingly, the type of asymptotic distribution of extreme values

[1] Independent and identically distributed.

does not entirely depend on the exact cumulative distribution function F. This major advantage of the EVT enables us, in a way, to 'neglect' the exact form of F.

Another reason why EVT-based models and methods can be more accurate in estimating tail-risk measures is that each tail of the distribution is estimated independently, hence being more flexible and taking into account possible skewness of the data[2]. The main criticism of the EVT, however, stems from the fact that the underlying probabilistic theory holds for i.i.d. samples, whereas financial time series are time-dependent. A naive application of the EVT to the raw time series of returns tends to produce poor estimates of the VaR and ES (see, for instance, Chavez-Demoulin et al., 2014). Consequently, there are two main approaches to modelling the tails of the time-varying conditional return distribution in the literature. First, we focus on an EVT-based model for standardised residuals, where the conditional mean and the conditional volatility are described by some other model (mainly a volatility model) () – presented for instance in McNeil and Frey (2000). This approach assumes that a volatility model removes the time dependence of a time series rendering standardised residuals i.i.d. The second approach involves modelling the behavior of extreme values directly and taking into consideration the dependence structure of the data (see, for instance, Chavez-Demoulin et al., Bee et al., 2019; Bień-Barkowska, 2020; Bień-Barkowska, 2024; Chavez-Demoulin et al., 2005; Chavez-Demoulin et al., 2014; Tomlinson et al., 2024). This approach is commonly defined as the duration between consecutive extreme events, and it considers the magnitude of large losses occurring over a high threshold. Bień-Barkowska (2024) proposed a discrete-duration version of the autoregressive conditional duration peaks-over-threshold model, where duration between the extremes is treated as discrete. On the other hand, these approaches in most cases do not consider the possibility of time-varying parameters to capture short-term shocks during changing market conditions (see Fuentes et al., 2023). Attempts were made to overcome this limitation by using a class of score-driven models introduced by Creal et al. (2013), which have become increasingly popular in recent years.

Researchers also tried to apply a score-driven model to extreme-events modelling. Massacci (2016) proposed a score-driven Generalized Pareto framework to model the magnitude of extremes using a one-factor model. Zhang and Schwaab (2016) criticised one-factor model as not justified empirically, and they introduced a score-driven framework based on two stages. Similarly, Bee et al. (2019) proposed a Peaks over Threshold approach based on realised measures obtained from intraday returns, including autoregressive terms using a score-driven frame-

[2] Skewness in financial time series is one of the properties that are visible in such data (see Hansen, 1994; Harvey & Siddique, 1999).

work. D'Innocenzo et al. (2024) also introduced a score-driven model with time-varying tail parameters, but with no pre-filtering for volatility. Lately, Fuentes et al. (2023) proposed a Marked Point Process model for extreme events with time-varying parameters, whose dynamics are functions of the observations through the score function of the predictive density and possibility to incorporate realised volatility measures. The use of realised volatility measures in the modelling framework has been gaining popularity in the literature recently (see, for instance; Bauwens & Xu, 2023; Bee et al., 2019; Yao et al., 2019). Empirical application of such approaches is limited, as it requires availability of intraday data, which is not common, and these type of data have other drawbacks (see for instance Fantazzini, 2011).

This paper introduces an extension of the first approach by incorporating information from volatility of extreme returns into the EVT-based model. The motivation behind such an approach is that time-varying volatility of returns is an intrinsic property of financial time series, hence also extreme observations show time-varying volatility. Therefore, extreme observations are not heterogeneous from the point of view of time and taking into account extreme time-varying volatility in an EVT-based model should be beneficial for tail-risk measures. We propose a model that uses a standard GARCH model to describe the conditional mean and variance and the Generalized Pareto Distribution (GPD) with the Parkinson estimates of the magnitudes of threshold exceedances to describe the dynamics of extreme values (referred to as the GARCH-GPD-P further in the text).

We carry out the Monte Carlo simulation based on the stochastic volatility (SV) model and analyse how efficient the proposed model is for VaR and ES estimation compared to three benchmarks, namely the GARCH models with the normal (Gaussian) and t-distributed errors and the model proposed by McNeil and Frey (2000), i.e. the combination of the GARCH model and EVT-based Peaks over Threshold method with the GPD. Additionally, we perform an empirical analysis for a relatively large sample of stock indices, currencies and cryptocurrencies to study their usefulness in empirical cases.

The paper further consists of: Section 2, describing the applied models (i.e. GARCH-GPD and the newly-proposed GARCH-GPD-P), Section 3, which provides information on Value-at-Risk and the Expected Shortfall and their backtesting procedure, Section 4 that compares the GARCH-GPD-P model against three benchmarks by carrying out a Monte Carlo simulation to analyse the effects of their specifications on the Value-at-Risk and Expected Shortfall forecasting, and Section 5, comparing the performance of the models to empirical financial time series, i.e. stock indices. The article's conclusions and summary are provided in Section 6.

2. Theoretical background

2.1. GARCH models

The GARCH model of Bollerslev (1986) is the most popular univariate volatility model, and it is based solely on closing prices. We apply this model in the paper as a benchmark for comparison reasons. The GARCH model describes the dynamics of the conditional variance of returns.

Let us assume that the ε_t is the univariate innovation process for the conditional mean (or, in a particular case, the return process) and can be written as:

$$\varepsilon_t | \psi_{t-1} \sim N(0, h_t), \tag{1}$$

where ψ_{t-1} is the set of all information available at time $t-1$, N is the conditional normal distribution, and h_t is the conditional variance. The GARCH(1,1) model is the one most frequently used in empirical studies. It may be presented as:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{2}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$.

The parameters of the GARCH model can be estimated by the quasi-maximum likelihood (QML) method. The log-likelihood function can be written as:

$$L(\boldsymbol{\theta}) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(\ln h_t + \frac{\varepsilon_t^2}{h_t}\right), \tag{3}$$

where $\boldsymbol{\theta}$ is a vector containing unknown parameters of the model, and n is the number of daily observations used in the estimation. The estimates obtained by the QML method are consistent and asymptotically normal (see Bollersle & Wooldridge, 1992; Straumann, 2005; Weiss, 1986).

Instead of the conditional normal distribution, the Student's t-distribution can be applied to better describe fatter tails and leptokurtosis of unconditional distributions of many empirical financial time series (Bollerslev, 1987). The log-likelihood function (Bollerslev, 1987) can be written as:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{n} \left(\ln\left[\Gamma\left(\frac{\upsilon+1}{2}\right)\right] - \ln\left[\Gamma\left(\frac{\upsilon}{2}\right)\right] - \frac{1}{2}\ln[\pi(\upsilon-2)] - \frac{1}{2}\ln(h_t) - \right.$$
$$\left. - \frac{\upsilon+1}{2}\ln\left[1 + \frac{\varepsilon_t^2}{(\upsilon-2)h_t}\right]\right), \tag{4}$$

where $\Gamma(\cdot)$ is the Gamma function and υ are the degrees of freedom parameter. To ensure that the second-order moment exists, the constraint $\upsilon > 2$ is imposed.

2.2. Peaks over Threshold (POT) approach

A natural choice for modelling extreme values is to focus on values that are in the tail of the distribution, i.e. the observations above some high threshold. In the Peaks over Threshold (POT) approach, we are interested in the exceedances over threshold u, conditional on the fact that u is exceeded. Let X_1, X_2 ... be a sequence of i.i.d. random variables, having a marginal distribution function F_u. As shown by Balkema and de Haan (1974) and Pickands (1975), the excess distribution over threshold u corresponding to a random variable X is

$$F_u(x) = P(X - u|X > u) = \frac{F(x+u) - F(u)}{1 - F(u)}, \qquad 0 \le x < x_{sup} - u, \tag{5}$$

where $x_{sup} = \sup\{x \in \mathbb{R}: F(x) < 1\}$. The asymptotic distribution of F_u is the GPD with shape parameter γ and scale parameter σ:

$$GPD_{\gamma,\sigma} = \begin{cases} 1 - \left(1 + \gamma\frac{x}{\sigma}\right)^{-\frac{1}{\gamma}}, \gamma \neq 0 \\ 1 - \exp\left(-\frac{x}{\sigma}\right), \gamma = 0 \end{cases}, \tag{6}$$

where $x \ge 0$ if $\gamma \ge 0$ and $0 \le x \le -\sigma/\gamma$ if $\gamma < 0$ and $\sigma > 0$, when $\gamma > 0$, F_u has a Pareto-type upper tail with a tail index $1/\gamma$. The assumption of i.i.d. is rather restrictive, but fortunately, Leadbetter et al. (1983) proved it for stationary random variables. An estimate of the tail probability can be obtained in the following way (McNeil & Frey, 2000):

$$H_{\hat{\gamma},\hat{\sigma}} = \left(1 + \hat{\gamma}\frac{x}{\hat{\sigma}}\right)^{-\frac{1}{\hat{\gamma}}}, \tag{7}$$

where $\hat{\gamma}$ and $\hat{\sigma}$ are the estimates of the GPD parameters.

This parametric approach consists of two steps:

1. given a sample of $X_1, \ldots, X_n$, choose a threshold u and set $Y_i = X_i - u$, where $i = 1, \ldots, N_u$ and N_u denotes the number of extreme values above the threshold u,
2. fit the GPD to the sequence $Y_1, \ldots, Y_{N_u}$ of exceedances to obtain estimates $\hat{\gamma}, \hat{\sigma}$ of the parameters γ, σ.

The parameters of GPD can be estimated by a maximum likelihood (Hosking & Wallis, 1987; Smith, 1985) with the log-likelihood function:

$$L(\gamma, \sigma) = -N_u ln\sigma - \left(1 + \frac{1}{\gamma}\right) \sum_{i=1}^{N_u} ln\left(1 + \frac{\gamma y_i}{\sigma}\right), \tag{8}$$

provided $(1 + \sigma^{-1}\gamma y_i) > 0$ for $i = 1, \ldots N_u$. Other estimation methods may be used, like probability-weighted moments (PWM) (Hosking et al., 1985). One drawback of the POT method is that the estimates of GPD are sensitive to the choice of threshold u. The choice of threshold u involves a trade-off between bias and variance for the estimates. There are different methods of choosing the threshold – for instance, on the basis of the mean excess plot, by minimising the mean squared error of the estimator (see Beirlant et al., 1996; Jansen & de Vries, 1991; Koedijk et al., 1990), or a widely-used approach that boils down to 10%–15% of the data points that fall in the tail of the distribution (see Smith, 1987). Chavez-Demoulin and Embrechts (2004) show that small variations in the value of the threshold typically have little impact on the estimation.

2.3. GARCH-POT approach

The POT approach is sometimes called the unconditional Peaks over Threshold method, as we fit GPD directly to observations that are above threshold u, disregarding the potentially time-varying mean and variance nature of the observations. The time-dependent structure of observations is assumed to be i.i.d., which in many cases is not true for financial time series. To mitigate this problem, McNeil and Frey (2000) proposed to filter the data by using the ARMA-GARCH model, and then to apply the POT approach to the standardised residuals that should be i.i.d. The main idea behind this method is the assumption that we are dealing with strictly stationary time series of the form $r_t = \mu_t + h_t^{1/2}\varepsilon_t$, with μ_t and h_t being the conditional mean, and variance and ε_t the strict white noise process of unknown distribution. This method will be further referred to in the text as GARCH-GPD, and involves two steps:

1. estimate the ARMA-GARCH(1,1) model with normally distributed errors to model the conditional mean and variance and obtain the standardised residuals $\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2}$;
2. from the standardised residuals $\tilde{\varepsilon}_t$, where $t = 1, \dots, n$, obtain extremes residuals that are above a high threshold u, for which the exceedances are $\{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\}$, and define threshold excesses as $\check{\varepsilon}_i = \tilde{\varepsilon}_i - u$, where $i = 1, \dots N_u$;
3. fit the GPD distribution to the extreme standardised residuals, i.e. $\check{\varepsilon}_i \sim GPD(\gamma, \sigma)$, to obtain estimates $\hat{\sigma}_0$, $\hat{\sigma}_1$ and $\hat{\gamma}$.

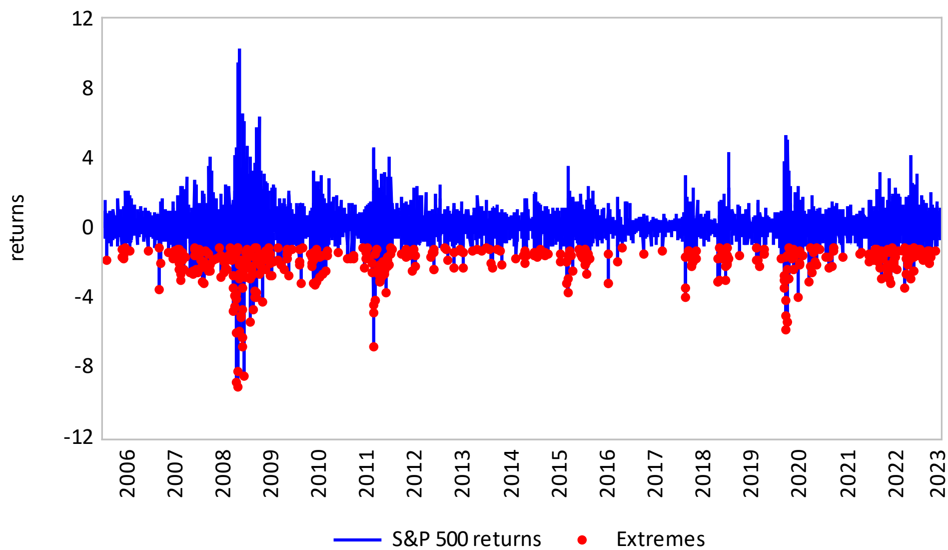
Importantly, Jalal and Rockinger (2008) show that even when the ARMA-GARCH model is misspecified, the GARCH-GPD approach provides good results, which indicates this method is relatively robust. The GARCH-GPD method has been present in the literature, and in most cases, has generated more accurate estimates of tails than other methods (see, Bali, 2007; Chan & Gray, 2006; Kuester et al., 2006).

The use of volatility model is not limited to the standard GARCH(1,1) model, as other specifications may be used, for instance the asymmetric GARCH models, i.e. GJR (Glosten et al., 1993; Pagan & Schwert, 1990), EGARCH (Nelson, 1991) or RGARCH (Molnár, 2016), where lagged squared residuals are replaced with the range-based volatility estimator, or even a CARR model (Chou, 2005), a popular univariate volatility model based on a price range.

2.4. GARCH-POT approach with GPD has a time-varying scale parameter

The unconditional POT approach assumes that the extremes are stationary, so the parameters γ, σ are constant over time. This is likely not the case for financial time series, as the extreme values used for the POT method come from different groups that are above a given threshold u. From an empirical point of view, volatility clustering is a major phenomenon behind financial time series, observing the grouping of high and low volatility across time. It means that clusters with high volatility will have more observations falling in the tail of the distribution, thus being more likely above threshold u than other clusters. We could expect that extreme observations above threshold u should be a part of high-volatility groups formed across the time frame and most likely distant in time from other groups. In EVT, this behavior is well known as the ability of extremes to create clusters. There are methods, like the extremal index (see, for instance, Embrechts et al., 2003, pp. 124–135; Ferro & Segers, 2003), to estimate how extreme observations form series. Figure 1 presents S&P returns with identified extreme values based on the 10th quantile of return distribution as a threshold. Not surprisingly, there are more extreme observations identified for the subperiods like 2008-2009 (financial crisis), 2011 (sovereign crisis), or 2020 (COVID-19 outbreak), and less extreme observations for subperiods 2006, 2014 or 2016–2017. In the literature, there are works employing a time-varying Generalised Pareto distribution with different covariates to model extremes (Bee et al., 2019; Chavez-Demoulin et al., 2005; Chavez-Demoulin et al., 2014; Massacci, 2016; Zhang & Schwaab, 2016), but these models describe extreme values and the dependence in the original data in a single framework. Modelling volatility itself has often proven to be a challenge; hence, it seems that modelling the conditional mean and the conditional variance together but separately from modelling extremes is a more appropriate approach. In this paper, we propose an extension of the GARCH-GPD model of McNeil and Frey, by extending GPD to include time-varying parameters to account for the dynamics of extreme observations.

Figure 1. S&P daily returns with extreme values from 3rd January 2006 to 31st May 2023. Red dots indicate days for which a threshold set at the 10th quantile of distribution is not exceeded



Source: author's work based on the data from www.finance.yahoo.com.

Following Coles (2001), the GPD with time-varying parameters σ_i and γ_i for a series of extremes x, where $i = 1, \dots, N_u$ (the number of extremes), can be written as[3]:

[3] It is worth emphasising that i here denotes time for the extremes and not the time for all observations of the underlying process.

$$GPD_{\gamma_i,\sigma_i} = \begin{cases} 1 - \left(1 + \gamma_i \dfrac{x_i}{\sigma_i}\right)^{-\frac{1}{\gamma_i}}, \gamma_i \neq 0 \\ 1 - \exp\left(-\dfrac{x_i}{\sigma_i}\right), \gamma_i = 0 \end{cases}, \tag{9}$$

where $x_i \geq 0$ if $\gamma_i \geq 0$ and $0 \leq x_i \leq -\sigma_i/\gamma_i$ if $\gamma_i < 0$ and $\sigma_i > 0$. The time-varying shape parameter γ_i is some function f_{γ_i} with a constant and covariates:

$$\gamma_i = f_{\gamma_i}\left(\boldsymbol{X}'_{\gamma_i}\boldsymbol{\gamma}\right), \tag{10}$$

where $\boldsymbol{X}'_{\gamma_i} = \left[1, \boldsymbol{X}'_{\gamma_i,1}, \dots, \boldsymbol{X}'_{\gamma_i,l}\right]$ is a vector of covariates and $\boldsymbol{\gamma} = [\gamma_0, \gamma_1, \dots, \gamma_l]$ is a vector of l parameters to be estimated.

Time-varying scale parameter σ_i is some function f_{σ_i} with a constant and covariates:

$$\sigma_i = f\left(\boldsymbol{X}'_{\sigma_i}\boldsymbol{\sigma}\right), \tag{11}$$

where $\boldsymbol{X}'_{\sigma_i} = \left[1, \boldsymbol{X}'_{\sigma_i,1}, \dots, \boldsymbol{X}'_{\sigma_i,k}\right]$ is a vector of covariates, and $\boldsymbol{\sigma} = [\sigma_0, \sigma_1, \dots, \sigma_k]$ is a vector of k parameters to be estimated. The parameters of time-varying GPD can be estimated by a maximum-likelihood method with the following log-likelihood function (see, Coles, 2001):

$$L(\gamma_i, \sigma_i) = -N_u ln\sigma_i - \left(1 + \frac{1}{\gamma_i}\right)\sum_{i=1}^{N_u} ln\left(1 + \frac{\gamma_i y_i}{\sigma_i}\right), \tag{12}$$

provided $(1 + {\sigma_i}^{-1}\gamma_i y_i) > 0$ for $i = 1, \dots N_u$.

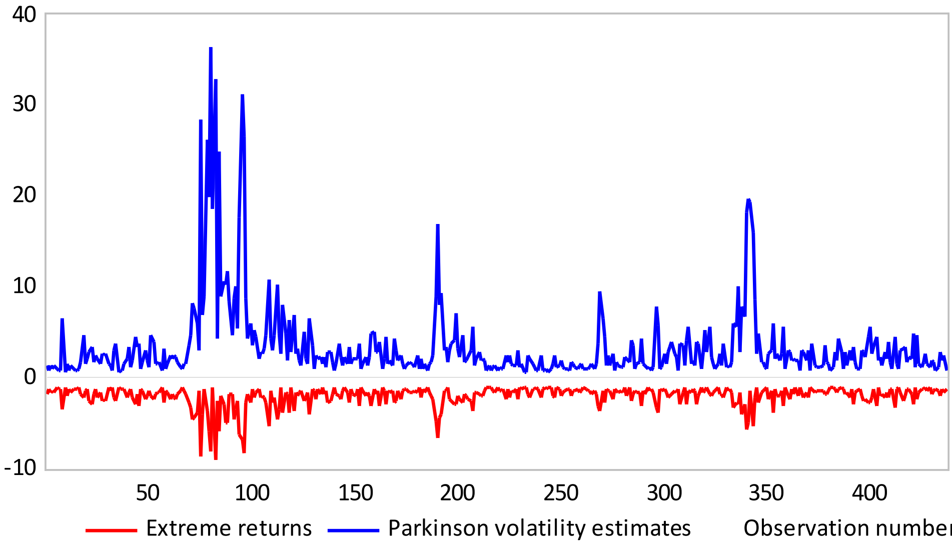
The simplest case of GPD_{γ_i,σ_i} is when there is only a constant for both shape and scale parameters, thus it reduces to the classical GPD given in (6). The question arises as to what covariates and functions f_{γ_i}, f_{σ_i} should be specified to model time--varying parameters. It is usually difficult to estimate time-varying shape parameter γ, so, advisably, it should be kept constant to stabilise the results (see Chavez--Demoulin et al., 2005). It means that we are going to consider the idea of the time-varying scale parameter σ_i only. A natural choice for f_{σ_i} can be a linear additive or logarithmic function, where the latter ensures that σ_i is always positive.

A more important decision to be made is with covariates, as these should, in theory, describe the dynamic behaviour of extreme observations. We propose to use a range-based estimator that can describe return volatility relatively accurately due to the use of high and low prices. A range-based estimator can show the correct volatility, especially on turbulent days with drops and recoveries in the markets, while the traditional close-to-close volatility indicates a low level. It should be even more pronounced for extreme observations, as these occur when market volatility is particularly high. We propose to use the Parkinson volatility estimator (Parkinson, 1980) in the form of

$$\sigma_{P,i}^2 = [ln(H_i/L_i)]^2/(4\ln 2), \tag{13}$$

where H_i and L_i are the high and low prices at a given day i. In the literature, there is growing evidence that the use of range-based volatility estimators can lead to more accurate conditional volatility and covariance estimates and forecasts, in both univariate (Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fałdziński et al., 2024; Fiszeder & Perczak, 2016; Molnár, 2012, 2016) and multivariate frameworks (Asai, 2013; Chou & Cai, 2009; Chou et al., 2009; Fiszeder et al., 2019; Fiszeder et al., 2023a, 2023b; Su & Wu, 2014). Moreover, there are range-based volatility models (based on a range instead of returns) that outperform classical models based on closing prices (see the reviews in Chou et al., 2015; Petropoulos et al., 2022). Different estimators based on daily low, high, or additionally open and closing prices can be employed (Garman & Klass, 1980; Rogers & Satchell, 1991; Yang & Zhang, 2000). The Garman-Klass estimator is sensitive to microstructure effects associated with low liquidity during the start of quotations, and Molnár (2016) showed that the Garman-Klass estimator does not improve results compared to the Parkinson estimator. On the other hand, the Rogers-Satchell estimator can take a zero value despite the high volatility during the day. It happens when the opening price is equal to the low price and the closing price is equal to the high price or vice versa, i.e., the opening price is equal to the high price and the closing price is equal to the low price. The Yang-Zhang estimator requires estimating an additional parameter and assumes constant variance over time, which is untrue. Moreover, the Yang-Zhang estimator cannot be estimated for a single day. For these reasons, we focus here on the Parkinson estimator.

Figure 2. S&P 500 extreme observations and Parkinson volatility estimates that are ordered consecutively



Source: author's work based on the data from www.finance.yahoo.com.

To justify the use of a range-based estimator, Figure 2 presents the Parkinson daily volatility estimates associated with extreme observations found for the S&P 500 index from the time-range presented in Figure 1, where extremes are ordered as they occurred in time (in total there are 438 extreme observations). The red solid line illustrates extreme returns, and the blue solid line Parkinson's volatility estimates. High and low Parkinson volatility estimates are concurrent with high and low extreme daily returns, and it seems to provide a good approximation of daily extreme-returns volatility. Therefore, we propose the following time-varying scale equation σ_i for GPD:

$$\sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2, \text{ where } i = 1, \dots, N_u, \tag{14}$$

where $\sigma_0 > 0$ and $\sigma_1 \geq 0$ to ensure that σ_i is positive. It is worth noting that the Parkinson's volatility estimates $\sigma_{P,i}^2$ are contemporaneous with extreme residuals. It is possible to consider the past Parkison volatility estimates, but concurrent values to extreme returns should be preferred as the contemporaneous values are available at a given time i and should provide a better fit than the past ones. In this regard, it is worth noting that extremes are a sub-sample of available observations.

The proposed method will be referred to further in the text as GARCH-GPD-P, and consists of the following steps:

1. estimate the ARMA-GARCH(1,1) model to obtain both the conditional mean μ_t and conditional variance h_t;
2. obtain the standardised residuals $\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2}$;
3. from the standardised residuals $\tilde{\varepsilon}_t$, where $t = 1, \dots, n$, obtain extremes residuals that are above a high threshold u, for which the exceedances are $\{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\}$, and define threshold excesses as $\check{\varepsilon}_i = \tilde{\varepsilon}_i - u$, where $i = 1, \dots N_u$;
4. fit GPD distribution to the extreme standardised residuals, i.e. $\check{\varepsilon}_i \sim GPD(\gamma, \sigma_i)$, where $\sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2$ and $\sigma_{P,i}^2$ is the Parkinson estimator at observation i (noting that $i \leq n$) to obtain estimates $\hat{\sigma}_0$, $\hat{\sigma}_1$ and $\hat{\gamma}$.

The GPD and GPD-P method rely on extremes as a sub-sample of all observations above threshold u. Given two samples that imperfectly overlap with each other, the sub-samples of extremes above threshold may have a perfect overlap, some overlap or, in edge case, no overlap in extremes. Consequently, the GPD-P estimates are based on the sub-sample of observations that is deemed as extreme at a particular time.

The proposed framework GARCH-GPD-P can be concisely formulated as:

$$r_t = \mu_t + \varepsilon_t, \varepsilon_t | \psi_{t-1} \sim N(0, h_t), t = 1, \dots, n, \tag{15}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{16}$$

$$\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2}, \tag{17}$$

$$\check{\varepsilon}_i = \tilde{\varepsilon}_i - u, \text{for } i = 1, \dots N_u, \text{where } \{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\}, \tag{18}$$

$$\check{\varepsilon}_i \sim GPD(\gamma, \sigma_i), \text{where } \sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2. \tag{19}$$

3. Value-at-Risk and Expected Shortfall

3.1. Value-at-Risk and Expected Shortfall estimation

Tail-based risk measures such as the Value-at-Risk (VaR) and the Expected Shortfall (ES) are mostly used in quantitative risk management, from the perspective of the regulatory and financial institution. The Basel Accords explicitly use VaR and ES as risk measures and oblige financial institutions to implement and report them to monitor risk and determine the amount of capital that is subject to regulatory supervision.

Let $\alpha \in [0,1]$ denote the coverage level (or probability level). The α level VaR is defined as $VaR_t(\alpha) = P(r_t \leq -VaR_t) = \alpha$, so the $VaR_t(\alpha)$ is the α quantile of the r_t returns distribution that is negative. The VaR has been criticised for not being able to show the average potential loss; it could only show whether losses were larger than the VaR. This was one of the reasons why the ES has been proposed to measure the size and the likelihood of losses. ES is defined as the expected loss given that the loss is greater than VaR, and it may be written as $ES_t(\alpha) = -E[-r_t > VaR_t(\alpha)]$. A more useful representation of $ES_t(\alpha)$ is:

$$ES_t(\alpha) = \frac{1}{\alpha}\int_0^\alpha VaR_t(u)\,du. \tag{20}$$

$ES_t(\alpha)$ comprises information from the left tail of the returns distribution, by integrating VaR from 0 to α. In practice, risk managers specify parametric conditional versions of VaR and ES. For the GARCH model, VaR and ES are given by:

$$VaR_{t,cond}(\alpha) = -\mu_t - \sqrt{h_t}F^{-1}(\alpha), \tag{21}$$

$$ES_{t,cond}(\alpha) = -\mu_t - \sqrt{h_t}m(\alpha), m(\alpha) = E[\varepsilon_t|\varepsilon_t \leq F^{-1}(\alpha)], \tag{22}$$

where $F^{-1}(\alpha)$ is the α-quantile of the inverse cumulative distribution function. In this paper, we are using the normal distribution and Student's *t*-distribution function with υ degrees of freedom. The driving force behind the VaR and ES estimates variability is the conditional variance (see So & Yu, 2006), as the conditional mean is, in most cases, close to zero (or omitted), and α-quantile of the inverse cumulative distribution function is used as a constant value (for instance, for the normal distribution it is -1.64 at a 5-percent probability level). Thus, to achieve better estimates of VaR and ES, we have to improve variance estimates, as a quantile from the normal or Student's *t*-distribution is constant at a given probability.

To obtain VaR and ES with the GPD approach, we need an inverse of the cumulative GPD function given by equation (7) and estimates $\hat{\gamma}$ and $\hat{\sigma}$. Then, the unconditional VaR and ES with GPD (following McNeil & Frey, 2000) are given as:

$$VaR_{unc}(\alpha) = \hat{u} + \frac{\hat{\sigma}}{\hat{\gamma}}\left[\left(\frac{n}{N_u}\alpha\right)^{-\hat{\gamma}} - 1\right], \tag{23}$$

$$ES_{unc}(\alpha) = \frac{VaR_{unc}(\alpha)}{1-\hat{\gamma}} + \frac{\hat{\sigma} - \hat{\gamma}\hat{u}}{1-\hat{\gamma}}, \tag{24}$$

where $\hat{u}$ is the threshold estimate, n is the number of observations, and N_u is the number of extremes.

Consequently, the unconditional VaR and ES with time-varying GPD_{γ_i,σ_i} can be written as:

$$VaR_{unc}(\alpha) = \hat{u} + \frac{\hat{\sigma}_i}{\hat{\gamma}_i}\left[\left(\frac{n}{N_u}\alpha\right)^{-\hat{\gamma}_i} - 1\right], \tag{25}$$

$$ES_{unc}(\alpha) = \frac{VaR_{unc}(\alpha)}{1-\hat{\gamma}_i} + \frac{\hat{\sigma}_i - \hat{\gamma}_i\hat{u}}{1-\hat{\gamma}_i}, \tag{26}$$

where $\hat{\sigma}_i$ and $\hat{\gamma}_i$ are estimates of σ_i and γ_i for $i = 1, \dots N_u$. In the proposed framework, we are using the latest available extreme for the unconditional VaR and ES calculation, i.e. for $i = N_u$. For VaR and ES calculation when a new extreme observation is available, VaR and ES estimates are impacted not only by the change in the conditional mean and the conditional variance, but also by the change in scale parameter $\hat{\sigma}_i$ through the change in the GPD-P quantile.

The conditional one-day-ahead VaR and ES with the GARCH-GPD and GARCH-GPD-P approaches are given by:

$$VaR_{t+1,cond}(\alpha) = -\mu_{t+1} - \sqrt{h_{t+1}}VaR_{unc}(\alpha), \tag{27}$$

$$ES_{t+1,cond}(\alpha) = -\mu_{t+1} - \sqrt{h_{t+1}}ES_{unc}(\alpha), \tag{28}$$

where μ_{t+1} and h_{t+1} are the one-day-ahead forecasts of the conditional mean and the conditional variance of returns, respectively.

The advantage of GPD and time-varying GPD-P lies in the fact that the unconditional VaR and ES are tail-based estimates depending on parameter estimates for GPD and GDP-P, respectively. The difference between GPD and time--varying GPD-P is that the latter takes into account the magnitudes of threshold exceedances measured by the Parkinson estimator, thus we can expect more accurate estimates of the unconditional VaR and ES. This is because the variability of extremes should be described more accurately by the time-varying scale σ_i parameter. In other words, to obtain better VaR and ES estimates for the GARCH-GPD or GARCH-GPD-P, we can improve either or both the conditional variance and tail-based estimates from the GPD or GPD-P.

3.2. Value-at-Risk and Expected Shortfall backtesting

There is already a wide spectrum of methods and models to estimate tail-based risk measures, like VaR and ES. The evaluation of forecasting accuracy is of great importance when it comes to risk measures, especially for practitioners and regulatory institutions, to ensure that financial institutions have adequate capital to deal with large unexpected losses. The literature provides information on many various ways to assess the accuracy of VaR estimates by developing statistical tests, methods and measures known as backtesting. We can divide backtesting methods into three categories: a) statistical tests verifying the validity of VaR assumptions, b) measures to assess VaR accuracy, and c) statistical tests to determine which of the competing models are superior to others.

The hit variable (or violation variable) associated with the ex-post observation of a $VaR_t(\alpha)$ at time t, denoted $I_t(\alpha)$, is defined as:

$$I_t(\alpha) = \mathbf{1}\big(r_t \leq -VaR_t(\alpha)\big), \tag{29}$$

where $\mathbf{1}(\cdot)$ is the indicator function. Kupiec (1995) shows that in order to assess the VaR validity it is possible to test whether the hit sequence $I_t(\alpha)$ follows the two conditions: a) unconditional coverage (UC) $P[I_t(\alpha) = 1] = E[I_t(\alpha)] = \alpha$, and b) independence property (IND), i.e. variable $I_t(\alpha)$ has to be independent of variable $I_{t-k}(\alpha), \forall k \neq 0$. These two conditions are necessary but not sufficient for the VaR definition. The most popular backtesting tests are: the unconditional coverage LR_{UC} proposed by Kupiec (1995) and the independence LR_{ind} and conditional coverage LR_{CC} tests by Christoffersen (1998). It has been documented that these tests have law power (see de la Pena et al., 2007; Pérignon & Smith, 2008; Pritsker, 2006). Alternatively, Candelon et al. (2011) proposed the unconditional, independence and conditional coverage tests (denoted here as J_{UC}, J_{IND} and J_{CC}, respectively) based on the duration of the hit sequence, and showed that their GMM-based tests are of greater statistical power than the classically used ones. Additionally, they encourage obtaining simulated p-values instead of asymptotic ones, by applying Dufour's approach (Dufour, 2006) to ensure the correct test size.

Besides testing the hit process, loss functions can be used to select a model that produces accurate Value-at-Risk estimates. Lopez (1998) suggested measuring the accuracy of VaR forecasts by the distance between the observed returns and the forecasted VaR. A model is penalised if a violation takes place and is preferred to another one because it gives a lower loss value. In the general form, Lopez proposes the following formula:

$$LF_t = \begin{cases} f(r_t, VaR_t(\alpha)) \text{ if } r_t < -VaR_t(\alpha) \\ g(r_t, VaR_t(\alpha)) \text{ if } r_t \geq -VaR_t(\alpha) \end{cases}, \tag{30}$$

where $f(x,y)$ and $g(x,y)$ are such that $f(x,y) \geq g(x,y)$. The best model is the one that minimizes $LF = \sum_{t=1}^{T} LF_t$. Lopez in 1998 proposed the following loss measure:

$$RLF(L) = \begin{cases} 1 + (VaR_t - r_t)^2 \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases}. \tag{31}$$

Sarma et al. (2003) and Caporin (2008) proposed loss functions from two perspectives: the regulator's loss function (RLF) and the firm's loss function (FLF).

$$RLF(C1) = \begin{cases} \left|1 - \left|\frac{r_t}{VaR_t}\right|\right| \text{ if } r_t < -VaR_t, \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{32}$$

$$RLF(C2) = \begin{cases} \frac{(|r_t| - |VaR_t|)^2}{VaR_t} \text{ if } r_t < -VaR_t\,, \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{33}$$

$$RLF(C3) = \begin{cases} |r_t - VaR_t| \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases}, \tag{34}$$

$$FLF(STS) = \begin{cases} (r_t - VaR_t)^2 \text{ if } r_t < -VaR_t \\ -ococVaR_t \text{ if } r_t \geq -VaR_t \end{cases}, OCOC \text{ is the opportunity cost of capital,} \tag{35}$$

$$FLF(C1) = \left|1 - \left|\frac{r_t}{VaR_t}\right|\right|, \tag{36}$$

$$FLF(C2) = \frac{(|r_t| - |VaR_t|)^2}{VaR_{t|}}, \tag{37}$$

$$FLF(C3) = |r_t - VaR_t|. \tag{38}$$

Şener et al. (2012) propose a loss function that penalises the magnitude of the errors, the autocorrelation between the errors, and excessive capital allocations. The penalisation measure takes the form:

$$PM(\varphi, VaR) = \frac{1}{T^*}[(1-\varphi)PM_{VS} + \varphi PM_{SS}], \tag{39}$$

where PM_{VS} and PM_{SS} is the penalisation measure for the violation space and the safe space, respectively, φ is the weighting parameter and T^* is the number of all negative returns. The weighting parameter φ is assumed to be set to the coverage level α, thus violations have more importance than non-violations, which is expected from the regulator's and financial institution perspectives. The penalisation measure for the violation space PM_{VS} can be written as:

$$PM_{VS} = \sum_{i=1}^{n_c-1} \sum_{j=1}^{n_c} \frac{1}{d_{i,i+j}} \left(\prod_{k=1}^{l_i} (1 + lf_{k,i}) \prod_{k=1}^{l_{i+j}} (1 + lf_{k,i+j}) - 1 \right), \tag{40}$$

where $lf_t = (VaR_t(\alpha) - r_t)$ given $r_t < -VaR_t(\alpha)$, n_c is the number of violation clusters, $d_{i,i+j}$ is the time between i-th and j-th violations clusters, and l_i is the length of violation cluster i.

The penalisation measure for the violation space PM_{VS} focuses on the magnitude of unexpected losses and clusters of unexpected losses (autocorrelation), and is calculated only for violations. On the other hand, the penalisation measure for the safe space PM_{SS} may be written as:

$$PM_{SS} = \sum_{t=1}^{T} \big(r_t - VaR_t(\alpha)\big)[\mathbf{1}(r_t > VaR_t(\alpha) | r_t < 0)], \tag{41}$$

where $\mathbf{1}$ is the indicator function and T is the number of all observations for which VaR forecasts have been obtained. This measure takes into account excessive capital allocation for returns that are not a violation and are negative. The idea behind the penalisation measure is to have the flexibility to capture both the regulator and risk manager's perspectives while being able to give different weights to each.

Furthermore, to determine which of the competing models produces superior VaR estimates, Sarma et al. (2003) proposed to use the Diebold and Mariano test (Diebold & Mariano, 1995), and Şener et al. (2012) introduced a predictive ability test for the penalisation measure $PM(\varphi, VaR)$ that does not require a benchmark model, thus allowing the simultaneous comparison of several models. The test is based on White's framework (White, 2000) as an extension of Diebold and Mariano test. The null hypothesis states that the loss series generated by any chosen forecasting method is statistically no worse than the others.

When it comes to backtesting of Expected Shortfall, the situation is quite different from Value-at-Risk, where the literature was scarce. More recently, Du and Escanciano (2016) introduced the unconditional DE_{UC} and the conditional DE_{IND} tests based on cumulative violations sequence. The cumulative violation process is defined as

$$H_t(\alpha) = \frac{1}{\alpha}\int_0^\alpha I_t(u)du, \tag{42}$$

where $H_t(\alpha)$ has a mean equal to $\alpha/2$. Then, the unconditional backtest UC_{ES} is a t-test for hypothesis $E[H_t(\alpha)] = \alpha/2$. The test statistic is given by:

$$DE_{UC} = \frac{\sqrt{n_f}(\bar{H}(\alpha) - \alpha/2)}{\sqrt{Var(H_t(\alpha))}} \sim N(0,1), \tag{43}$$

where $\bar{H}(\alpha)$ denotes the sample mean of $H_t(\alpha)$, n_f is the number of ES estimates and $Var(H_t(\alpha))$ is the variance of $H_t(\alpha)$ with the standard normal asymptotic distribution $N(0,1)$. The conditional backtest of independence DE_{IND} is based on the lag-j autocovariance and autocorrelation of $H_t(\alpha)$ for $j \geq 0$ that are defined as follows:

$$cov_{n_f,j} = \frac{1}{n_f - j}\sum_{t=1+j}^{n_f}(H_t(\alpha) - \alpha/2)\big(H_{t-j}(\alpha) - \alpha/2\big) \text{ and } \rho_{n_f,j} = \frac{cov_{n_f,j}}{cov_{n_f,0}}. \tag{44}$$

The test statistic is given as:

$$DE_{IND}(m) = n_f \sum_{j=1}^{m} \hat{\rho}_{n_f,j}, \tag{45}$$

where $\hat{\rho}_{n_f,j}$ is the sample estimate of $\rho_{n_f,j}$ with the limiting chi-square distribution χ_m^2 with m degrees of freedom.

4. Monte Carlo simulation

We conduct a Monte Carlo simulation to analyse the finite sample properties of the proposed model, i.e. the GARCH-GPD-P versus the competing models (the GARCH model with normally and Student's t-distributed errors denoted as GARCH-n and GARCH-t, respectively, and McNeil and Frey's GARCH-GPD). We choose the stochastic volatility (SV) model as the data-generating process due to its flexibility, and because this model is relatively often used for simulation purposes in the literature (see for instance Alizadeh et al., 2002; Buescu et al., 2013; Molnár, 2016; Shu & Zhang, 2006). The main advantage of the SV model over the GARCH one is that it assumes two innovation processes (for the conditional mean and the conditional volatility). In the SV model, the volatility is a random variable, hence this model can be more flexible than the GARCH model. It is believed that the SV model is more effective in describing empirical properties of financial time series (see Danielsson, 1994; Kim et al., 1998). Assuming the SV model as the data generating process does not favour any of the competing models.

Daily volatility is simulated by the stochastic volatility model that can be given as (see Melino & Turnball, 1990; Taylor, 1990):

$$ln(P_t/P_{t-1}) = \mu_{sv,t} + \sigma_{sv,t}\varepsilon_t, \tag{46}$$

$$ln\left(\sigma^2_{sv,t}\right) = \alpha_{sv} + \phi_{sv} ln\sigma^2_{sv,t-1} + \sigma_\eta \eta_t, \tag{47}$$

where ε_t and η_t are mutually independent and i.i.d. following the normal distribution with the zero mean and unit variance $N(0,1)$. We assume the following set of values for the parameters: $\mu_{sv,t} = 0.001$, $\alpha_{sv} = 0.02$, $\phi_{sv} = 0.95$ and $\sigma^2_\eta = 0.065$. These values are consistent with the ones observed empirically for the stochastic volatility model. As we need to obtain not only daily close prices, but also low and high prices, we simulate intraday price paths following the geometric Brownian motion based on the simulated daily volatility and mean from the stochastic volatility model.

We simulate 1,600 daily price paths with their volatilities following the SV model (Equations 46 and 47), where for each day we generate 100,000 intraday prices based on the geometric Brownian motion. The first 100 observations are dropped to remove the impact of the starting values. Then, we use the next 500 observations (from 101 to 600) to estimate the parameters of all four competing models (the GARCH-n, GARCH-t, GARCH-GPD and GARCH-GPD-P). This step involves obtaining the Parkinson volatility estimates based on simulated high and low prices and estimating the conditional VaR and ES for the next day by Equations (21), (22), (27) and (28), where one-day-ahead forecasts of the conditional mean and the conditional volatility are used. For the GARCH-GPD and the GARCH-GPD-P models, we set the threshold as the 12-percent cut-off point of the most negative standardised residuals. The threshold was set on the basis of the mean excess plot for the empirical times series used in Section 3. We repeat this process for each subsequent day by applying the rolling window approach, where one observation from the beginning of the sample is removed and one observation is added to the end of the sample, thus obtaining a fixed size of 500 observations in the sample. This way, we have 1,000 VaR and ES daily estimates for one iteration of the simulation. They are backtested using methods and measures described in subsection 3.2 for 5-percent and 10-percent coverage levels. Lastly, we repeat the process above 1,000 times, which is the number of iterations in the Monte Carlo simulation. The final results presented in the paper are the averages for all 1,000 iterations. In total, we obtain and evaluate 1,000,000 VaR and ES estimates as a basis for the backtesting procedures.

4.1. Evaluation of models based on the Monte Carlo simulation

For in-sample comparisons, we are going to focus on the results of two models, i.e. the GARCH-GPD and GARCH-GPD-P, as the GARCH-n and GARCH-t models are benchmarks for risk measure purposes. As described in Section 4, the parameters of all the models are estimated 1,000 times for each of the 1,000 repetitions of the Monte Carlo simulation based on the rolling window approach. For all the repetitions, we compute the average and standard deviation of the estimated parameters and the robust standard errors which are presented in Table 1. Scale parameter σ for the GPD and σ_1 for the GPD-P are highly significant. Moreover, the constant scale parameter for the GPD-P model is considerably lower than the σ scale parameter for the GPD. We perform the likelihood ratio test for each estimated model for all repetitions and the average values are presented in Table 1. The null hypothesis is rejected even at a high significance level indicating that the GPD-P model is better fitted to the extreme observations than the GPD model. It means that the information comprised of high and low prices associated with extreme observations provides considerable insight into the dynamic behaviour of the extremes.

The out-of-sample analysis involves the evaluation of the VaR and ES forecasts at 5-percent and 10-percent probability levels. For each repetition in the simulation, we evaluate the 1,000 obtained VaR and ES forecasts and we backtest them by testing their statistical properties, calculating the loss measures and testing the superiority of the VaR forecasts against the others. We repeat this process for all 1,000 iterations and compute the average of the obtained results.

Table 1. The results of the parameter estimates for the GPD and GPD-P for Monte Carlo simulation

Statistics	GPD			GPD-P				LM p-value
	σ	γ	ln L	σ_0	σ_1	γ	ln L	
Mean	0.6475*	–0.0922	–	0.1171	0.0590*	–0.2181*	–	0.000
	(0.1169)	(0.1139)	27.3616	(0.0448)	(0.0175)	(0.1091)	4.6664	0*
St. dev.	0.1169	0.1139	7.0	0.2708	0.0533	0.1074	6.9	0.003
	(0.0310)	(0.0274)	713	(0.0658)	(0.0591)	(0.0872)	171	2

Note. * indicates that the null hypothesis is rejected at a 5-percent significance level, the robust Huber-White standard errors are reported in parentheses, St. dev. – the standard deviation, ln L – logarithm of the likelihood function, LM p-value is the p-value from the likelihood ratio test based on the logarithm of the likelihood function for GPD vs GPD-P.

Source: author's work.

Table 2. The results of backtesting tests for VaR(10%) and VaR(5%) based on the Monte Carlo simulation

VaR coverage level	Statistic	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
		p-value	p-value	p-value	p-value
10%	LR_{UC}	0.3153	0.5606	0.6408	0.6623
	LR_{IND}	0.5434	0.5716	0.5648	0.5576
	LR_{CC}	0.3872	0.6062	0.6538	0.6591
	J_{UC}	0.3334	0.5546	0.6258	0.6644
	J_{IND}	0.3519	0.5557	0.5809	0.5931
	J_{CC}	0.3413	0.5476	0.5744	0.5898
5%	LR_{UC}	0.5845	0.5351	0.6268	0.6985
	LR_{IND}	0.5015	0.5121	0.4897	0.5012
	LR_{CC}	0.5796	0.5361	0.5952	0.6425
	J_{UC}	0.5838	0.5091	0.6133	0.6913
	J_{IND}	0.5733	0.5757	0.5740	0.5746
	J_{CC}	0.5638	0.5758	0.5737	0.5814

Note. * indicates that the null hypothesis is rejected at a 5-percent significance level. LR_{UC} is the unconditional coverage test proposed by Kupiec (1995), LR_{IND}, LR_{CC} are the independence and conditional coverage tests, respectively, proposed by Christoffersen (1998). J_{UC}, J_{IND}, J_{CC} are the unconditional coverage, independence and conditional coverage tests, respectively, proposed by Candelon et al. (2011). For J_{IND} and J_{CC}, the number of moments is fixed to 5, p-values for J_{UC}, J_{IND}, J_{CC} are obtained through Dufour's (2006) Monte Carlo procedure involving 10,000 repetitions.

Source: author's work.

Table 2 shows the results of testing statistical properties of VaR at 10-percent and 5-percent coverage levels. At both levels, all the competing models seem to perform relatively well as the null hypothesis is not rejected for all the tests, although the p-values for the GARCH-GPD-P and GARCH-GPD are generally higher than for the GARCH-n and the GARCH-t models. Table 3 presents the results of the loss functions used for VaR forecast evaluation. To that end, we utilise the following measures split into two groups, i.e. the regulator's loss functions (RLF) – $RLF(L)$ by Lopez (1998), $RLF(STS)$ by Sarma et al. (2003), $RLF(C1)$, $RLF(C2)$ and $RLF(C3)$, all three proposed by Caporin (2008), and the FLFs – $FLF(STS)$ by Sarma et al. (2003), $FLF(C1)$, $FLF(C2)$ and $FLF(C)$, all proposed by Caporin (2008). At a 10-percent coverage level, the GARCH-n model leads to the smallest values of the regulator's loss functions, but at the same time, the FLFs are the highest across the models. The GARCH-GPD-P and GARCH-GPD perform quite similarly for all loss functions, although the values of loss functions are lower for the GARCH-GPD-P model. There are two cases ($FLF(C2)$ and $FLF(C3)$) where the GARCH-GPD-P model have the lowest values of all models. The poorer performance at lower coverage levels is not surprising as the EVT-based methods are designed to accurately model high tails, i.e. 5%, 1% or even 0.5%. At a 5-percent coverage level, we can observe that the GARCH-GPD-P model produces the best estimates of VaR according to all regulators' loss functions. On the other hand, we can see that the proposed model may lead to some overestimation based on the firm's loss functions. This is in line with the empirical observation from other studies where the POT approach is applied.

Table 3. The average results of the loss measures for VaR(10%) and VaR(5%) based on the Monte Carlo simulation

VaR coverage level	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
10%	RLF(L)	**111.0527**	126.4374	127.7731	127.5317
	RLF(STS)	**22.2427**	25.3874	25.6931	25.3156
	RLF(C1)	**43.0243**	52.2956	53.2825	53.0290
	RLF(C2)	**30.1174**	36.6573	37.3801	37.2902
	RLF(C3)	**31.3574**	35.7206	36.1238	36.0265
	FLF(STS)	**56.4412**	57.0688	57.1631	57.0823
	FLF(C1)	572.6937	**569.5876**	569.7450	569.6610
	FLF(C2)	322.0481	306.4964	305.5258	**303.8595**
	FLF(C3)	809.4938	772.2083	769.1470	**766.4545**
	PM	**0.0297**	0.0313	0.0310	0.0303
	PM(VS)	**5.9594**	7.8347	7.0591	7.0421
	PM(SS)	184.0021	165.2148	163.6847	**161.3238**
5%	RLF(L)	62.3923	66.0560	63.7472	**58.9353**
	RLF(STS)	12.1923	12.9660	12.5772	**11.0953**
	RLF(C1)	18.5921	20.2220	19.4369	**18.4008**
	RLF(C2)	12.9681	14.1229	13.5645	**12.5804**
	RLF(C3)	17.3157	18.3933	17.8284	**16.1446**
	FLF(STS)	57.9073	**57.5883**	57.8911	61.5843
	FLF(C1)	607.8039	**603.7517**	606.3970	620.8263
	FLF(C2)	432.7660	**420.4871**	428.5252	509.8466
	FLF(C3)	993.6257	**975.5646**	986.9144	1 082.4124
	PM	0.0309	0.0304	0.0308	**0.0294**
	PM(VS)	1.7541	1.9833	1.8841	**1.0103**
	PM(SS)	276.3196	**267.1751**	272.9081	273.3008

Note. The lowest values of loss functions are marked in bold. RLF(L) is the loss function proposed by Lopez (1998), RLF(STS), FLF(STS) are the loss functions proposed by Sarma et al. (2003), RLF(C1), RLF(C2), RLF(C3), FLF(C1), FLF(C2) and FLF(C3) are the loss functions proposed by Caporin (2008), PM, PM_{VS} and PM_{SS} are the penalisation measure, the penalisation measure for the violation space and the penalisation measure for the safe space proposed, respectively, by Şener et al. (2012).

Source: author's work.

The best values of the firm's loss functions are obtained for the GARCH-t model. It is worth noting that the GARCH-GPD-P model has the best value of the penalisation measure, mainly because in case of violations, the GARCH-GPD-P model is the least underestimated.

Table 4 shows the results of the predictive ability test of Şener et al. (2012) for VaR(5%) and VaR(10%). At both levels, we do not reject the null hypothesis, but we may see that the GARCH-GPD-P and the GARCH-t have the highest p-values at a the 5-percent and 10-percent probability, respectively. This means that it is difficult to find significant statistical differences in VaR forecasting among the tested models.

Table 4. The average p-values of the predictive ability test (Şener et al., 2012) for VaR(5%) and VaR(10%) based on the penalisation measure: the Monte Carlo simulation

VaR coverage level	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
10%	0.6551	0.8687	0.7258	0.7938
5%	0.3895	0.7878	0.6010	0.8529

Source: author's work.

Table 5 presents the results of backtesting for the Expected Shortfall at the 10-percent and 5-percent levels. At both levels, we do not reject the null hypothesis for the unconditional and independent tests, although we may observe that the p-values for the GARCH-GPD-P are the highest, thus indicating that this model may produce better properties of ES. The mean of cumulative violation process H_t for the GARCH-GPD-P is closer to the desired level (i.e. $\alpha/2$) than any other competing model. t suggests that the most accurate forecasts of ES come from the GARCH-GPD-P model.

Table 5. The results of backtesting for ES(10%) and ES(5%) based on Du and Escanciano (2016): the Monte Carlo simulation

ES coverage level	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean H_t	p-value	Mean H_t	p-value	Mean H_t	p-value	Mean H_t
10%	DE_{UC}	0.5758	0.0497	0.5583	0.0520	0.6524	0.0515	0.6989	0.0510
	DE_{IND}	0.4870	–	0.4680	–	0.4702	–	0.5240	–
5%	DE_{UC}	0.3746	0.0290	0.5043	0.0273	0.6033	0.0263	0.6551	0.0245
	DE_{IND}	0.5587	–	0.5505	–	0.6755	–	0.7904	–

Note. For independence test DE_{IND}, we calculate the statistics up to 5 lags. DE_{UC}, DE_{IND} are the unconditional coverage and independence tests, respectively, proposed by Du and Escanciano (2016), H_t is the cumulative violation process.

Source: author's work.

All in all, it is impossible to select the best model for VaR and ES forecasting. This is also a prevailing conclusion from other studies that compare risk measures from different perspectives (see for instance Abad et al., 2014; Nieto & Ruiz, 2016). The performance of the GARCH-GPD-P model in the Monte Carlo simulation indicates that it has the advantage over other competing models at a higher probability level (5%), where it yields more accurate VaR and ES forecasts.

5. Analysis of stock indices, currencies and cryptocurrencies

5.1. Data

We apply the analysed models to real financial data, i.e. five stock indices, three currencies and four cryptocurrencies. The set of data consists of three classes of assets: five selected U.S. stocks: Amazon, Apple, Google, Microsoft and NVIDIA, three currencies: EUR-USD, GBP-USD, USD-JPY and four cryptocurrencies: BTC--USD, ETH-USD, LTC-USD and XRP-USD. The dataset comprises daily data spanning over 16.5 years, i.e. from 3rd January 2006 to 31st May 2023 (4,382 observations) for stocks, from 3rd January 2006 to 31st May 2023 (4,512 observations) for currencies, from 3rd January 2015 to 31st May 2023 (3,073 observations) for BTC-USD, 3rd January 2016 to 31st May 2023 (2,708 observations) for LTC-USD and 3rd January 32018 to 31st May 2023 (1,977 observations) for ETH-USD and XRP--USD. These long periods consist of high-volatility events (like the financial crisis, the European sovereign debt crisis and COVID-19), but also low-volatility periods, where the latter is more prominent over time. Table 6 presents the descriptive statistics for the logarithmic returns calculated as $r_t = 100\ln(c_t/c_{t-1})$, where c_t is a closing price at time t. All return series appear to have heavy tails and they do not follow normal distribution. The time series show non-zero skewness and kurtosis greater than three. In the majority of the cases, stocks and cryptocurrencies time series are autocorrelated, whereas currencies do not seem to be autocorrelated. The three groups of time series share similarities, but also differences, such as higher volatility for cryptocurrencies and lower volatility for currencies compared with the stocks volatility. These three asset classes provide the opportunity to show the performance of the proposed model across somewhat different groups of time series.

Table 6. Summary statistics of the daily returns

Time series	Mean	Standard deviation	Minimum	Maximum	Skewness	Excess kurtosis	Ljung-Box
Amazon	0.0896	2.4206	23.8621	−24.6182	0.4308*	15.5456*	8.2049
Apple	0.0958	2.0509	13.0194	−19.7470	−0.2751*	9.0581*	21.5800*
Google	0.0553	1.8849	18.2251	−12.3685	0.2457*	11.2674*	18.4530*
Microsoft	0.0572	1.7748	17.0626	−15.9453	−0.0420	12.1607*	68.4090*
NVIDIA	0.1090	3.1091	26.0876	−36.7109	−0.3207*	12.3159*	7.4354
EURSUD	−0.0022	0.5741	3.41572	−2.94799	0.0635	5.4851*	2.1372
GBP/USD	−0.0072	0.6133	3.130041	−9.50501	−1.0561*	18.3437*	11.6850
USD/JPY	0.0037	0.6255	5.23658	−4.13554	−0.2202*	8.3409*	8.6468
BTC/USD	0.1452	3.8282	22.5119	−46.473	−0.7935*	14.1622*	9.1718
ETH/USD	0.0448	4.9436	23.06952	−55.0732	−1.0068*	13.4112*	20.6500*
LTC/USD	0.1201	5.4522	51.14174	−44.9062	0.2625*	14.3996*	22.3370*
XRP/USD	−0.0775	5.7847	44.47556	−55.0503	−0.0698	16.3753*	6.0308

Note. The sample period is 3rd January 2006 to 31st May 2023, * indicates that the null hypothesis is rejected at a 5-percent significance level, Ljung-Box – the Ljung-Box statistic for 5 lags.

Source: author's work based on the data from www.finance.yahoo.com site.

5.2. In sample evaluation based on empirical data

Firstly, we evaluate the proposed model, i.e. the GARCH-GPD-P against GARCH-GPD for the whole range of data. The estimation results of the GPD-P and the GPD are presented in Table 7. Parameter σ_1, responsible for the dynamics of extremes based on the Parkinson volatility estimates is highly significant and positive for all time series. This means that the dynamic behaviour of extreme values occurs and takes part in explaining the tail of the distribution. The σ_0 estimates in the GPD-P are considerably lower (in many cases, two to three times lower) than those obtained for the GPD. We compare the likelihood functions of the competing models and for all the considered time series, the likelihood ratio test indicates that GPD-P is significantly better fitted to the data (extreme observations) than the GPD.

Table 7. The results of the parameter estimates for the GPD and GPD-P for stock indices

Time series	GPD			GPD-P				LM p-value
	σ	γ	ln L	σ_0	σ_1	γ	ln L	
Amazon	0.5994* (0.0359)	0.0166 (0.0493)	−221.1060	0.2320* (0.0484)	0.0704* (0.0080)	−0.2350* (0.0303)	−151.3465	0.0000
Apple	0.6480* (0.0369)	−0.0583 (0.0391)	−222.4223	0.2484* (0.0373)	0.0838* (0.0111)	−0.2255* (0.0289)	−172.1086	0.0000
Google	0.5659* (0.0432)	0.1332* (0.0547)	−246.9619	0.1102* (0.0349)	0.1373* (0.0137)	−0.1838* (0.0368)	−166.4483	0.0000
Microsoft	0.6010* (0.0408)	0.0339 (0.0434)	−229.8312	0.1581* (0.0350)	0.1480* (0.0132)	−0.2454* (0.0312)	−145.2000	0.0000
NVIDIA	0.5789* (0.0402)	0.0186 (0.0444)	−206.6774	0.2151* (0.0399)	0.0361* (0.0045)	−0.1859* (0.0343)	−146.0543	0.0000

Table 7. The results of the parameter estimates for the GPD and GPD-P for stock indices (cont.)

Time series	GPD			GPD-P				LM p-value
	σ	γ	ln L	σ_0	σ_1	γ	ln L	
EURSUD	0.5222* (0.0308)	0.0313 (0.0463)	−172.1332	0.1897* (0.0337)	0.6258* (0.0736)	−0.1426* (0.0308)	−114.2083	0.0000
GBP/USD	0.5895* (0.0338)	0.0519 (0.0585)	−236.0500	0.2599* (0.0507)	0.7039* (0.0778)	−0.3022* (0.0292)	−157.9306	0.0000
USD/JPY	0.5611* (0.0353)	0.1114* (0.0435)	−240.6455	0.1887* (0.0447)	0.5820* (0.0740)	−0.1669* (0.0339)	−158.1730	0.0000
BTC/USD	0.6908* (0.0639)	0.1837* (0.0748)	−249.8602	0.2351* (0.0582)	0.0220* (0.0027)	−0.2401* (0.0435)	−181.7658	0.0000
ETH/USD	0.7550* (0.0774)	0.1130 (0.0814)	−164.7198	0.1974* (0.0679)	0.0171* (0.0015)	−0.4946* (0.0566)	−99.9482	0.0000
LTC/USD	0.6707* (0.0593)	0.1149 (0.0660)	−193.9026	0.0556 (0.0380)	0.0156* (0.0018)	−0.3292* (0.0346)	−121.0097	0.0000
XRP/USD	0.6322* (0.0730)	0.2089* (0.0808)	−148.5824	0.2201* (0.0661)	0.0125* (0.0023)	−0.2655* (0.0586)	−110.2027	0.0000

Note. Robust Huber-White standard errors are reported in parentheses, * indicates that the null hypothesis is rejected at a 5-percent significance level, ln L is the logarithm of the likelihood function, LM p-value is the p-value from the likelihood ratio test based on the logarithm of the likelihood function for GPD vs GPD-P.

Source: author's work.

5.3. Forecasting Value-at-Risk

In this subsection, we compare the proposed model (the GARCH-GPD-P) with the GARCH-GPD and two benchmarks, namely the GARCH-n and the GARCH-t, for VaR forecasting. We formulate out-of-sample one-day-ahead forecasts of the conditional VaR (5-percent and 10-percent coverage level) based on the GARCH-n, GARCH-t, GARCH-GPD, and GARCH-GPD-P models, where parameters are estimated each day on the basis of a rolling sample of two fixed sizes: 500 (approximately two years) and 1,000. Then, the first observation from the sample is dropped and one is added to the end of the sample (the rolling window approach) to obtain the VaR forecasts. This process is repeated iteratively until all the observations are exhausted, i.e. until 31st May 2023. Table A1 in the Appendix summarises the forecasting start and end dates as the number of forecasts used in the empirical study. We present the results only for the first group (500 observations used for the parameters estimation), as the results for the second group are similar and do not change the conclusions.

For backtesting purposes, we evaluate the VaR forecasts by testing their statistical properties, calculating loss measures and testing the superiority of VaR forecasts over the other ones. The statistical adequacy of VaR forecasts is verified by: the unconditional coverage LR_{UC} proposed by Kupiec (1995), independence LR_{ind} and conditional coverage LR_{CC} tests designed by Christoffersen (1998), unconditional coverage J_{uc}, independence J_{ind} and the conditional coverage J_{cc} tests devised by Candelon et al. (2011). Under Basel Accords (Basel Committee on Banking Supervision, 2011, 2019), financial institutions that report too many violations in the previous year, need to apply additional capital charges directly linked to the number of these violations. It means that the unconditional coverage property is of paramount importance from the regulators and financial institutions' point of view. In other words, rejecting the null hypothesis of the unconditional coverage test would result in too many violations and additional capital charges. A model leading to such an outcome is by far undesirable for the market participants, regulators and financial institutions.

Firstly, Table A2, Table A3 and Table A4 (Appendix) present the results of the statistical properties of VaR for 10% and Table A5, Table A6 and Table A7 (Appendix) for 5%. Generally speaking, VaR forecasts from the GARCH-GPD-P, GARCH-GPD and GARCH-t models have better statistical properties than the ones obtained from the GARCH-n. Only VaR forecasts from the GARCH-GPD-P model meet both criteria, i.e. the unconditional coverage and independence properties at a 5-percent significance level for both coverage levels. In many cases, VaR forecasts from the GARCH-n model have a significantly different number of violations and are not independent across time.

Secondly, we evaluate methods for VaR forecasting based on the same set of loss functions that are used in the simulation. Moreover, we calculate penalisation measure PM and its components, i.e. the penalisation measure for violation space $PM(VS)$ and safe space $PM(SS)$ proposed by Şener et al. (2012). The results for VaR(10%) are shown in Tables 8–10 and for VaR(5%) in Tables 11–13. At a 10-percent coverage level, in many cases (mainly stocks and currencies), the GARCH-GPD-P model generates VaR forecasts that lead to the smallest loss functions from the regulator's perspective (RLF measures). The second most accurate model in terms of the regulator's loss functions is the GARCH-n, especially for cryptocurrencies. For the firm's loss functions (FLFs) it is difficult to indicate a single best model, but the GARCH-t model seems to be the most prominent. The lowest values of penalisation measure *PM* are obtained for the proposed GARCH-GPD-P model (in the case of stocks and currencies) and for the GARCH-n model (in the case of cryptocurrencies). It is not surprising that for such a low coverage level as 10%, the standard GARCH model can produce more accurate VaR forecasts, as EVT-based methods are believed to be better at describing extreme quantiles such as 5%, 1%, 0.5% or even higher.

For a 5-percent coverage level, the situation is different, as the GARCH-GPD-P generates the most accurate VaR forecasts based on many of the regulator's loss functions for all three asset classes. When it comes to the FLFs, VaR forecasts from the GARCH-t model have the lowest values in most cases. For all the selected time series, penalisation measure *PM* is also the smallest for the GARCH-GPD-P model. The second most accurate model for the *PM* is either the GARCH-GPD or the GARCH-n model. It seems the proposed model tends to overestimate the VaR because for most FLFs, other models produce more accurate results.

At a 5-percent coverage level, the results show that the GARCH-GPD-P is generally better than the competing models. The probable reason is that the use of high and low prices in the form of the Parkinson estimator for extreme observations generates a quick reaction to what is happening in the markets. If there is a jump in volatility, it will have an immediate reaction on the time-varying scale parameter in the GPD, thus producing higher VaR estimates. In turbulent times, this mechanism is going to provide more accurate VaR estimates and result in a smaller number of violations (as reported in the unconditional coverage tests). On the other hand, in periods of low volatility, it could lead to the VaR overestimation.

Table 8. The results of the loss measures for VaR(10%): stocks

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	RLF(L)$\cdot 10^{-3}$	1.1673	1.2924	1.3647	**1.1618**
	RLF(STS) $\cdot 10^{-3}$	0.8163	0.9174	0.9657	**0.7798**
	RLF(C1) $\cdot 10^{-3}$	**0.1795**	0.2090	0.2187	0.1973
	RLF(C2) $\cdot 10^{-3}$	**0.3658**	0.4299	0.4515	0.3891
	RLF(C3) $\cdot 10^{-3}$	**0.3730**	0.4092	0.4296	0.3757
	FLF(STS) $\cdot 10^{-3}$	1.2030	1.2905	1.3210	**1.1609**
	FLF(C1) $\cdot 10^{-3}$	2.1919	2.1958	2.1856	**2.1835**
	FLF(C2) $\cdot 10^{-3}$	3.5947	3.5611	**3.4529**	3.6330
	FLF(C3) $\cdot 10^{-3}$	9.3595	9.1635	**8.8889**	9.2737
	PM	0.2811	0.3544	0.3809	**0.2226**
	$PM_{VS} \cdot 10^{-3}$	0.4681	0.6243	0.6851	**0.3496**
	$PM_{SS}\cdot 10^{-3}$	2.0649	1.9534	**1.8303**	2.0350
Apple	RLF(L) $\cdot 10^{-3}$	1.0551	1.1739	1.0762	**0.9370**
	RLF(STS) $\cdot 10^{-3}$	0.6661	0.7519	0.6822	**0.5700**
	RLF(C1) $\cdot 10^{-3}$	0.2029	0.2406	0.2101	**0.1886**
	RLF(C2) $\cdot 10^{-3}$	0.3341	0.3974	0.3468	**0.2999**
	RLF(C3) $\cdot 10^{-3}$	0.3535	0.3956	0.3602	**0.3186**
	FLF(STS) $\cdot 10^{-3}$	0.9882	1.0542	1.0026	**0.9133**
	FLF(C1) $\cdot 10^{-3}$	2.1956	**2.2009**	2.2012	2.2084
	FLF(C2) $\cdot 10^{-3}$	2.9798	**2.8910**	2.9837	3.1468
	FLF(C3) $\cdot 10^{-3}$	7.9454	**7.6484**	7.9225	8.2488
	PM	0.2115	0.2631	0.2235	**0.1598**
	$PM_{VS} \cdot 10^{-3}$	0.3289	0.4370	0.3529	**0.2200**
	$PM_{SS}\cdot 10^{-3}$	1.5980	**1.4580**	1.5882	1.7500
Google	RLF(L) $\cdot 10^{-3}$	0.9718	1.0987	1.0865	**0.9658**
	RLF(STS) $\cdot 10^{-3}$	0.6158	0.6957	0.6865	**0.5968**
	RLF(C1) $\cdot 10^{-3}$	**0.1955**	0.2421	0.2322	0.2104
	RLF(C2) $\cdot 10^{-3}$	**0.3624**	0.4444	0.4325	0.3802
	RLF(C3) $\cdot 10^{-3}$	**0.3176**	0.3621	0.3516	**0.3176**
	FLF(STS) $\cdot 10^{-3}$	0.9213	0.9759	0.9727	**0.9118**
	FLF(C1) $\cdot 10^{-3}$	2.2319	**2.2209**	2.2231	2.2441
	FLF(C2) $\cdot 10^{-3}$	2.8904	**2.7375**	2.7851	3.0595
	FLF(C3) $\cdot 10^{-3}$	7.3364	**6.9504**	7.0520	7.5121
	PM	0.1650	0.2005	0.1931	**0.1581**
	$PM_{VS} \cdot 10^{-3}$	0.2464	0.3286	0.3110	**0.2272**
	$PM_{SS}\cdot 10^{-3}$	1.6669	**1.4705**	1.5216	1.7676

Table 8. The results of the loss measures for VaR(10%): stocks (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Microsoft	RLF(L) · 10^{-3}	0.8631	1.0015	0.9621	**0.8597**
	RLF(STS) · 10^{-3}	0.5061	0.5885	0.5591	**0.4797**
	RLF(C1) · 10^{-3}	**0.1958**	0.2477	0.2333	0.2128
	RLF(C2) · 10^{-3}	**0.3208**	0.4028	0.3785	0.3345
	RLF(C3) · 10^{-3}	0.2977	0.3435	0.3299	**0.2951**
	FLF(STS) · 10^{-3}	0.7930	0.8514	0.8283	**0.7691**
	FLF(C1) · 10^{-3}	2.2076	2.2072	**2.2033**	2.2148
	FLF(C2) · 10^{-3}	2.7333	**2.6242**	2.6446	2.8044
	FLF(C3) · 10^{-3}	7.0240	**6.6725**	6.7629	7.0730
	PM	0.1489	0.1790	0.1682	**0.1344**
	PM_{VS} · 10^{-3}	0.2148	0.2824	0.2592	**0.1845**
	PM_{SS}· 10^{-3}	1.4652	**1.2994**	1.3404	1.4999
NVIDIA	RLF(L) · 10^{-3}	1.9801	2.2310	2.1285	**1.8861**
	RLF(STS) · 10^{-3}	1.6191	1.8300	1.7335	**1.5191**
	RLF(C1) · 10^{-3}	**0.1858**	0.2202	0.2136	0.1941
	RLF(C2) · 10^{-3}	0.5282	0.6300	0.5992	**0.5263**
	RLF(C3) · 10^{-3}	0.5082	0.5726	0.5511	**0.5006**
	FLF(STS) · 10^{-3}	2.1426	2.3193	2.2342	**2.0558**
	FLF(C1) · 10^{-3}	2.2109	**2.2057**	2.2092	2.2115
	FLF(C2) · 10^{-3}	4.8917	**4.7013**	4.7701	5.0555
	FLF(C3) · 10^{-3}	12.6114	**12.1108**	12.2747	12.8274
	PM	0.4731	0.6044	0.5293	**0.3832**
	PM_{VS} · 10^{-3}	0.8069	1.0835	0.9285	**0.6206**
	PM_{SS}· 10^{-3}	2.7036	**2.4547**	2.5349	2.8170

Note. The lowest values of loss functions are marked in bold. RLF(L) is the loss function proposed by Lopez (1998), RLF(STS), FLF(STS) are the loss functions proposed by Sarma et al. (2003), RLF(C1), RLF(C2), RLF(C3), FLF(C1), FLF(C2) and FLF(C3) are the loss functions proposed by Caporin (2008), PM, PM_{VS} PM_{SS} are the penalisation measure, the penalisation measure for the violation space and the penalisation measure for the safe space, respectively, proposed by Şener et al. (2012).

Source: author's work.

Table 9. The results of the loss measures for VaR(10%): currencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
EUR/USD	RLF(L)· 10^{-3}	0.4436	0.4858	0.4693	**0.4365**
	RLF(STS) · 10^{-3}	0.0686	0.0728	0.0703	**0.0625**
	RLF(C1) · 10^{-3}	0.1699	0.1874	0.1814	**0.1640**
	RLF(C2) · 10^{-3}	0.1005	0.1089	0.1055	**0.0939**
	RLF(C3) · 10^{-3}	0.1105	0.1188	0.1150	**0.1038**
	FLF(STS) · 10^{-3}	0.2030	0.2020	**0.2014**	0.2017
	FLF(C1) · 10^{-3}	2.2485	**2.2368**	2.2382	2.2568
	FLF(C2) · 10^{-3}	1.2112	**1.1740**	1.1836	1.2616
	FLF(C3) · 10^{-3}	3.1510	**3.0790**	3.0995	3.2246
	PM	0.0304	0.0313	0.0304	**0.0283**
	PM_{VS} · 10^{-3}	0.0271	0.0308	0.0284	**0.0207**
	PM_{SS}· 10^{-3}	0.6984	**0.6619**	0.6714	0.7321

Table 9. The results of the loss measures for VaR(10%): currencies (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
GBP/USD	RLF(L) · 10^{-3}	**0.5191**	0.5710	0.5731	0.5307
	RLF(STS) · 10^{-3}	**0.1651**	0.1800	0.1721	0.1567
	RLF(C1) · 10^{-3}	**0.1867**	0.2268	0.2142	0.1948
	RLF(C2) · 10^{-3}	**0.2045**	0.2516	0.2266	0.2073
	RLF(C3) · 10^{-3}	0.1334	0.1480	0.1446	**0.1310**
	FLF(STS) · 10^{-3}	0.3045	0.3104	**0.3031**	0.3080
	FLF(C1) · 10^{-3}	2.2694	2.2597	**2.2530**	2.2617
	FLF(C2) · 10^{-3}	1.3585	1.3087	**1.3058**	1.5933
	FLF(C3) · 10^{-3}	3.2785	3.1399	**3.1500**	3.5113
	PM	0.0390	0.0456	0.0407	**0.0367**
	PM_{VS} · 10^{-3}	0.0433	0.0614	0.0504	**0.0318**
	PM_{SS}· 10^{-3}	0.7517	**0.6763**	0.6862	0.8779
USD/JPY	RLF(L) · 10^{-3}	**0.4808**	0.5446	0.5400	0.4915
	RLF(STS) · 10^{-3}	0.1268	0.1496	0.1410	**0.1225**
	RLF(C1) · 10^{-3}	**0.1774**	0.2454	0.2216	0.1991
	RLF(C2) · 10^{-3}	**0.1550**	0.2339	0.1880	0.1637
	RLF(C3) · 10^{-3}	**0.1305**	0.1541	0.1496	0.1330
	FLF(STS) · 10^{-3}	**0.2651**	0.2762	0.2682	0.2692
	FLF(C1) · 10^{-3}	2.3518	2.3696	**2.3459**	2.3673
	FLF(C2) · 10^{-3}	1.4067	1.3781	**1.3441**	1.5687
	FLF(C3) · 10^{-3}	3.3052	**3.1276**	3.1338	3.4714
	PM	0.0369	0.0448	0.0401	**0.0368**
	PM_{VS} · 10^{-3}	0.0376	0.0587	0.0487	**0.0344**
	PM_{SS}· 10^{-3}	0.7497	**0.6608**	0.6657	0.8057

Note. As in Table 8.
Source: author's work.

Table 10. The results of the loss measures for VaR(10%): cryptocurrencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
BTC/USD	RLF(L)· 10^{-3}	**4.9515**	6.3602	6.3294	5.9225
	RLF(STS) · 10^{-3}	**4.7695**	6.0912	6.0794	5.6985
	RLF(C1) · 10^{-3}	**0.1526**	0.2963	0.2445	0.2315
	RLF(C2) · 10^{-3}	**1.2292**	2.0489	1.8486	1.6898
	RLF(C3) · 10^{-3}	**0.5781**	0.7856	0.7752	0.7212
	FLF(STS) · 10^{-3}	**5.3468**	6.5286	6.5325	6.2507
	FLF(C1) · 10^{-3}	**1.7127**	1.8737	1.7573	1.7743
	FLF(C2) · 10^{-3}	7.0829	6.9557	**6.7321**	6.5502
	FLF(C3) · 10^{-3}	13.9326	**11.6914**	11.9423	11.7111
	PM	**1.4267**	3.5851	3.8839	2.6811
	PM_{VS} · 10^{-3}	**1.6313**	4.4137	4.7848	3.2228
	PM_{SS}· 10^{-3}	3.2737	**2.2540**	2.3797	3.1668

Table 10. The results of the loss measures for VaR(10%): cryptocurrencies (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
ETH/USD	RLF(L) $\cdot 10^{-3}$	5.2748	6.3902	6.3386	**5.2233**
	RLF(STS) $\cdot 10^{-3}$	5.1718	6.2482	6.1946	**5.1053**
	RLF(C1) $\cdot 10^{-3}$	**0.0701**	0.1124	0.1105	0.0851
	RLF(C2) $\cdot 10^{-3}$	**0.8087**	1.1761	1.2140	0.8351
	RLF(C3) $\cdot 10^{-3}$	**0.4167**	0.5482	0.5357	0.4409
	FLF(STS) $\cdot 10^{-3}$	**5.5904**	6.5766	6.5308	5.5920
	FLF(C1) $\cdot 10^{-3}$	0.9177	**0.9107**	0.9187	0.9516
	FLF(C2) $\cdot 10^{-3}$	4.6726	**4.1980**	4.3716	4.3941
	FLF(C3) $\cdot 10^{-3}$	10.0032	**8.5455**	8.6827	9.4151
	PM	**1.8349**	4.9249	4.5914	1.9632
	$PM_{VS} \cdot 10^{-3}$	**1.2472**	3.5957	3.3420	1.3051
	$PM_{SS} \cdot 10^{-3}$	2.3948	**1.7145**	1.7918	2.4200
LTC/USD	RLF(L) $\cdot 10^{-3}$	**7.4716**	9.3778	10.3435	7.6661
	RLF(STS) $\cdot 10^{-3}$	**7.3226**	9.1648	10.1135	7.5031
	RLF(C1) $\cdot 10^{-3}$	**0.0947**	0.1589	0.1760	0.1107
	RLF(C2) $\cdot 10^{-3}$	**1.0165**	1.5178	1.8399	1.1282
	RLF(C3) $\cdot 10^{-3}$	**0.6411**	0.8814	0.9372	0.6621
	FLF(STS) $\cdot 10^{-3}$	**8.0707**	9.7507	10.6799	8.4103
	FLF(C1) $\cdot 10^{-3}$	1.4017	**1.3917**	1.4153	1.3962
	FLF(C2) $\cdot 10^{-3}$	8.3638	**7.4050**	7.8857	7.7415
	FLF(C3) $\cdot 10^{-3}$	17.3272	14.7533	**14.5387**	14.5567
	PM	**1.6890**	3.8954	4.9088	1.8033
	$PM_{VS} \cdot 10^{-3}$	**1.7267**	4.3624	5.5461	1.7733
	$PM_{SS} \cdot 10^{-3}$	4.5184	3.2019	**3.1100**	5.1608
XRP/USD	RLF(L) $\cdot 10^{-3}$	**5.7677**	7.8465	7.4876	6.1725
	RLF(STS) $\cdot 10^{-3}$	**5.6807**	7.7065	7.3516	6.0565
	RLF(C1) $\cdot 10^{-3}$	**0.0566**	0.1129	0.1093	0.0865
	RLF(C2) $\cdot 10^{-3}$	**0.7561**	1.3285	1.2855	0.9463
	RLF(C3) $\cdot 10^{-3}$	**0.3675**	0.5594	0.5405	0.4531
	FLF(STS) $\cdot 10^{-3}$	**6.2024**	8.0598	7.7326	6.6928
	FLF(C1) $\cdot 10^{-3}$	0.9725	**0.9530**	0.9623	0.9934
	FLF(C2) $\cdot 10^{-3}$	6.4486	**5.4519**	5.6352	5.6517
	FLF(C3) $\cdot 10^{-3}$	11.7213	**8.8927**	9.3912	10.2841
	PM	**1.8440**	9.2295	8.6622	5.2533
	$PM_{VS} \cdot 10^{-3}$	**1.2419**	6.9741	6.5271	3.7641
	$PM_{SS} \cdot 10^{-3}$	3.2520	**1.8737**	2.1079	3.9691

Note. As in Table 8.

Source: author's work.

Table 11. The results of the loss measures for VaR(5%): stocks

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	RLF(L) · 10^{-3}	0.6325	0.6477	0.6669	**0.5874**
	RLF(STS) · 10^{-3}	0.4245	0.4377	0.4569	**0.3834**
	RLF(C1) · 10^{-3}	**0.0772**	0.0820	0.0801	0.0865
	RLF(C2) · 10^{-3}	**0.1573**	0.1647	0.1638	0.1588
	RLF(C3) · 10^{-3}	0.2003	0.2086	0.2118	**0.1939**
	FLF(STS) · 10^{-3}	**0.9444**	0.9628	0.9686	0.9768
	FLF(C1) · 10^{-3}	2.3212	2.3297	**2.3190**	2.3463
	FLF(C2) · 10^{-3}	4.8079	4.9064	**4.7376**	6.0414
	FLF(C3) · 10^{-3}	11.4991	11.6156	**11.3582**	12.8568
	PM	0.1342	0.1446	0.1493	**0.1323**
	PM_{VS} · 10^{-3}	0.1100	0.1292	0.1448	**0.0681**
	PM_{SS}· 10^{-3}	3.1390	3.1812	**3.0669**	3.2644
Apple	RLF(L) · 10^{-3}	0.5681	0.5972	0.5112	**0.4457**
	RLF(STS) · 10^{-3}	0.3471	0.3632	0.3062	**0.2487**
	RLF(C1) · 10^{-3}	0.0882	0.0941	**0.0750**	0.0767
	RLF(C2) · 10^{-3}	0.1376	0.1463	0.1161	**0.1135**
	RLF(C3) · 10^{-3}	0.1932	0.2034	0.1708	**0.1518**
	FLF(STS) · 10^{-3}	0.7835	0.7898	**0.7666**	0.7736
	FLF(C1) · 10^{-3}	2.3089	**2.2974**	2.3450	2.3933
	FLF(C2) · 10^{-3}	3.9964	**3.9017**	4.2822	5.3016
	FLF(C3) · 10^{-3}	9.7129	**9.5570**	10.1199	11.3066
	PM	0.1131	0.1171	0.1084	**0.1050**
	PM_{VS} · 10^{-3}	0.0924	0.1043	0.0730	**0.0428**
	PM_{SS}· 10^{-3}	2.4387	**2.3648**	2.6329	2.6291
Google	RLF(L) · 10^{-3}	0.5593	0.5958	0.5433	**0.4846**
	RLF(STS) · 10^{-3}	0.3583	0.3758	0.3473	**0.2926**
	RLF(C1) · 10^{-3}	0.0926	0.1015	**0.0882**	0.0931
	RLF(C2) · 10^{-3}	0.1725	0.1888	0.1650	**0.1646**
	RLF(C3) · 10^{-3}	0.1877	0.1981	0.1816	**0.1570**
	FLF(STS) · 10^{-3}	0.7687	0.7725	**0.7654**	0.8018
	FLF(C1) · 10^{-3}	2.3650	**2.3413**	2.3773	2.4475
	FLF(C2) · 10^{-3}	3.8830	**3.7322**	3.9813	5.5264
	FLF(C3) · 10^{-3}	8.9989	**8.7753**	9.1343	10.8414
	PM	0.1034	0.1043	0.1032	**0.1009**
	PM_{VS} · 10^{-3}	0.0779	0.0860	0.0743	**0.0416**
	PM_{SS}· 10^{-3}	2.4986	**2.3821**	2.5583	2.5763
Microsoft	RLF(L) · 10^{-3}	0.4737	0.5225	0.4683	**0.4385**
	RLF(STS) · 10^{-3}	0.2767	0.2975	0.2753	**0.2385**
	RLF(C1) · 10^{-3}	0.0877	0.0990	**0.0868**	0.0917
	RLF(C2) · 10^{-3}	0.1443	0.1589	0.1434	**0.1392**
	RLF(C3) · 10^{-3}	0.1662	0.1805	0.1617	**0.1513**
	FLF(STS) · 10^{-3}	**0.6650**	0.6741	0.6666	0.6929
	FLF(C1) · 10^{-3}	2.3389	**2.3233**	2.3427	2.3870
	FLF(C2) · 10^{-3}	3.6320	**3.5223**	3.6758	4.7211
	FLF(C3) · 10^{-3}	8.6167	**8.4472**	8.6615	9.8578
	PM	0.0933	0.0966	0.0929	**0.0894**
	PM_{VS} · 10^{-3}	0.0654	0.0760	0.0637	**0.0394**
	PM_{SS}· 10^{-3}	2.2326	**2.1549**	2.2510	2.1770

Table 11. The results of the loss measures for VaR(5%): stocks (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
NVIDIA	RLF(L) $\cdot 10^{-3}$	1.1063	1.1609	1.0660	**0.9396**
	RLF(STS) $\cdot 10^{-3}$	0.9133	0.9609	0.8700	**0.7356**
	RLF(C1) $\cdot 10^{-3}$	0.0822	0.0863	**0.0796**	0.0834
	RLF(C2) $\cdot 10^{-3}$	0.2360	0.2495	0.2248	**0.2139**
	RLF(C3) $\cdot 10^{-3}$	0.2849	0.2983	0.2749	**0.2517**
	FLF(STS) $\cdot 10^{-3}$	1.6190	1.6527	1.5851	**1.5472**
	FLF(C1) $\cdot 10^{-3}$	2.3288	**2.3164**	2.3338	2.3716
	FLF(C2) $\cdot 10^{-3}$	6.5116	**6.3493**	6.6322	8.2795
	FLF(C3) $\cdot 10^{-3}$	15.4668	**15.2363**	15.6374	17.4874
	PM	0.2165	0.2293	0.2033	**0.1877**
	$PM_{VS} \cdot 10^{-3}$	0.2181	0.2499	0.1877	**0.1259**
	$PM_{SS} \cdot 10^{-3}$	4.1113	**3.9930**	4.1836	4.1431

Note. As in Table 8.
Source: author's work.

Table 12. The results of the loss measures for VaR(5%): currencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
EUR/USD	RLF(L) $\cdot 10^{-3}$	0.2222	0.2214	0.2236	**0.2160**
	RLF(STS) $\cdot 10^{-3}$	0.0352	0.0354	0.0346	**0.0300**
	RLF(C1) $\cdot 10^{-3}$	0.0707	0.0721	0.0712	**0.0703**
	RLF(C2) $\cdot 10^{-3}$	0.0429	0.0429	0.0428	**0.0410**
	RLF(C3) $\cdot 10^{-3}$	0.0560	0.0569	0.0555	**0.0498**
	FLF(STS) $\cdot 10^{-3}$	0.2169	0.2158	**0.2157**	0.2354
	FLF(C1) $\cdot 10^{-3}$	2.4041	2.3947	**2.3936**	2.4555
	FLF(C2) $\cdot 10^{-3}$	1.6682	**1.6502**	1.6591	1.8337
	FLF(C3) $\cdot 10^{-3}$	3.8794	**3.8549**	3.8669	3.9032
	PM	0.0299	0.0297	0.0295	**0.0280**
	$PM_{VS} \cdot 10^{-3}$	0.0069	0.0071	0.0064	**0.0041**
	$PM_{SS} \cdot 10^{-3}$	1.0597	**1.0470**	1.0522	1.0436
GBP/USD	RLF(L) $\cdot 10^{-3}$	0.3286	0.3450	0.3162	**0.3098**
	RLF(STS) $\cdot 10^{-3}$	0.1176	0.1260	0.1152	**0.1038**
	RLF(C1) $\cdot 10^{-3}$	0.0839	0.0983	0.0804	**0.0792**
	RLF(C2) $\cdot 10^{-3}$	0.1152	0.1370	0.1121	**0.1097**
	RLF(C3) $\cdot 10^{-3}$	0.0762	0.0821	0.0728	**0.0634**
	FLF(STS) $\cdot 10^{-3}$	0.3030	0.3061	**0.3024**	0.3646
	FLF(C1) $\cdot 10^{-3}$	2.4199	**2.4085**	2.4235	2.4602
	FLF(C2) $\cdot 10^{-3}$	1.8040	**1.7435**	1.8210	1.9052
	FLF(C3) $\cdot 10^{-3}$	4.0239	**3.9294**	4.0424	4.0484
	PM	0.0343	0.0351	0.0342	**0.0328**
	$PM_{VS} \cdot 10^{-3}$	0.0137	0.0183	0.0130	**0.0060**
	$PM_{SS} \cdot 10^{-3}$	1.1262	**1.0713**	1.1336	1.1398

Table 12. The results of the loss measures for VaR(5%): currencies (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
USD/JPY	RLF(L) $\cdot 10^{-3}$	**0.2648**	0.2975	0.2818	0.2672
	RLF(STS) $\cdot 10^{-3}$	0.0788	0.0875	0.0808	**0.0662**
	RLF(C1) $\cdot 10^{-3}$	**0.0786**	0.0974	0.0860	0.0856
	RLF(C2) $\cdot 10^{-3}$	0.0755	0.1035	0.0801	**0.0719**
	RLF(C3) $\cdot 10^{-3}$	0.0743	0.0804	0.0781	**0.0699**
	FLF(STS) $\cdot 10^{-3}$	0.2646	0.2682	**0.2623**	0.2677
	FLF(C1) $\cdot 10^{-3}$	2.4877	2.4875	**2.4722**	2.4275
	FLF(C2) $\cdot 10^{-3}$	1.8474	1.8174	**1.8035**	1.9426
	FLF(C3) $\cdot 10^{-3}$	4.0593	**3.9858**	3.9910	4.0351
	PM	0.0338	0.0355	0.0334	**0.0308**
	$PM_{VS} \cdot 10^{-3}$	0.0116	0.0170	0.0124	**0.0085**
	$PM_{SS} \cdot 10^{-3}$	1.1205	**1.0833**	1.0872	1.1430

Note. As in Table 8.
Source: author's work.

Table 13. The results of the loss measures for VaR(5%): cryptocurrencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
BTC/USD	RLF(L) $\cdot 10^{-3}$	3.5872	4.1160	3.8117	**3.4967**
	RLF(STS) $\cdot 10^{-3}$	3.4682	3.9650	3.6807	**3.3617**
	RLF(C1) $\cdot 10^{-3}$	0.0849	0.1307	0.0952	**0.0711**
	RLF(C2) $\cdot 10^{-3}$	0.7562	0.9925	0.8195	**0.7209**
	RLF(C3) $\cdot 10^{-3}$	0.3717	0.4469	0.4038	**0.3143**
	FLF(STS) $\cdot 10^{-3}$	**4.2347**	4.6415	4.4130	4.6220
	FLF(C1) $\cdot 10^{-3}$	1.7660	1.8090	**1.7594**	1.8124
	FLF(C2) $\cdot 10^{-3}$	8.9334	**8.1458**	8.5988	8.9265
	FLF(C3) $\cdot 10^{-3}$	17.1626	**15.5972**	16.5849	17.0877
	PM	0.5468	0.8304	0.6360	**0.5281**
	$PM_{VS} \cdot 10^{-3}$	0.4400	0.8360	0.5659	**0.4015**
	$PM_{SS} \cdot 10^{-3}$	4.7747	**4.0613**	4.5249	4.4202
ETH/USD	RLF(L) $\cdot 10^{-3}$	4.0074	4.4719	4.1703	**3.1127**
	RLF(STS) $\cdot 10^{-3}$	3.9374	4.3889	4.0963	**3.0517**
	RLF(C1) $\cdot 10^{-3}$	**0.0353**	0.0470	0.0399	0.0354
	RLF(C2) $\cdot 10^{-3}$	0.4727	0.5584	0.4973	**0.3214**
	RLF(C3) $\cdot 10^{-3}$	0.2754	0.3308	0.2982	**0.2444**
	FLF(STS) $\cdot 10^{-3}$	4.4912	4.8781	4.6284	**4.0703**
	FLF(C1) $\cdot 10^{-3}$	0.9677	**0.9407**	0.9620	1.0239
	FLF(C2) $\cdot 10^{-3}$	6.0753	**5.2895**	5.8584	5.8557
	FLF(C3) $\cdot 10^{-3}$	12.3425	**11.1961**	11.9597	11.4522
	PM	0.8659	1.1373	0.9703	**0.8122**
	$PM_{VS} \cdot 10^{-3}$	0.4635	0.6946	0.5494	**0.2641**
	$PM_{SS} \cdot 10^{-3}$	3.5066	**2.9737**	3.3584	3.9539

Table 13. The results of the loss measures for VaR(5%): cryptocurrencies (cont.)

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
LTC/USD	RLF(L) · 10^{-3}	5.3013	5.8459	6.4325	**3.9071**
	RLF(STS) · 10^{-3}	5.2053	5.7269	6.3185	**3.8301**
	RLF(C1) · 10^{-3}	0.0468	0.0613	0.0639	**0.0423**
	RLF(C2) · 10^{-3}	0.5601	0.6532	0.7493	**0.3916**
	RLF(C3) · 10^{-3}	0.4089	0.4843	0.5103	**0.3270**
	FLF(STS) · 10^{-3}	6.1904	6.6076	7.2043	**5.7100**
	FLF(C1) · 10^{-3}	1.4763	**1.4332**	1.4513	1.6000
	FLF(C2) · 10^{-3}	10.8787	**9.5686**	9.8804	10.2211
	FLF(C3) · 10^{-3}	21.4234	**19.5871**	19.7093	18.9232
	PM	0.7684	0.9737	1.0589	**0.7526**
	$PM_{VS} \cdot 10^{-3}$	0.5474	0.8363	0.9315	**0.2353**
	$PM_{SS} \cdot 10^{-3}$	6.5813	**5.6306**	5.7035	5.4774
XRP/USD	RLF(L) · 10^{-3}	4.4567	5.5485	5.1969	**3.0706**
	RLF(STS) · 10^{-3}	4.4077	5.4825	5.1319	**3.0116**
	RLF(C1) · 10^{-3}	**0.0309**	0.0462	0.0422	0.0356
	RLF(C2) · 10^{-3}	0.4579	0.6333	0.5935	**0.3698**
	RLF(C3) · 10^{-3}	0.2601	0.3353	0.3136	**0.2392**
	FLF(STS) · 10^{-3}	5.0882	6.0244	5.7262	**4.6218**
	FLF(C1) · 10^{-3}	1.0286	**0.9887**	1.0041	1.0570
	FLF(C2) · 10^{-3}	8.2378	**6.4892**	7.1847	6.7012
	FLF(C3) · 10^{-3}	14.5282	**11.9674**	12.9762	13.1000
	PM	0.9431	1.5521	1.3568	**0.8611**
	$PM_{VS} \cdot 10^{-3}$	0.4785	1.0102	0.8356	**0.2856**
	$PM_{SS} \cdot 10^{-3}$	4.6401	**3.4052**	3.8779	4.8478

Note. As in Table 8.
Source: author's work.

Tables 14 and 15 provide a summary of the models with the lowest loss measure for all time series used in the empirical analysis, for VaR at a 10-percent and a 5-percent coverage level, respectively. At a 10-percent probability level, the GARCH-GPD-P and GARCH-n models resulted in 49 cases (out of 144) with the lowest values of the loss measures. At a 5-percent probability level, the GARCH-GPD-P model resulted in 78 cases with the lowest values of the loss measures.

Thirdly, we apply a predictive ability test for penalisation measure $PM(\varphi, VaR)$ proposed by Şener et al. (2012) to verify the obtained results statistically. Rejecting the null hypothesis means that a given model is less effective in terms of VaR forecasting measured by the penalisation measure than any other competing model. Tables 16 and 17 present the results of the predictive ability test for VaR(10-percent) and VaR(5%). At a 10% coverage level, we do not reject the null hypothesis for the GARCH-GPD-P and the GARCH-n models (in almost all cases). This means that the differences in VaR forecasts from GARCH-n and GARCH-GPD-P across all four competing models are statistically significant. At a 5-percent coverage level, we can see that the GARCH-GPD-P and GARCH-t models are significantly more accurate than other models. These results are in line with the outcome obtained for the loss functions.

Table 14. The results of the loss measures for VaR(10%): a model with the lowest loss measure

	Amazon	Apple	Google	Microsoft	NVIDIA	EURSUD	GBPUSD	USDJPY	BTCUSD	ETHUSD	LTCUSD	XRPUSD
RLF(L)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n
RLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n
RLF(C1)	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
RLF(C2)	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
RLF(C3)	GARCH-n	GARCH-GPD-P	GARCH-n/GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
FLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
FLF(C1)	GARCH-GPD-P	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-t	GARCH-t	GARCH-t
FLF(C2)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t
FLF(C3)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t
PM	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n
PM_{VS}	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n
PM_{SS}	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t

Note. As in Table 8.

Source: author's work.

Table 15. The results of the loss measures for VaR(5%): a model with the lowest loss measure

	Amazon	Apple	Google	Microsoft	NVIDIA	EURSUD	GBPUSD	USDJPY	BTCUSD	ETHUSD	LTCUSD	XRPUSD
RLF(L)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(C1)	GARCH-n	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n
RLF(C2)	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(C3)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
FLF(STS)	GARCH-n	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-GPD-P	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
FLF(C1)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t
FLF(C2)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t
FLF(C3)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t
PM	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
PM_{VS}	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
PM_{SS}	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t

Source: author's work.

Table 16. The *p*-values of the predictive ability test (Şener et al., 2012) for VaR(10%) based on the penalisation measure

Assets	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	0.6180	0.0000*	0.0000*	1.0000
Apple	0.0335*	0.0000*	0.0000*	1.0000
Google	1.0000	0.0000*	0.0000*	1.0000
Microsoft	1.0000	0.0000*	0.0000*	1.0000
NVIDIA	0.8460	0.0000*	0.0000*	1.0000
EUR/USD	0.0000*	0.0000*	0.0000*	1.0000
GBP/USD	1.0000	0.0000*	0.0000*	0.5023
USD/JPY	1.0000	0.0000*	0.0000*	0.6388
BTC/USD	1.0000	0.0000*	0.0000*	0.0000*
ETH/USD	1.0000	0.0000*	0.0000*	1.0000
LTC/USD	1.0000	0.0000*	0.0000*	1.0000
XRP/USD	1.0000	0.0000*	0.0000*	0.0000*

Note. * indicates that the null hypothesis is rejected at a 5-percent significance level.
Source: author's work.

Table 17. The *p*-values of the predictive ability test (Şener et al., 2012) for VaR(5%) based on the penalisation measure

Assets	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	0.0000*	0.0000*	0.0000*	1.0000
Apple	0.0000*	0.3537	0.0000*	1.0000
Google	0.0000*	1.0000	0.0000*	0.8814
Microsoft	0.0000*	1.0000	0.0000*	1.0000
NVIDIA	0.0000*	1.0000	0.0001*	1.0000
EUR/USD	0.0000*	1.0000	0.0000*	1.0000
GBP/USD	0.0000*	1.0000	0.0001*	1.0000
USD/JPY	0.2369	0.0000*	0.0000*	1.0000
BTC/USD	0.0000*	0.0000*	0.0000*	1.0000
ETH/USD	0.0000*	0.0148*	0.0000*	1.0000
LTC/USD	0.0000*	0.1938	0.0000*	1.0000
XRP/USD	0.0000*	0.0000*	0.0000*	1.0000

Note. * indicates that the null hypothesis is rejected at a 5-percent significance level.
Source: author's work.

5.4. Forecasting an expected shortfall

In this subsection, we compare the proposed model (the GARCH-GPD-P) against the GARCH-GPD and two benchmarks, the GARCH-n and the GARCH-t, for the Expected Shortfall forecasting. The forecasting procedure is similar to the one for VaR in subsection 3.3.

Tables 18 and 19 present the results of the ES statistical properties for a 10-percent and a 5-percent coverage level, respectively. At a 10-percent and 5-percent probability level, only ES forecasts from the GARCH-GPD-P model result in the not-rejection of the null hypothesis. On the other hand, the GARCH-n model leads to the failing of the unconditional coverage property in five cases and the independence property in three cases, the GARCH-t model leads to the failing of the independence property in three cases and the GARCH-GPD model leads to the failing of the independence property in two cases, at a 10-percent probability level. At a 5-percent probability, the GARCH-n model leads to the failing of the unconditional coverage property in six cases and the independence property in two cases, the GARCH-t model leads to the failing of the unconditional property in four cases and the independence property in three cases, and the GARCH-GPD model leads to the failing of the unconditional property and the independence property in one case. The results indicate that the ES forecasts obtained from the GARCH-GPD-P model are better than those of the other competing models. This is partly confirmed by the mean of cumulative violations H_t that in theory should be equal to $\alpha/2$. The mean of cumulative violation process H_t for the GARCH-GPD-P is closer to the desired level than any other competing model.

Table 18. The results of backtesting for ES(10%) based on Du and Escanciano (2016) tests

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean H_t	*p*-value	Mean H_t	*p*-value	Mean H_t	*p*-value	Mean H_t
Amazon	DE_{UC}	0.7284	0.0510	0.5887	0.0515	0.4208	0.0523	0.1586	0.0460
	DE_{IND}	0.0010*	–	0.0003*	–	0.0012*	–	0.5472	–
Apple	DE_{UC}	0.0126*	0.0570	0.0004*	0.0600	0.5367	0.0517	0.1700	0.0459
	DE_{IND}	0.4125	–	0.2759	–	0.3879	–	0.9371	–
Google	DE_{UC}	0.6399	0.0513	0.0670	0.0552	0.7052	0.0508	0.8382	0.0506
	DE_{IND}	0.0445*	–	0.0206*	–	0.0449*	–	0.3718	–
Microsoft	DE_{UC}	0.5150	0.0518	0.0504	0.0555	0.4172	0.0523	0.2488	0.0468
	DE_{IND}	0.1564	–	0.0764	–	0.2140	–	0.3858	–
Nvidia	DE_{UC}	0.6284	0.0514	0.1995	0.0536	0.1572	0.0545	0.6640	0.0512
	DE_{IND}	0.0205*	–	0.0207*	–	0.0661	–	0.5348	–
EUR/USD	DE_{UC}	0.5672	0.0517	0.2451	0.0534	0.2319	0.0535	0.2739	0.0532
	DE_{IND}	0.7575	–	0.2791	–	0.8661	–	0.4593	–
GBP/USD	DE_{UC}	0.4309	0.0477	0.7011	0.0511	0.5965	0.0484	0.8642	0.0495
	DE_{IND}	0.5643	–	0.4843	–	0.2327	–	0.7765	–
USD/JPY	DE_{UC}	0.0245*	0.0433	0.3083	0.0470	0.3649	0.0473	0.3156	0.0470
	DE_{IND}	0.3724	–	0.0529	–	0.1788	–	0.4759	–
BTC/USD	DE_{UC}	0.0390*	0.0420	0.0527	0.0527	0.5175	0.0475	0.9848	0.0499
	DE_{IND}	0.5416	–	0.4537	–	0.5555	–	0.6605	–
ETH/USD	DE_{UC}	0.0447*	0.0447	0.6197	0.0528	0.4724	0.0460	0.7224	0.0520
	DE_{IND}	0.3114	–	0.2075	–	0.9701	–	0.1606	–
LTC/USD	DE_{UC}	0.1379	0.0437	0.7778	0.0512	0.6524	0.0519	0.6753	0.0518
	DE_{IND}	0.4783	–	0.4317	–	0.4118	–	0.4350	–
XRP/USD	DE_{UC}	0.0277*	0.0376	0.9871	0.0499	0.6980	0.0522	0.7553	0.0482
	DE_{IND}	0.8502	–	0.8329	–	0.8027	–	0.9801	–

Note. * indicates that the null hypothesis is rejected at a 5% significance level. For the independence test DE_{IND}, we calculate statistics up to 5 lags. DE_{UC}, DE_{IND} are the unconditional coverage and independence tests, respectively, proposed by Du and Escanciano (2016). H_t is the cumulative violation process.

Source: author's work.

Table 19. The results of backtesting for ES(5%) based on Du and Escanciano (2016) tests

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean $H_t(5\%)$	p-value	Mean $H_t(5\%)$	p-value	Mean $H_t(5\%)$	p-value	Mean $H_t(5\%)$
Amazon	DE_{UC}	0.0157*	0.0299	0.2135	0.0275	0.0464*	0.0290	0.4256	0.0246
	DE_{IND}	0.0039*	–	0.0276*	–	0.0673	–	0.8085	–
Apple	DE_{UC}	0.0000*	0.0347	0.0001*	0.0329	0.2871	0.0272	0.5590	0.0262
	DE_{IND}	0.8345	–	0.7643	–	0.9919	–	0.2187	–
Google	DE_{UC}	0.0003*	0.0323	0.0075*	0.0304	0.5245	0.0263	0.7205	0.0257
	DE_{IND}	0.3295	–	0.5126	–	0.2664	–	0.6382	–
Microsoft	DE_{UC}	0.0052*	0.0307	0.0176*	0.0298	0.1282	0.0283	0.2330	0.0274
	DE_{IND}	0.3715	–	0.3548	–	0.6375	–	0.2524	–
Nvidia	DE_{UC}	0.0070*	0.0305	0.1460	0.0280	0.3193	0.0270	0.5386	0.0263
	DE_{IND}	0.0005*	–	0.0003*	–	0.0227*	–	0.7602	–
EUR/USD	DE_{UC}	0.3532	0.0269	0.7455	0.0256	0.8299	0.0254	0.7884	0.0245
	DE_{IND}	0.3936	–	0.5435	–	0.6237	–	0.2494	–
GBP/USD	DE_{UC}	0.0055*	0.0306	0.0179*	0.0297	0.8459	0.0254	0.9584	0.0249
	DE_{IND}	0.0719	–	0.0037*	–	0.0669	–	0.5972	–
USD/JPY	DE_{UC}	0.4784	0.0264	0.7762	0.0244	0.8108	0.0245	0.9918	0.0250
	DE_{IND}	0.8998	–	0.2625	–	0.6726	–	0.1678	–
BTC/USD	DE_{UC}	0.0897	0.0292	0.1403	0.0287	0.1071	0.0290	0.8934	0.0247
	DE_{IND}	0.2789	–	0.0257*	–	0.6941	–	0.1793	–
ETH/USD	DE_{UC}	0.4789	0.0273	0.3073	0.0284	0.2041	0.0208	0.6237	0.0234
	DE_{IND}	0.1311	–	0.1568	–	0.7552	–	0.1719	–
LTC/USD	DE_{UC}	0.7242	0.0260	0.6869	0.0261	0.5138	0.0270	0.9312	0.0252
	DE_{IND}	0.3733	–	0.5664	–	0.6328	–	0.3182	–
XRP/USD	DE_{UC}	0.4526	0.0225	0.5390	0.0230	0.2253	0.0210	0.3609	0.0220
	DE_{IND}	0.3556	–	0.1299	–	0.9194	–	0.3142	–

Note. As in Table 18.
Source: author's work.

6. Conclusions

The high and low prices and their range are believed to provide additional and useful information regarding the volatility of returns. Therefore, incorporating such prices in volatility models can lead to better estimates and forecasts of the conditional variance and covariance, but they may also be used to obtain more accurate estimates of risk measures. There is a growing body of literature showing that range-based models or models that use range-based estimators may outperform standard volatility models (see, see e.g. Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fałdziński et al., 2024; Fiszeder & Fałdziński, 2019; Fiszeder & Perczak, 2016; Fiszeder et al., 2019; Molnár, 2016; Xie, 2019). However, high and low prices are rarely used to describe the volatility of extreme observations. It seems natural that high and low prices provide additional insight into the dynamic behaviour of the returns that are at the tails of their distribution. In this paper, we propose an extension of the GARCH-GPD approach of McNeil and Frey (2000), by incorporating a range-based estimator to describe the magnitudes of threshold exceedances. We thus extend the Generalised Pareto Distribution by adding a meaningful covariate. The proposed model, the GARCH-GPD-P, is compared to the GARCH-GPD and two standard benchmarks, i.e. the GARCH model with the normal and *t*-distributed errors.

We evaluate the competing models on the basis of the Monte Carlo simulation and empirical time series. For the simulated time series, the GARCH-GPD-P is able to produce more accurate VaR and ES forecasts, especially at higher coverage levels (e.g. 5%). At lower coverage levels, the differences in risk measures forecasting are not significant and it is difficult to determine which model is the best. As regards empirical time series, there is even stronger evidence that the proposed GARCH-GPD-P model is able to perform more efficiently for high probabilities than the other competing models. For the Expected Shortfall forecasting, it seems to be of particular use as we obtained the most accurate estimates for the GARCH-GPD-P model.

This study can be extended in the future to better describe returns that are not extreme observations but are forecasted by the GARCH-GPD-P model. One potential way to achieve this goal that is considered in the literature is to combine several VaR forecasting procedures (see Jeon & Taylor, 2013; McAleer et al., 2010, 2013).

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Appendix

Table A1. Summary of forecasting

Time series	First forecast date (1,000 obs. estimation)	First forecast date (500 obs. estimation)	Forecast end date	Number of forecasts (1,000 obs. estimation)	Number of forecasts (500 obs. estimation)
Amazon	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Apple	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Google	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Microsoft	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
NVIDIA	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
EURSUD	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
GBP/USD	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
USD/JPY	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
BTC/USD	Sep 27th, 2017	May 15th, 2016	May 31st, 2023	2,073	2,573
ETH/USD	Sep 27th, 2020	May 16th, 2019	May 31st, 2023	977	1,477
LTC/USD	Sep 27th, 2018	May 15th, 2017	May 31st, 2023	1,708	2,208
XRP/USD	Sep 27th, 2020	May 16th, 2019	May 31st, 2023	977	1,477

Source: author's work.

Table A2. The results of backtesting tests for VaR(10%): stocks

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
AMAZON	LR_{UC}	4.0791	0.0434	0.5038	0.4778	0.3311	0.5650	0.1105	0.7395
	LR_{IND}	2.4336	0.1188	9.5546	0.0020*	7.0744	0.0078*	13.2247	0.0003*
	LR_{CC}	6.5127	0.0385	10.0584	0.0065*	7.4056	0.0247*	13.3352	0.0013*
	J_{UC}	4.1365	0.0392*	0.4398	0.5072	0.3868	0.5213	0.0784	0.7839
	J_{IND}	11.8428	0.0199*	25.2281	0.0075*	16.5276	0.0134*	17.4157	0.0122*
	J_{CC}	19.0070	0.0019*	26.6976	0.0096*	16.7889	0.0176*	17.6504	0.0166*
APPLE	LR_{UC}	0.0018	0.9659	3.1891	0.0741	0.0959	0.7569	1.3078	0.2528
	LR_{IND}	0.0000	0.9986	0.0987	0.7535	0.1254	0.7232	0.8083	0.3686
	LR_{CC}	0.0018	0.9991	3.2877	0.1932	0.2213	0.8953	2.1161	0.3471
	J_{UC}	0.0092	0.9235	3.1811	0.0745	0.1301	0.7184	1.2320	0.2670
	J_{IND}	6.1819	0.0925	6.6702	0.0484*	6.4365	0.0806	2.5328	0.3655
	J_{CC}	6.1249	0.2943	9.4953	0.0909	6.0720	0.1646	4.6959	0.4541
GOOGLE	LR_{UC}	5.5084	0.0189*	0.6183	0.4317	0.0257	0.8726	1.3650	0.2427
	LR_{IND}	3.9694	0.0463*	1.7509	0.1858	1.9726	0.1602	3.1286	0.0769
	LR_{CC}	9.4778	0.0087*	2.3692	0.3059	1.9983	0.3682	4.4937	0.1057
	J_{UC}	5.7103	0.0159*	0.6895	0.4167	0.0469	0.8290	1.2840	0.2575
	J_{IND}	12.7326	0.0201*	4.7349	0.1478	19.3361	0.0100*	2.4504	0.3812
	J_{CC}	37.3224	0.0057*	5.3820	0.2064	18.4694	0.0147*	4.2231	0.3127
MICROSOFT	LR_{UC}	5.2328	0.0222*	0.8091	0.3684	0.1100	0.7402	0.1270	0.7216
	LR_{IND}	0.9430	0.3315	1.2956	0.2550	1.3378	0.2474	1.7420	0.1869
	LR_{CC}	6.1758	0.0456*	2.1047	0.3491	1.4478	0.4849	1.8690	0.3928
	J_{UC}	5.4046	0.0199*	0.8834	0.3520	0.1489	0.6999	0.0902	0.7711
	J_{IND}	7.2537	0.0632	5.1827	0.1177	7.4774	0.0575	0.9888	0.7242
	J_{CC}	21.0741	0.0123*	5.4394	0.1979	6.5939	0.1199	1.1588	0.8418
NVIDIA	LR_{UC}	3.5074	0.0611	0.4528	0.5010	0.0257	0.8726	1.1061	0.2929
	LR_{IND}	0.0068	0.9341	0.0442	0.8335	0.1928	0.6606	0.4240	0.5150
	LR_{CC}	3.5142	0.1725	0.4970	0.7800	0.2185	0.8965	1.5300	0.4653
	J_{UC}	3.7415	0.0570	0.4496	0.4907	0.0274	0.8659	1.1375	0.2896
	J_{IND}	7.2300	0.0629	20.5038	0.0081*	15.0130	0.0148*	1.1509	0.6834
	J_{CC}	18.4533	0.0164*	16.5907	0.0147*	14.1027	0.0260*	2.7342	0.5162

Note. * indicates that the null hypothesis is rejected at a 5-percent significance level. LR_{UC} is the unconditional coverage test proposed by Kupiec (1995), LR_{IND}, LR_{CC} are the independence and conditional coverage tests, respectively, proposed by Christoffersen (1998). J_{UC}, J_{IND}, J_{CC} are the unconditional coverage, independence and conditional coverage tests, respectively, proposed by Candelon et al. (2011). For J_{IND} and J_{CC}, the number of moments is fixed to 5, *p*-values for J_{UC}, J_{IND}, J_{CC} are obtained by Dufour's (2006) Monte Carlo procedure based on 10,000 repetitions.

Source: author's work.

Table A3. The results of backtesting tests for VaR(10%) – currencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	P-value	Value	P-value	Value	P-value	Value	P-value
EUR/USD	LR_{UC}	1.9391	0.1638	0.3823	0.5364	0.0134	0.9078	2.0916	0.0932
	LR_{IND}	1.2127	0.2708	0.0097	0.9214	0.3539	0.5519	0.1695	0.6806
	LR_{CC}	3.1518	0.2068	0.3920	0.8220	0.3673	0.8322	2.2611	0.3229
	J_{UC}	2.0339	0.1561	0.3746	0.5408	0.0135	0.9080	2.1980	0.1330
	J_{IND}	2.4774	0.3704	1.6541	0.5510	4.1887	0.1751	2.7031	0.3405
	J_{CC}	5.5107	0.1979	1.8269	0.6987	4.2697	0.2952	6.0168	0.1628
GBP/USD	LR_{UC}	6.2695	0.0123*	0.2628	0.6082	0.0003	0.9874	2.0172	0.1555
	LR_{IND}	1.0918	0.2961	1.2412	0.2652	0.8109	0.3678	1.2425	0.2650
	LR_{CC}	7.3613	0.0252*	1.5040	0.4714	0.8112	0.6666	3.2597	0.1960
	J_{UC}	6.8452	0.0088*	0.2674	0.6050	0.0002	0.9893	2.1179	0.1397
	J_{IND}	4.2050	0.1815	2.4302	0.3894	0.6305	0.8594	0.7408	0.82187
	J_{CC}	14.3086	0.0240*	3.0071	0.4712	0.6293	0.9513	2.9588	0.4771
USD/JPY	LR_{UC}	6.3998	0.0114*	0.1070	0.7436	0.0134	0.9078	2.9427	0.0863
	LR_{IND}	3.3961	0.0654	5.0150	0.0251*	3.7122	0.0540	1.6874	0.1939
	LR_{CC}	9.7949	0.0075*	5.1219	0.0772	3.7256	0.1552	4.6301	0.0988
	J_{UC}	6.6806	0.0085*	0.0759	0.0784	0.0040	0.9417	2.9232	0.0833
	J_{IND}	1.8227	0.5077	7.2026	0.0599	6.0349	0.0906	5.5492	0.1085
	J_{CC}	9.3026	0.0613	7.3839	0.1065	6.0673	0.1701	9.2916	0.0618

Note. As in table A2.

Source: author's work.

Table A4. The results of backtesting tests for VaR(10%): cryptocurrencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
BTC/USD	LR_{UC}	26.9925	0.0000*	0.5834	0.4450	0.2321	0.6300	0.3449	0.5570
	LR_{IND}	6.3575	0.0117*	4.7939	0.0286*	2.7762	0.0957	0.0261	0.8716
	LR_{CC}	33.3500	0.0000*	5.3773	0.0680	3.0083	0.2222	0.3710	0.8307
	J_{UC}	33.5185	0.0001*	0.6637	0.4079	0.1757	0.6737	0.2858	0.5811
	J_{IND}	5.2001	0.1149	12.8658	0.0179*	9.3562	0.0331*	3.9553	0.1962
	J_{CC}	64.7621	0.0037*	13.4171	0.0275*	9.5760	0.0528	4.8211	0.2450
ETH/USD	LR_{UC}	16.6330	0.0000*	0.2473	0.6190	0.1038	0.7474	2.8762	0.0899
	LR_{IND}	0.1029	0.7484	0.1566	0.6923	0.0002	0.9885	0.4652	0.4952
	LR_{CC}	16.7358	0.0002*	0.4039	0.8171	0.1040	0.9493	3.3414	0.1881
	J_{UC}	20.4027	0.0001*	0.1716	0.6716	0.0559	0.8165	2.8502	0.0940
	J_{IND}	4.3022	0.1410	1.2979	0.5918	1.5783	0.5162	1.8815	0.4897
	J_{CC}	34.5089	0.0062*	1.5168	0.7297	1.6893	0.6895	6.0781	0.1630
LTC/USD	LR_{UC}	28.9569	0.0000*	0.3094	0.5780	0.4208	0.5166	0.0276	0.8681
	LR_{IND}	2.5518	0.1102	1.1678	0.2799	3.1923	0.0740	0.9282	0.9558
	LR_{CC}	31.5086	0.0000*	1.4772	0.4778	3.6131	0.1642	0.9558	0.6201
	J_{UC}	38.4433	0.0001*	0.3174	0.5903	0.4089	0.5206	0.0076	0.9127
	J_{IND}	1.0381	0.6958	1.8565	0.4593	4.4049	0.1498	2.9923	0.2693
	J_{CC}	47.2257	0.0053*	2.3091	0.5715	4.5803	0.2515	3.0298	0.4354
XRP/USD	LR_{UC}	32.0366	0.0000*	0.4531	0.5009	1.0550	0.3044	1.1727	0.2788
	LR_{IND}	0.7016	0.4022	0.2634	0.6078	0.0211	0.8846	0.9941	0.3187
	LR_{CC}	32.7383	0.0000*	0.7165	0.6989	1.0761	0.5839	2.1668	0.3384
	J_{UC}	45.0011	0.0001*	0.3537	0.5651	0.9285	0.3272	1.2701	0.2522
	J_{IND}	0.7966	0.7484	1.0683	0.6641	2.3646	0.3541	2.8026	0.2927
	J_{CC}	61.3815	0.0029*	1.5788	0.7218	3.7696	0.3306	3.8069	0.3303

Note. As in Table A2.

Source: author's work.

Table A5. The results of backtesting tests for VaR(5%): stocks

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
AMAZON	LR_{UC}	0.0224	0.8810	0.5991	0.4389	0.2143	0.6434	0.2143	0.6434
	LR_{IND}	5.6770	0.0172*	4.2360	0.0396*	7.8459	0.0051*	2.6128	0.1060
	LR_{CC}	5.6994	0.0579	4.8351	0.0891	8.0602	0.0178*	2.8272	0.2433
	J_{UC}	0.0515	0.8233	0.6948	0.3938	0.2847	0.5960	0.2847	0.5935
	J_{IND}	4.9644	0.1125	5.440	0.1166	7.4888	0.0474*	4.7132	0.1280
	J_{CC}	4.9285	0.2174	4.9982	0.2178	6.7962	0.1182	4.2881	0.2814
APPLE	LR_{UC}	1.3452	0.2461	4.9403	0.0262*	0.1055	0.7454	0.2343	0.6283
	LR_{IND}	0.1057	0.7452	0.0158	0.9001	0.0004	0.9845	0.5244	0.4690
	LR_{CC}	1.4508	0.4841	4.9561	0.0839	0.1058	0.9485	0.7587	0.6843
	J_{UC}	1.4385	0.2279	4.7290	0.0291*	0.0609	0.7908	0.1669	0.6873
	J_{IND}	2.3000	0.3695	0.3083	0.9429	2.3675	0.3566	1.6148	0.5066
	J_{CC}	3.4301	0.3773	5.2756	0.1901	2.7586	0.4780	1.7967	0.6730
GOOGLE	LR_{UC}	0.1635	0.6860	0.5991	0.4389	0.6474	0.4211	0.3829	0.5361
	LR_{IND}	1.1627	0.2809	1.3238	0.2499	1.6162	0.2036	0.8293	0.3625
	LR_{CC}	1.3262	0.5153	1.9230	0.3823	2.2635	0.3225	1.2122	0.5455
	J_{UC}	0.1072	0.7456	0.6948	0.3875	0.5448	0.4677	0.4684	0.4959
	J_{IND}	21.4308	0.0087*	40.4976	0.0031*	17.5822	0.0118*	0.8869	0.7313
	J_{CC}	26.3640	0.0098*	27.5929	0.0082*	28.2465	0.0086*	1.1398	0.8248
MICROSOFT	LR_{UC}	0.0521	0.8195	1.0188	0.3145	0.1481	0.7003	0.5232	0.4695
	LR_{IND}	1.0671	0.3016	0.9807	0.3220	1.2124	0.2708	1.7593	0.1847
	LR_{CC}	1.1192	0.5714	1.9925	0.3693	1.3606	0.5065	2.2825	0.3194
	J_{UC}	0.0925	0.7792	1.1114	0.2941	0.2094	0.6445	0.2847	0.6009
	J_{IND}	3.5941	0.2025	3.2414	0.2389	10.1449	0.0247*	4.0428	0.1580
	J_{CC}	3.4580	0.3708	3.7736	0.3289	8.6268	0.0666*	3.9920	0.2986
NVIDIA	LR_{UC}	0.4851	0.4861	1.5291	0.2163	0.0940	0.7592	0.5991	0.4389
	LR_{IND}	0.2172	0.6411	0.0940	0.7592	0.0832	0.7729	0.2803	0.5965
	LR_{CC}	0.7023	0.7039	1.6231	0.4442	0.1772	0.9152	0.8795	0.6442
	J_{UC}	0.4737	0.4807	1.4475	0.2207	0.0949	0.7589	0.5822	0.4320
	J_{IND}	6.2601	0.0726	10.6862	0.0241*	5.1656	0.1013	0.6773	0.8045
	J_{CC}	5.7040	0.1675	10.0402	0.0507	4.8906	0.2194	1.0570	0.8488

Note. As in Table A2.

Source: author's work.

Table A6. The results of backtesting tests for VaR(5%): currencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
EUR/USD	LR_{UC}	0.9921	0.3192	1.1452	0.2846	0.7194	0.3963	1.1452	0.2846
	LR_{IND}	0.0135	0.9074	0.0439	0.8341	0.1560	0.6929	0.9439	0.3313
	LR_{CC}	1.0056	0.6048	1.1891	0.5518	0.8754	0.6455	2.0891	0.3518
	J_{UC}	1.0411	0.3045	1.2063	0.2737	0.7494	0.3954	0.0925	0.7655
	J_{IND}	6.8731	0.0639	1.2510	0.6170	4.2890	0.1583	1.0700	0.6629
	J_{CC}	10.6620	0.0465*	3.3908	0.3906	6.4944	0.1316	1.0976	0.8293
GBP/USD	LR_{UC}	0.5862	0.4439	1.7761	0.1826	0.0022	0.9624	0.1662	0.6835
	LR_{IND}	0.0004	0.9846	0.1071	0.7434	0.8959	0.3439	0.2623	0.6085
	LR_{CC}	0.5865	0.7458	1.8832	0.3900	0.8981	0.6382	0.4286	0.8071
	J_{UC}	0.5658	0.0452*	1.6718	0.1932	0.0022	0.9566	0.1631	0.6738
	J_{IND}	9.5343	0.0330*	9.1258	0.0318*	4.5468	0.1411	0.5752	0.8484
	J_{CC}	8.7007	0.0690	9.6805	0.0561	4.5092	0.2585	0.6444	0.9337
USD/JPY	LR_{UC}	1.1452	0.2846	0.4570	0.4990	0.0008	0.9769	0.0008	0.9769
	LR_{IND}	0.0177	0.8942	0.3870	0.5339	0.5052	0.4772	0.1308	0.7176
	LR_{CC}	1.1629	0.5591	0.8439	0.6558	0.5060	0.7765	0.1317	0.9363
	J_{UC}	1.0411	0.2995	0.5396	0.4640	0.0102	0.9230	0.0102	0.9349
	J_{IND}	0.0303	0.9999	2.7671	0.3096	0.5864	0.8472	1.7285	0.4937
	J_{CC}	1.1197	0.8346	3.2947	0.4106	0.5890	0.9472	1.7212	0.6941

Note. As in Table A2.
Source: author's work.

Table A7. The results of backtesting tests for VaR(10%): cryptocurrencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
BTC/USD	LR_{UC}	26.9925	0.0000*	0.5834	0.4450	0.2321	0.6300	0.3449	0.5570
	LR_{IND}	6.3575	0.0117*	4.7939	0.0286*	2.7762	0.0957	0.0261	0.8716
	LR_{CC}	33.3500	0.0000*	5.3773	0.0680	3.0083	0.2222	0.3710	0.8307
	J_{UC}	33.5185	0.0001*	0.6637	0.4079	0.1757	0.6737	0.2858	0.5811
	J_{IND}	5.2001	0.1149	12.8658	0.0179*	9.3562	0.0331*	3.9553	0.1962
	J_{CC}	64.7621	0.0037*	13.4171	0.0275*	9.5760	0.0528	4.8211	0.2450
ETH/USD	LR_{UC}	16.6330	0.0000*	0.2473	0.6190	0.1038	0.7474	2.8762	0.0899
	LR_{IND}	0.1029	0.7484	0.1566	0.6923	0.0002	0.9885	0.4652	0.4952
	LR_{CC}	16.7358	0.0002*	0.4039	0.8171	0.1040	0.9493	3.3414	0.1881
	J_{UC}	20.4027	0.0001*	0.1716	0.6716	0.0559	0.8165	2.8502	0.0940
	J_{IND}	4.3022	0.1410	1.2979	0.5918	1.5783	0.5162	1.8815	0.4897
	J_{CC}	34.5089	0.0062*	1.5168	0.7297	1.6893	0.6895	6.0781	0.1630
LTC/USD	LR_{UC}	28.9569	0.0000*	0.3094	0.5780	0.4208	0.5166	0.0276	0.8681
	LR_{IND}	2.5518	0.1102	1.1678	0.2799	3.1923	0.0740	0.9282	0.9558
	LR_{CC}	31.5086	0.0000*	1.4772	0.4778	3.6131	0.1642	0.9558	0.6201
	J_{UC}	38.4433	0.0001*	0.3174	0.5903	0.4089	0.5206	0.0076	0.9127
	J_{IND}	1.0381	0.6958	1.8565	0.4593	4.4049	0.1498	2.9923	0.2693
	J_{CC}	47.2257	0.0053*	2.3091	0.5715	4.5803	0.2515	3.0298	0.4354
XRP/USD	LR_{UC}	32.0366	0.0000*	0.4531	0.5009	1.0550	0.3044	1.1727	0.2788
	LR_{IND}	0.7016	0.4022	0.2634	0.6078	0.0211	0.8846	0.9941	0.3187
	LR_{CC}	32.7383	0.0000*	0.7165	0.6989	1.0761	0.5839	2.1668	0.3384
	J_{UC}	45.0011	0.0001*	0.3537	0.5651	0.9285	0.3272	1.2701	0.2522
	J_{IND}	0.7966	0.7484	1.0683	0.6641	2.3646	0.3541	2.8026	0.2927
	J_{CC}	61.3815	0.0029*	1.5788	0.7218	3.7696	0.3306	3.8069	0.3303

Note. As in Table A2.
Source: author's work.

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Polish-Slovak-Ukrainian Scientific Seminars – 30 years of cooperation

Józef Pociecha[a]

The close collaboration between Polish and Slovak academic statisticians began with the establishment of the independent Slovak Republic on 1st January 1993. On 3rd January 1993, Professor Józef Pociecha from the Department of Statistics at Krakow University of Economics, then headed by Professor Aleksander Zeliaś, visited the Department of Statistics at the University of Economics in Bratislava, headed by Eva Sodomová, Associate Professor, to discuss the possibilities of deepening collaboration between the departments. The initial idea, introduced by Viera Pacaková, Associate Professor, was to carry out a joint research project. After further discussions, it was concluded that the project would concern labour market statistical investigations in the transformed economies of both countries and be coordinated by the Central European University in Budapest. All relevant documents were prepared, but eventually our research project was not accepted.

In order not to waste the work and time spent on designing the initial research project, a new idea emerged of a joint Polish-Slovak scientific seminar on quantitative methods and empirical research in the area of economic investigations. The seminar's aim was to bring together economists, statisticians and mathematicians from Krakow University of Economics and the University of Economics in Bratislava. This joint seminar provided an opportunity for both parties to present the results of their most recent research. The seminar also offered a forum for discussion about the possibility to undertake special joint research projects.

The 1st Slovak-Polish Scientific Seminar, held on 27th–31st March 1995, was organised by the Department of Statistics at the University of Economics in Bratislava. The seminar focused on one topic: the labour market and unemployment in Slovakia and in Poland. 'Problems of measurement of labour forces and their utilisation in transforming economies' was the theme of the seminar. The members of the cooperating statistics departments from Bratislava and Kraków presented a total of 11 papers.

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The 2nd Polish-Slovak Scientific Seminar focused on 'Statistical methods of the analysis of socio-economic aspects of the labour market in Poland and Slovakia'. It was organised by the Department of Statistics at Krakow University of Economics and took place on 15th–18th November 1995. In addition to the eleven papers delivered by authors from both universities, there was one guest presentation by Professor Serguei Gerasymenko from the Kiev State University of Economics. Professor Gerasymenko suggested extending collaboration to three partner institutions, i.e. the departments of statistics in Bratislava, Kraków and Kiev. The proposal was accepted, giving rise to a three-partner collaboration.

The 3rd Polish-Slovak-Ukrainian Scientific Seminar titled 'Research on labour markets and the level of life in Poland, Slovakia and Ukraine: methods and results', was held in Kraków on 14th–15th November 1996. The main topic of the seminar, which was the labour market and unemployment, was extended with topics connected with the former one: quality of life, consumer behaviour and poverty. The authors from Krakow University of Economics, the University of Economics in Bratislava and the Kiev State University of Economics presented 11 papers on the above-mentioned issues.

'Statistical methods in socio-economic investigations: theory and applications' was the main theme of the 4th edition of the seminar. It was held in Bratislava on 10th–12th November 1997. The topic of the seminar was expanded to include a general overview of the statistical methods applied to socioeconomic research. In addition to the four groups of issues discussed during the previous seminar, papers on forecasting, actuarial statistics, demographic investigations and other research problems were presented. 17 papers were delivered by the authors from the University of Economics in Bratislava, Krakow University of Economics and guest participants from the Slovak Academy of Sciences, the Tinbergen Institute in Rotterdam, the Agricultural University in Nitra, the University of Žilina, INFOSTAT Bratislava and the Institute of Labour, Social Affairs and Family.

The 5th Ukrainian-Polish-Slovak Scientific Seminar relating to 'Economic and social statistics in transition', was organised by the Kiev National University of Economics and held in Kiev on 20th–22nd October 1998. The seminar focused on three main topics: quality of life, multivariate statistical analysis, and macroeconomic policy and indicators. Besides the 11 papers presented by authors from the three cooperating universities, there was a guest speaker from the Scientific and Research Institute of the State Statistics Service of Ukraine.

The 6th Polish-Slovak-Ukrainian Scientific Seminar on the 'Statistical methods in socio-economic research: theory and applications' took place in Kraków on 8th–10th November 1999. The following groups of issues were discussed: theoretical and methodological aspects of scientific research, problems relating to the investigation of market processes, economic and statistical challenges relating to transition processes in Central and Eastern European countries, and demographic research. 17 papers were presented by the authors from the three cooperating universities. Moreover, guest participants from the Lviv Academy of Commerce, the Ivan Franko Lviv National University and the University of Rijeka presented the results of their research.

The 7th Slovak-Polish-Ukrainian-Czech Scientific Seminar concerning 'Analysis and international comparisons of the social consequences of transformation processes in post-communist countries', was organised by the University of Economics in Bratislava and held in Svätý Jur on 15th–18th November 2000. Five main groups of issues were discussed and 18 papers on following topics delivered: the economic conditions of social changes, quality of life, methods and results of international comparisons, labour market and consumer behaviour. In addition to scientists working at the three cooperating universities, speakers from the University of Economics in Prague, the Lviv Academy of Commerce and the Wrocław University of Economics presented their findings.

The 8th Ukrainian-Polish-Slovak Scientific Seminar on 'Problems of economic statistics in transition countries', was held in Kiev on 24th–26th October 2001. Four major themes were discussed: methodological and informational foundations of statistics, contemporary problems of applied statistics, statistical aspects of demographic investigations, and education in statistics. Besides the regular participants from Ukraine, Poland and Slovakia, guest authors from the Berlin School of Economics, the Cabinet of Ministers of Ukraine, the State Statistics Committee of Ukraine, the Scientific Research Institute of Statistics, the National Academy of Management, the Lviv Academy of Commerce, and the Ivan Franko Lviv National University presented their papers.

The 9th Polish-Slovak-Ukrainian Scientific Seminar focusing on 'International comparisons of socioeconomic consequences of transition processes in Central and Eastern European countries', was organised by Krakow University of Economics and held in Krynica on 6th–8th November 2002. The four main groups of issues discussed at the seminar included economic processes and methods of their comparison, statistical methods for political and social investigations, quantitative economic research in theory and practise, ageing process and insurance problems, and demographic analysis and its applications. The authors of the 20 delivered presentations came from the Bratislava, Kraków, Kiev and Lviv cooperating universities. There were also guest speakers from the National Academy of Management in Kiev and from Świętokrzyska Academy in Kielce.

The 10th Slovak-Polish-Ukrainian Scientific Seminar focusing on the 'Education of quantitative mathematical and statistical methods at economic universities referring to future needs', organised by the University of Economics in Bratislava, was held in Svätý Jur on 4th–7th November 2003. 17 papers were presented which set direction in new approaches to teaching statistical disciplines, the role of innovations in the study programme, theoretical and methodical aspects of the application of statistical methods in demographic, social and economic investigations. Pacaková and Pociecha also presented an occasional paper 'Nine Polish-Slovak and Polish-Slovak-Ukrainian Scientific Seminars: the main ideas and methods of socioeconomic investigations' (2003). The seminar was attended by scientists from Krakow University of Economics, the Kyiv National University of Economics, Wrocław University of Economics, Katowice University of Economics, the Lviv Academy of Commerce, the Kyiv National Academy of Management and the University of Economics in Bratislava.

The 11th Ukrainian-Polish-Slovak Scientific Seminar on 'Statistics in management of social and economic development' was held in Kiev on 20th–24th October 2004. Statisticians from nine Ukrainian institutions, Krakow University of Economics and the University of Economics in Bratislava took part in the event. 25 papers were presented on topics relating to the problems of macroeconomic statistics, household surveys, provision of informative support to the national development strategy, and the analysis and forecasting of demographic processes. All the presentations were dedicated to the then-current problems of statistical methodology and practice of statistics and a great majority of the recommendations may have been successfully applied in statistical analysis and forecasting of the economic processes occurring in transition countries.

The 12th Polish-Slovak-Ukrainian Scientific seminar presenting 'A comparative analysis of the socioeconomic consequences of transition processes in Central and Eastern European countries' was held in Krynica on 8th-10th November 2005. The seminar brought together Polish, Slovak and Ukrainian economists, statisticians and demographers from the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman (the Kyiv National University of Economics until 2005), the Lviv Academy of Commerce, the Odessa State University of Economics, the Doneck National University and Krakow University of Economics. The 17 papers that were delivered at the seminar focused on five main groups of topics: economic process analysis, demographic research, quantitative economic investigations, social and business statistics, and labour market analysis.

The 13th Slovak-Polish-Ukrainian Scientific Seminar on the 'Education of quantitative mathematical and statistical methods at economic universities referring to future needs', organised by the Department of Statistics of the University of Economics in Bratislava, was held in Svätý Jur on 7th–10th November 2006. Scientists from Krakow University of Economics, the Kyiv National Economic

University named after Vadym Hetman, Wrocław University of Economics, Katowice University of Economics, the Odessa State University of Economics, the Doneck National University and the University of Economics in Bratislava attended the seminar. This three-day event set direction in the new approaches to teaching quantitative methods, innovations in study programmes, the role of quantitative methods as a subject of study in economics and management and the restructuring process of the higher education system in the EU according to the Bologna Process. The theoretical aspects and applications of the statistical methods of demographic, social and economic analyses of various phenomena were also discussed.

The 14th Ukrainian-Polish-Slovak Scientific Seminar, addressing the issue of 'Statistics in management of social and economic development', organised by the Odessa State University, took place in Odessa on 24th–28th September 2007. Statisticians from 12 institutions (nine Ukrainian, one Polish and two Slovak) delivered a total of 20 presentations that focused on four main topics: economic and human development in Central and Eastern European countries, various aspects of demography and social statistics, the theory and methodology of statistical analysis and statistical methods of market analysis. This four-day event also provided an excellent opportunity for the participants to exchange opinions on the issues of statistical methodology and practice. The formulated recommendations proved useful in the area of statistical analysis and forecasting contemporary trends in the transition processes that the considered countries were undergoing.

The 15th Polish-Slovak-Ukrainian Scientific Seminar was held in Kraków on 21st–24th October 2008. Participants from the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Lviv Academy of Commerce, the Odessa State University of Economics, the Kharkov National University of Economics, the Doneck National University and Krakow University of Economics attended the event. The 20 papers which were presented at the seminar covered six groups of issues: quantitative methods in economics, quantitative investigations methodology, comparative studies on economies in transition, demographic investigations, social and economic statistics, statistical methods and their applications. The seminar gave rise to an initiative to exchange experiences gained in the course of the scientific work of the participating universities.

The 16th Slovak-Polish-Ukrainian Scientific Seminar on 'Quantitative methods in socio-economic analysis', organised by the Department of Statistics in Bratislava, was held in Kucisdorf Valley on 27th–30th October 2009. 16 presentations were made during the seminar, referring to the topical problems of statistical analysis in various economic and social contexts. The contributions focused on issues of statistical analyses in the economic, social and demographic areas, and their applications in banking and insurance institutions (mathematical modelling of credit risk, bankruptcy prediction), macroeconomic indicators (capital mobility, pro-, anti- and acyclic factors of economy), social and demographic analyses (household budget analysis, employment and unemployment analysis) and applications in other sectors of economic life (mathematical methods in management, quality control methods).

The 17th Ukrainian-Polish-Slovak Scientific Seminar was held in Lviv on 22nd–24th September 2010. The host institution was the Lviv Academy of Commerce. Researchers from Krakow University of Economics, the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Ivan Franko Lviv National University, the Odessa State University of Economics and the Doneck National University attended the seminar and delivered 21 papers. The event focused on the contemporary problems of transformation processes. The relevant socioeconomic issues were divided into the four main groups: multi-dimensional macro- and microeconomic issues, the economic policy in transformation economies, quantitative methods in social research, and regional issues in statistical research.

The 18th Polish-Slovak-Ukrainian Scientific Seminar was devoted to the 'Statistical analysis of the economic and social consequences of transition processes in Central and Eastern European countries'. The event was organised by the Department of Statistics of Krakow University of Economics and took place in Krynica on 25th–28th October 2011. Statisticians, econometricians and demographers from the University of Economics in Bratislava, Kyiv National Economic University named after Vadym Hetman, the Ivan Franko National University of Lviv, the Lviv Academy of Commerce, the Odessa National University of Economics, the Doneck National University and Krakow University of Economics attended the seminar. During the event, 22 papers were presented, focusing on six main groups of problems: the statistical analysis of international economic processes, economic investigations of the Ukrainian economy, statistical tools in socioeconomic investigations, issues relating to business statistics, methods of demographic analysis and statistical methods for process analysis.

The 19th Slovak-Polish-Ukrainian Scientific Seminar on 'Quantitative methods in socioeconomic analysis', organised by the Department of Statistics of the University of Economics in Bratislava, took place in Svätý Jur on 23rd–27th October 2012. The event was attended by representatives of Krakow University of Economics, the Kyiv National Economic University named after Vadym Hetman, the Lviv Academy of Commerce and the University of Economics in Bratislava. The main topics of the 19 presentations were: macroeconomic analysis methods, special analysis methods, demographic research, labour market analysis, household income analysis, consumer opinion analysis, financial analyses, risk analysis and product design quality assessment.

The 20th, jubilee, Ukrainian-Polish-Slovak Scientific Seminar discussed 'The role of statistics in the modern economy model development'. The meeting was organised in Kyiv on 5th–7th November 2013 by Professor Igor G. Mantsurov, head of the Department of Statistics of the Kyiv National Economic University named after Vadym Hetman. The seminar was attended by representatives of Krakow University of Economics, the University of Economics in Bratislava, the University of Pardubice in the Czech Republic, the Kyiv National Economic University named after Vadym Hetman, the State Statistics Service of Ukraine, the Odessa National Economic University, the Donetsk National University, the Oles Honchar Dnipropetrovsk National University and the Lviv Academy of Commerce. During the event, 23 contributions were made, and among them, Pociecha and Sodomová presented 'Twenty years of Polish-Slovak-Ukrainian Scientific Seminars: ideas and investigations' (2014).

The 21st Polish-Slovak-Ukrainian Scientific Seminar themed 'Quantitative methods for the analysis of economic and social consequences of transition processes in Central and Eastern European countries', organised by the Department of Statistics at Krakow University of Economics, took place on 21st–24th October 2014 in Szczawnica. 25 papers were presented during the event. Participants from Krakow University of Economics, the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Lviv Academy of Commerce, the Lviv Institute of Banking, the University of Banking of the National Bank of Ukraine, and the Scientific and Research Institute of Economics at the Ministry of Economic Development and Trade of Ukraine attended the seminar. The main topics of the presentations were: efficiency measures of educational and scientific systems, economic investigations of the Ukrainian economy, methods of demographic analysis, household budget studies and business statistics.

The 22nd Slovak-Polish-Ukrainian Scientific Seminar relating to 'Statistical methods in socioeconomic research: theory and applications' was organised by the University of Economics in Bratislava in Virt on 20th–23rd October 2015. Participants from the University of Economics in Bratislava, Krakow University of Economics, the Kyiv National Economic University named after Vadym Hetman, the Odessa National Economic University and the Lviv Academy of Commerce attended the seminar, during which 18 papers were presented. The main topics included: sampling methods in socio-economic studies, the influence of the global crisis on the economic stability of Central and Eastern Europe, challenges posed by the then-current crisis in Ukraine, demographic future of Ukraine, diversity of household expenditure, situation of women on the labour market, the logistic regression method in the research of the financial standing and business statistics methods.

The 23rd Ukrainian-Polish-Slovak Scientific Seminar, focusing on 'The role of statistics in the modern economy model development', organised by the Kyiv National Economic University named after Vadym Hetman and Research Institute for System Statistical Studies Information Systems and Technologies, took place on 12th–13th October 2016 in Kyiv. Representatives of Krakow University of Economics, Nicolaus Copernicus University in Torun (Poland), the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Oles Honchar Dnipropetrovsk National University, the National Academy of Statistics, Accounting and Audit, Kryvyi Rih National University, the Donetsk National University, the Odessa National Economic University, the Kyiv National University of Trade and Economics and the Lviv University of Trade and Economics attended the seminar and 33 papers were delivered.

The 24th Polish-Slovak-Ukrainian Scientific Seminar devoted to 'Statistical methods in socioeconomic research: theory and applications' was held in Dobczyce on 10th–13th October 2017. 22 papers were presented by participants from the Krakow University of Economics, the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Lviv University of Trade and Economics and the Lviv State University of Internal Affairs. The topics of presentations focused on: the contemporary problems of statistical analysis, social statistics, the statistical methods for economic investigations, methods of demographic research, statistical analysis of the global trends in higher education, quality of life, actuarial statistics and statistical methods in auditing.

The 25th Slovak-Polish-Ukrainian Scientific Seminar, addressing 'Quantitative methods in socioeconomic research: theory and applications', took place in Bratislava on 10th–12th October 2018. Participants from the University of Economics in Bratislava, Krakow University of Economics and the Kyiv National Economic University named after Vadym Hetman presented 15 papers. The main topics of the presentations included the philosophical foundations of statistics, contemporary problems of statistical analysis, social statistics, living standards of the population, bankruptcy prediction methods, statistical methods for economic investigations, methods of demographic research, statistical analysis of quality of life, and actuarial statistics.

The 26th Ukrainian-Polish-Slovak Scientific Seminar on the 'The role of statistics in the development of the modern economic systems' was organised by Professor Igor Mantsurov in Kyiv on 8th–9th October 2019. Representatives of the Krakow University of Economics, the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman, the Institute for Demography and Social Studies of the National Academy of Sciences of Ukraine, the

Institute for Economics and Forecasting of NAS of Ukraine, and the Ivan Franko Lviv National University attended the meeting, delivering a total of 28 papers. Digital development and its diversity, social statistics, statistical methods for economic research, demographic problems in Ukraine, labour market analysis and statistical analysis of the global trends in the EU economy were the main topics discussed.

The 27th Polish-Slovak-Ukrainian Scientific Seminar concerning 'Statistical methods in a modern economy: theory and applications', was organised on-line after the COVID-19 pandemic outbreak by Krakow University of Economics on 19th October 2021. Participants from Krakow University of Economics, the University of Economics in Bratislava, the Kyiv National Economic University named after Vadym Hetman and the Lviv University of Trade and Economics attended the meeting and delivered 17 presentations. They concerned the paradigms of statistical inference and statistical learning, household budget studies, statistical analysis of the Ukrainian agricultural sector, business demography methods, population ageing and rising costs of old-age pensions, statistical analysis of the young generations' environmental behaviour and labour market analysis.

The main theme of the 28th Slovak-Polish-Ukrainian Scientific Seminar was 'Quantitative methods in socioeconomic analysis: theory and applications'. The meeting was organised by the University of Economics in Bratislava on 20th–21st October 2022. Participants from the University of Economics in Bratislava, Krakow University of Economics and the Kyiv National Economic University named after Vadym Hetman attended the event, during which 10 papers were delivered. The presentations referred to problems with using artificial intelligence and machine learning in socioeconomic investigations, the application of statistical learning methods to the estimation of missing data to supplement databases, keyword analysis based on multivariate statistical methods, modelling the dynamics of innovation diffusion in industry, household budget research and a selection of issues relating to sampling methods.

The 29th Ukrainian-Polish-Slovak Scientific Seminar on 'Quantitative methods in socioeconomic analysis: theory and applications', held on 19th October 2023, hosted by the Kyiv National Economic University named after Vadym Hetman, was organised on-line due to the war in Ukraine. 12 papers were presented by participants from the Kyiv National Economic University named after Vadym Hetman, the University of Economics in Bratislava and Krakow University of Economics. The main issues discussed at the seminar included statistical inference and statistical learning: two sides of the approximation of the real world, methods and results of household budget analysis, analyses of the impact of the Russian invasion on the agricultural sector of Ukraine's economy, higher education in

Ukraine under martial law, the modelling of grain production in regions of the world in the context of global food security, and the analysis of the demographic security of Ukraine.

The current, jubilee, 30th Polish-Slovak-Ukrainian Scientific Seminar themed 'Statistical methods in modern economy: theory and applications' was organised by Krakow University of Economics on 24th–25th October 2024. The seminar gathered participants from the four collaborating universities: Krakow, Bratislava, Kiev and Lviv. 16 papers were presented on methods of household budget research, poverty research, consumer behaviour on the market, Bayesian modelling and forecasting, bankruptcy forecasting, diffusion of innovations in the EU, the analysis of the situation in the food economy and agricultural sector in Ukraine.

30 years of uninterrupted scientific meetings of statisticians in three countries of Central and Eastern Europe have made possible:

- an exchange of information on scientific research, the methods used and its results, conducted in the partnering departments of statistics;
- the improvement of the level and culture of statistical knowledge in Poland, Slovakia and Ukraine;
- the scientific promotion of the participants of these seminars;
- establishing close personal contacts and sometimes friendships among the attendees.

The participants of these scientific meetings were also able to see the shortcomings and limitations of the previous editions of the seminar, and these included:

- a failure to undertake a joint international project, which initially inspired the partnership;
- too few joint publications inspired by the results presented during the seminars in international journals;
- inability to expand the seminars to include other partners from Central and Eastern Europe;
- discontinuation of the publication of the seminar proceedings with the full content of the papers, which was usually done until 2013;
- low effectiveness of the publication of the seminar papers in well-known scientific journals.

The participants of the seminars also see certain challenges which, once overcome, are likely to offer an opportunity to develop these Central and Eastern European statistical seminars. These include:

- using English as the exclusive language at the conferences;
- transforming the seminars into conferences bringing together statisticians, econometricians and economists from Central and Eastern European countries;

- attracting researchers from Western Europe specialising in issues relating to Central and Eastern European countries to attend the seminars;
- establishing stronger ties between universities which organise the seminars and academic centres of Western Europe;
- successfully applying for funds in EU institutions which would support the organisation of the seminars.

The future will show whether the Polish-Slovak-Ukrainian scientific seminars will continue as an effective platform for scientific cooperation between statisticians from Central and Eastern Europe.

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