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# LETTER FROM THE EDITOR

Dear Readers, Contributors and Friends of our Journal,

It is with great pleasure that we present to you the first issue of *Przegląd Statystyczny. Statistical Review* of the year 2025, which also marks the start of an exciting new phase in the development of our Journal. With a new Editorial Board now in place, we move forward with fresh energy, clear goals and a renewed commitment to advancing high-quality research in the area of quantitative methods.

Our Journal focuses on the broadly understood quantitative methods, covering statistics, operations research, econometrics, mathematical economics, data analysis, finance and methodologies related to artificial intelligence. These disciplines play a vital role in solving real-world problems across economic, technological and social domains, as they lead to a better understanding of the complexity of these problems and thus support evidence-based decision-making.

In 2025, one of our key priorities is to strengthen the Journal's academic profile and visibility. We are committed to building a recognised and trusted scientific brand and for that purpose, we are going to take strategic steps toward the inclusion in the most important international indexing databases such as Scopus and the Web of Science. We believe this aim is achievable through continued collaboration with our authors, reviewers and the research community.

We would like to warmly invite you to become part of this journey. Whether you are working on theoretical developments, applied research or methodological innovations in quantitative methods, we encourage you to submit your work and take part in shaping the future of this growing and reputable journal.

On behalf of the Editorial Board,
Krzysztof Echaust
Editor-in-Chief

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# Bayesian analysis of recursive SVAR models with overidentifying restrictions

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**Abstract.** The paper provides a Bayesian methodological framework for the estimation of structural vector autoregression (SVAR) models with recursive identification schemes that allows for the inclusion of overidentifying restrictions. The proposed framework enables the researcher (i) to elicit the prior on non-zero contemporaneous relations between economic variables and (ii) to derive an analytical expression for the posterior distribution and marginal data density. We illustrate our methodological framework by estimating a New-Keynesian SVAR model for Poland.

**Keywords:** structural VAR, Bayesian inference, overidentifying restrictions

**JEL:** C11; C32; E47

## 1. Introduction

Structural vector autoregression (SVAR) models remain a standard tool used for analysing the dynamic propagation of economic shocks. Despite the extensive debate on the 'appropriate' structuralisation of vector autoregression (VAR) models held in the 1980s and 1990s, recursive identification schemes continue to be widely used both in the academic literature and policy analysis, particularly to investigate the effects of monetary shocks (e.g. Christiano et al., 1999, 2005; Uhlig, 2005).[1] This paper contributes to the literature by proposing an analytically tractable prior setup for recursive VARs with potentially overidentifying restrictions that is well-suited to get guidance from the economic theory. We illustrate how these methodological advances can be applied to estimate an SVAR model with the prior centred on the three-equation New-Keynesian model.

The most general approach to dealing with Bayesian SVAR models is arguably that of Waggoner and Zha (2003, WZ). Their algorithm for drawing from the posterior is

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[1] The fact that recursive models deserve extra attention is strengthened by the remark of Sims (2003): 'I personally find the arbitrary triangular ordering a more transparent data summary'.

very efficient under any identifying scheme. In particular, it allows for exact sampling under the triangular identifying scheme. A potential question then arises whether any new special treatment of recursive SVAR models (with overidentifying restrictions) is needed. The answer is yes for two reasons. Firstly, the efficiency of the algorithm in WZ comes at the cost of transparency. Secondly, as stated by WZ (see Waggoner & Zha, 2003, footnote 6), it is not well-suited to incorporate prior beliefs about the coefficients of a model. The reason of the above is that WZ normalise the variances of the disturbances in the SVAR model, which means that the coefficients lose their intuitive interpretation.

The alternative to WZ, designed to incorporate prior beliefs about the structural coefficients of a model, was proposed by Baumeister and Hamilton (2015, BH). This method describes the contemporaneous relations among endogenous variables and was applied to model the dynamics of the oil (Baumeister & Hamilton, 2019) or natural gas market (Rubaszek et al., 2021). The BH approach is very flexible although it comes at the expense of switching from exact sampling to the use of Markov Chain Monte Carlo (MCMC) techniques.

In this paper, we propose a prior for the SVAR model that normalises the coefficients of the contemporaneous relations and at the same time allows for exact sampling. As a consequence, we can directly specify the prior on contemporaneous relations so that inference becomes more intuitive compared to WZ and the setup well-suited to use theoretic economic models as in BH. The advantage of our prior compared to BH is that it shares some convenient features with the standard Normal-Wishart prior (Kadiyala & Karlsson, 1997; Sims & Zha, 1998) such as exact sampling from the posterior and an analytical form of the marginal data density (MDD) in the case of overidentified recursive models. The former characteristic can be useful in the context of the growing literature on large Bayesian VARs (Bańbura et al., 2010; Crump et al., 2025), which could be broadened to large Bayesian SVARs, whereas the latter considerably facilitates setting up a hierarchical prior, similarly to what was done for example in Giannone et al. (2015). The second advantage of our prior is that it allows the researcher to distinguish between the lags of the same and different variables, to centre the prior on the contemporaneous relations present in the economic model, and to impose overidentifying restrictions. In this sense, our prior can be treated as an extension of the Sims and Zha (1998) framework.

The structure of the article is as follows. Section 2 outlines the specification of the proposed prior and derives an analytical expression for the posterior and marginal data density (MDD). Section 3 presents an empirical illustration of our framework based on the New-Keynesian model as described by Orphanides (2003). Section 4 concludes and provides possible avenues for future research. Finally, the Appendix shows that our prior is a generalisation of the standard Normal-Wishart prior for VAR models.

## 2. Structural Bayesian VAR model

We consider an SVAR model of the following form:

$$Ay_t = B_{(1)}y_{t-1} + B_{(2)}y_{t-2} + \cdots + B_{(P)}y_{t-P} + B_{(0)} + \epsilon_t, \tag{1}$$

where $y_t$ is an $N \times 1$ vector of observations, $A$ and $B_{(p)}$ for $p \geq 1$ are $N \times N$ matrices of coefficients, $B_{(0)}$ is the vector of constants and $\epsilon_t \sim \mathcal{N}(0, \Omega)$ is the error term. For covariance matrix $\Omega$, we assume that it is diagonal with $\omega_n$ elements. To simplify the notation, we rewrite (1) as:

$$Ay_t = Bx_t + \epsilon_t, \tag{2}$$

where $x_t = [y_{t-1}{}' \; y_{t-2}{}' \; \dots \; y_{t-P}{}' \; 1]'$ is a $K$-dimensional vector and $B =$ $= \left[B_{(1)} \; B_{(2)} \; \dots B_{(P)} \; B_{(0)}\right]$ a matrix of size $N \times K$ with $K = PN + 1$.

The $n$-th equation of (2) can be written as:

$$A_n y_t = B_n x_t + \epsilon_{nt} \tag{3}$$

with $A_n = [a_{n1} \; a_{n2} \; \dots \; a_{nN}]$ and $B_n = [b_{n1} \; b_{n2} \; \dots \; b_{nK}]$ representing the $n$-th rows of matrices $A$ and $B$, respectively.

We impose the following restrictions on the $A$ matrix:

1. The elements on the diagonal satisfy $a_{nn} = 1$ (normalisation);
2. The determinant is $|A| = 1$;
3. There are $M_n$ free parameters of $A_n$, which are estimated (gathered in row vector $\tilde{A}_n$) and $N - (M_n + 1)$ parameters set to zero.

Following Waggoner and Zha (2003), we write down these restrictions as:

$$A_n = \left[1 \; \tilde{A}_n\right] S_n \tag{4a}$$

$$\tilde{A}_n = A_n S_n^* \tag{4b}$$

where $S_n$ and $S_n^*$ are selection matrices consisting of zeros and ones of size $(M_n + 1) \times N$ and $N \times M_n$, respectively.

The assumption that $|A| = 1$ means that our framework is suitable for a lower or upper triangular $A$ (or restricted subsets). Given this limitation, we will show that this setup is well-designed to introduce contemporaneous relations and overidentifying restrictions.

## 2.1. Prior specification

We propose the prior specification of the following form:[2]

$$p(\Omega) = \prod_{n=1}^{N} p\left(\omega_n\right) \equiv \prod_{n=1}^{N} \mathcal{IG}\left(\underline{v}_{1n}, \underline{v}_{2n}\right), \tag{5a}$$

$$p(A|\Omega) = \prod_{n=1}^{N} p\left(\tilde{A}_n|\Omega\right) \equiv \prod_{n=1}^{N} \mathcal{N}\left(\underline{A}_n, \omega_n \underline{F}_n\right), \tag{5b}$$

$$p(B|A,\Omega) = \prod_{n=1}^{N} p\left(B_n|A,\Omega\right) \equiv \prod_{n=1}^{N} \mathcal{N}\left(\underline{B}_n, \omega_n \underline{G}_n\right), \tag{5c}$$

where $\mathcal{N}$ stands for the normal pdf and the inverted gamma $\mathcal{IG}$ pdf is defined as:

$$\mathcal{IG}(v_1, v_2) := p(x) = v_2^{v_1}[\Gamma(v_1)]^{-1} x^{-(v_1+1)} \exp\{-v_2/x\},\ v_1, v_2 > 0. \tag{6}$$

The underlined parameters are fixed and depend on a set of hyperparameters, the values of which are chosen so that for the exactly identified models, our prior corresponded to that of the standard Wishart-Normal prior.[3]

For $p(\Omega)$, we suggest the following setting:

$$\begin{aligned} \underline{v}_{1n} &= 1/2\left(\underline{v} - (N - M_n - 1)\right) \\ \underline{v}_{2n} &= 1/2\left(\underline{v} - N - 1\right)\hat{\sigma}_n^2, \end{aligned} \tag{7}$$

where $\{\hat{\sigma}_n^2 : n = 1,2,\dots,N\}$ are estimated variances of the residuals from univariate autoregressions and $\underline{v}$ is the first hyperparameter.

In the case of $p(A|\Omega)$, we need to set $\underline{A}_n$ and $F_n$. The choice of the former depends on the underlying economic model. For the latter, we suggest:

$$\underline{F}_n = S_n^{*\prime} \operatorname{diag}\left(\left(\frac{\lambda_0}{\hat{\sigma}_1}\right)^2, \left(\frac{\lambda_0}{\hat{\sigma}_2}\right)^2, \dots, \left(\frac{\lambda_0}{\hat{\sigma}_N}\right)^2\right) S_n^*, \tag{8}$$

with $\lambda_0$ being the second hyperparameter.

[2] If $M_n$=0 and $\tilde{A}_n$ is the empty matrix, we set $p\left(\tilde{A}_n|\Omega\right)$ to unity.

[3] In Appendix A, we show that the standard Wishart-Normal prior is a specific case of our prior specification.

Finally, for $p(B|A,\Omega)$, we follow closely and set:

$$\underline{B}_n = A_n \underline{B}_*, \tag{9}$$

where $\underline{B}_*$ is an $N \times K$ matrix of the following form:

$$\underline{B}_* = [\ \underbrace{D}_{NxN}\ \ \underbrace{0}_{NxN}\ \ \underbrace{0}_{NxN}\ \dots\ \underbrace{0}_{NxN}\ \ \underbrace{0}_{Nx1}\ ]. \tag{10}$$

The usual practice is to assume that $D = \mathrm{diag}(1,1,\dots,1)$ so that the prior is concentrated on $N$ random walk (RW) processes. In the next section, we show that it might be justified to select a non-standard form of $\underline{B}_*$ so that the prior is concentrated on the underlying economic model.

As regards $\omega_n \underline{G}_n$, we assume it to be a diagonal matrix with elements corresponding to the prior variance of the coefficient for variable $y_{j,t-p}$:

$$\begin{array}{ll} \omega_n \left(\dfrac{\lambda_1}{\hat{\sigma}_j \times p^{\lambda_4}}\right)^2 & \text{if } a_{nj} \text{ is a free element in } A_n \\ \omega_n \left(\dfrac{\lambda_1 \lambda_2}{\hat{\sigma}_j \times p^{\lambda_4}}\right)^2 & \text{otherwise.} \end{array} \tag{11}$$

Hyperparameter $\lambda_1$ controls the overall tightness, $\lambda_2 \in (0,1)$ differentiates between variables with and without a contemporaneous impact on $y_{nt}$ and $\lambda_4$ is the lag decay. Finally, the prior variance for the constant term in the $n$-th equation is:

$$\omega_n \lambda_3^2, \tag{12}$$

where for large values of hyperparameter $\lambda_3$, the prior for the constant term is diffuse.

## 2.2. Posterior draw

Let $Y = [y_1\ y_2\ \dots\ y_T]'$ and $X = [x_1\ x_2\ \dots\ x_T]'$ be observation matrices of size $T \times N$ and $T \times K$, respectively, where $T$ is the sample size. The likelihood function is:[4]

$$p(Y\,\Big|A,B,\Omega) = (2\pi)^{-\frac{NT}{2}}\Big|\,\Omega|^{-\frac{T}{2}} etr\{-1/2\,\Omega^{-1}(AY' - BX')(AY' - BX')'\}, \tag{13}$$

where $etr\{\Lambda\} = exp(tr\{\Lambda\})$ is the exponent of the matrix trace.

[4] NB. We assume that |A|=1.

The algorithm of drawing from the posterior:

$$p(A, B, \Omega|Y) = p(\Omega|A, B, Y)p(B|A, Y)p(A|Y) \tag{14}$$

consists of three steps:
i. draw $A$ from $p(A|Y)$;
ii. draw $B$ from $p(B|A, Y)$;
iii. draw $\Omega$ from $p(\Omega|A, B, Y)$.

An appealing feature of our prior setup is that distributions $p(\Omega|A, B, Y)$, $p(B|A, Y)$ and $p(A|Y)$ have an analytical form and there is no need to resort to MCMC techniques. In what follows, we derive the exact formulas.

**Posterior $p(\Omega|A, B, Y)$**

The Bayes formula implies that:

$$p(\Omega|A, B, Y) \propto p(Y|A, B, \Omega)p(B|A, \Omega)p(A|\Omega)p(\Omega). \tag{15}$$

By substituting (5) and (13) to (15), given the diagonal form of $\Omega$, it can be derived that:

$$\omega_n|A, B, Y \sim \mathcal{IG}(\overline{v}_{1n}, \overline{v}_{2n}) \tag{16}$$

with:[5]

$$\begin{aligned} \overline{v}_{1n} &= \underline{v}_{1n} + T + K + M_n/2 \\ \overline{v}_{2n} &= \underline{v}_{2n} + \frac{(A_nY' - B_nX')(A_nY' - B_nX')' + (B_n - \underline{B}_n)\underline{G}_n^{-1}(B_n - \underline{B}_n)' + (\tilde{A}_n - \underline{A}_n)\underline{F}_n^{-1}(\tilde{A}_n - \underline{A}_n)'}{2} \end{aligned} \tag{17}$$

The diagonal form of $\Omega$ also means that:

$$p(\Omega|A, B, Y) = \prod_{n=1}^{N} p\,(\omega_n|A, B, Y). \tag{18}$$

[5] To simplify the notation, if $M_n = 0$, the term $(\tilde{A}_n - \underline{A}_n)\underline{F}_n^{-1}(\tilde{A}_n - \underline{A}_n)'$ drops out in all formulas of this section.

**Posterior $p(B|A,Y)$**

We start the computation of $p(B|A,Y)$ by noticing that:

$$p(A_n, B_n|Y) \propto \left( (B_n - \overline{B}_n)\overline{G}_n^{-1}(B_n - \overline{B_n})' + \varsigma_n \right)^{-\overline{v}_{1n}} \tag{19}$$

with:

$$\begin{aligned} \overline{B}_n &= \left(\underline{B}_n \underline{G}_n^{-1} + A_n Y'X\right)\overline{G}_n \\ \overline{G}_n &= \left(X'X + \underline{G}_n^{-1}\right)^{-1} \\ \varsigma_n &= A_n Y'YA_n' + \left(\tilde{A}_n - \underline{A}_n\right)\underline{F}_n^{-1}\left(\tilde{A}_n - \underline{A}_n\right)' + \underline{B}_n \underline{G}_n^{-1} \underline{B}_n' - \overline{B}_n \overline{G}_n^{-1} \overline{B}'_n + 2\underline{v}_{2n}. \end{aligned} \tag{20}$$

The result above follows from two observations. First, it is possible to calculate the joint distribution:

$$p(A,B|Y) = \frac{p(A,B,\Omega|Y)}{p(\Omega|A,B,Y)}. \tag{21}$$

The denominator is given by (16)-(18), whereas the nominator can be computed with (5) and (13) as $p(A,B,\Omega|Y) \propto p(Y|A,B,\Omega)p(B|A,\Omega)p(A|\Omega)p(\Omega)$. The second observation is that, given the structure of model (1), it is possible to decompose $p(A,B|Y)$ into:

$$p(A,B|Y) = \prod_{n=1}^{N} p\left(A_n, B_n|Y\right). \tag{22}$$

With (19) and (20), it can be shown that:

$$B_n|A_n, Y \sim t_K\left(\overline{B}_n, \overline{G}_n, \varsigma_n, g_n\right), \tag{23}$$

where $g_n = T + M_n + 2\underline{v}_{1n}$. Here, $t_K(\mu, \Sigma, \theta, \gamma)$ denotes $K$-dimensional $t$-Student pdf with $\gamma$ degrees of freedom:

$$\begin{aligned} &t_K(\mu, \Sigma, \theta, \gamma) := p(x) = \\ &= (\gamma\pi)^{-\frac{K}{2}}|\Sigma|^{-\frac{1}{2}}\frac{\Gamma\left((\gamma+K)/2\right)}{\Gamma(\gamma/2)}\theta^{\frac{\gamma+K}{2}}\{\theta\,(x-\mu)\Sigma^{-1}(x-\mu)'\}^{-\frac{\gamma+K}{2}}. \end{aligned} \tag{24}$$

Finally, by analogy to (21), the conditional distribution $p(B|A,Y)$ is:

$$p(B|A,Y)=\prod_{n=1}^{N}p\left(B_n|A_n,Y\right). \tag{25}$$

**Posterior $p(A|Y)$**

Let us define:

$$\begin{gathered}R_n=\begin{bmatrix}R_{n,11} & R_{n,12}\\ R_{n,21} & R_{n,22}\end{bmatrix}=\\ =S_n\left[Y'Y+\underline{B}_*\underline{G}_n^{-1}\underline{B}_*'-\left(\underline{B}_*\underline{G}_n^{-1}+Y'X\right)\overline{G}_n\left(\underline{B}_*\underline{G}_n^{-1}+Y'X\right)'\right]S_n',\end{gathered} \tag{26}$$

where $R_{n,11}$ is a scalar and $R_{n,22}$ an $M_n\times M_n$ matrix so that:

$$\left[1\ \tilde{A}_n\right]R_n\left[1\ \tilde{A}_n\right]'=A_nY'YA_n'+\underline{B}_n\underline{G}_n^{-1}\underline{B}_n'-\overline{B}_n\overline{G}_n^{-1}\overline{B}_n'. \tag{27}$$

Distribution $p\left(\tilde{A}_n|Y\right)$ can be computed by integrating out $B_n$ from $p(A_n,B_n|Y)$, which is given by (19). The result is a multivariate $t$-Student:

$$\tilde{A}_n|Y\sim t_{M_n}\left(\overline{A}_n,\overline{F}_n,\chi_n,f_n\right), \tag{28}$$

where:

$$\begin{aligned}\overline{A}_n &=\left(\underline{F}_n^{-1}\underline{A}_n'-R_{n,21}\right)'\overline{F}_n,\\ \overline{F}_n &=\left(R_{n,22}+\underline{F}_n^{-1}\right)^{-1},\\ \chi_n &=R_{n,11}+\underline{A}_n\underline{F}_n^{-1}\underline{A}_n'-\overline{A}_n\overline{F}_n^{-1}\overline{A}_n'+2\underline{v}_{2n},\\ f_n &=T+2\underline{v}_{1n}.\end{aligned} \tag{29}$$

Finally, posterior $p(A|Y)$ is:[6]

$$p(A|Y)=\prod_{n=1}^{N}p\left(\tilde{A}_n|Y\right). \tag{30}$$

[6] For $M_n=0$, we set $p\left(\tilde{A}_n|Y\right)$ to unity.

## 2.3. Marginal data density

Another advantageous feature of our prior setup is that there is an analytical form of the marginal data density. To derive it, we need to calculate the following integral:

$$p(Y) = \int p(Y|A,B,\Omega)p(B|A,\Omega)p(A|\Omega)p(\Omega)dAdBd\Omega. \tag{31}$$

We start by evaluating $p(Y|A,\Omega) = \int p(Y|A,B,\Omega)p(B|A,\Omega)dB$. The combination of (5c) and (13) leads to:

$$\begin{aligned} p(Y|A,B,\Omega) \quad p(B|A,\Omega) &= (2\pi)^{-NT/2}|\Omega|^{-T/2} etr\{-1/2\,\Omega^{-1}(AY'-BX')(AY'-BX')'\} \times \\ &\times (2\pi)^{-\frac{NK}{2}} \prod_{n=1}^{N} |\underline{G}_n|^{-0.5}\, \omega_n^{-K/2} \exp\{-1/2\,\omega_n^{-1}(B_n-\underline{B}_n)\underline{G}_n^{-1}(B_n-\underline{B}_n)'\}. \end{aligned} \tag{32}$$

Integrating out $B$ yields:

$$\begin{aligned} p(Y|A,\Omega) = \\ = \kappa_1 \prod_{n=1}^{N} \omega_n^{-T/2} \exp\{-\frac{1}{2}\omega_n^{-1}\left(A_nY'YA_n' + \underline{B}_n\underline{G}_n^{-1}\underline{B}_n' - \overline{B}_n\overline{G}_n^{-1}\overline{B}_n'\right)\}, \end{aligned} \tag{33}$$

where $\kappa_1 = (2\pi)^{-NT/2}\prod_{n=1}^{N}\left(|\overline{G}_n|/|\underline{G}_n|\right)^{0.5}$.

Next, we calculate $p(A,Y) = \int p(A,\Omega,Y)d\Omega = \int p(Y|A,\Omega)p(A|\Omega)p(\Omega)d\Omega$. By combining (5a), (5b) and (33), we obtain:

$$\begin{aligned} p(A,\Omega,Y) &= \kappa_1 \prod_{n=1}^{N} \omega_n^{-T/2} \exp\{-\frac{1}{2}\omega_n^{-1}\left(A_nY'YA_n' + \underline{B}_n\underline{G}_n^{-1}\underline{B}_n' - \overline{B}_n\overline{G}_n^{-1}\overline{B}_n'\right)\} \times \\ &\times \prod_{n=1}^{N} (2\pi)^{-\frac{M_n}{2}} |\underline{F}_n|^{-0.5} \omega_n^{-\frac{M_n}{2}} \exp\{-(\tilde{A}_n-\underline{A}_n)\underline{F}_n^{-1}(\tilde{A}_n-\underline{A}_n)'/2\omega_n\} \times \\ &\times \prod_{n=1}^{N} [\Gamma(\underline{v}_{1n})]^{-1} (\underline{v}_{2n})^{\underline{v}_{1n}} \omega_n^{-(\underline{v}_{1n}+1)} \exp\{-\underline{v}_{2n}/\omega_n\}. \end{aligned} \tag{34}$$

Integrating out Ω from (34) yields:

$$\begin{aligned} p(A,Y) &= \kappa_1\kappa_2 \prod_{n=1}^{N} \Gamma\left(\frac{g_n}{2}\right)(\varsigma_n)^{-\frac{g_n}{2}} = \\ &= \kappa_1\kappa_2 \prod_{n=1}^{N} \Gamma\left(\frac{g_n}{2}\right)\left((\tilde{A}_n - \overline{A}_n)\overline{F}_n^{-1}(\tilde{A}_n - \overline{A}_n)' + \chi_n\right)^{-\frac{g_n}{2}}, \end{aligned} \tag{35}$$

where $\kappa_2 = 2^{NT/2}\prod_{n=1}^{N}\pi^{-M_n/2}\left|\underline{F}_n\right|^{-0.5}\Gamma\left(\underline{v}_{1n}\right)^{-1}\left(2\underline{v}_{2n}\right)^{\underline{v}_{1n}}$.

In the last step, we compute integral $p(Y) = \int p(A,Y)dA$. Let us notice that

$$\begin{aligned} &\int \Gamma\left(\frac{g_n}{2}\right)\left((\tilde{A}_n - \overline{A}_n)\overline{F}_n^{-1}(\tilde{A}_n - \overline{A}_n)' + \chi_n\right)^{-g_n/2} d\tilde{A}_n = \\ &= \pi^{\frac{M_n}{2}}\Gamma(f_n/2)\left|\overline{F}_n\right|^{0.5}|\chi_n|^{-\frac{f_n}{2}}. \end{aligned} \tag{36}$$

As a result, the marginal data density is:

$$\begin{aligned} p(Y) &= \kappa_1\kappa_2 \prod_{n=1}^{N} \pi^{\frac{M_n}{2}}\Gamma(f_n/2)\left|\overline{F}_n\right|^{0.5}|\chi_n|^{-\frac{f_n}{2}} = \\ &= \pi^{-NT/2}\prod_{n=1}^{N}\left(\frac{\left|\overline{F}_n\right|\left|\overline{G}_n\right|}{\left|\underline{F}_n\right|\left|\underline{G}_n\right|}\right)^{0.5} \times \frac{\Gamma\left(\frac{T}{2} + \underline{v}_{1n}\right)}{\Gamma\left(\underline{v}_{1n}\right)} \times \left(2\underline{v}_{2n}\right)^{\underline{v}_{1n}}\chi_n^{-\left(\underline{v}_{1n} + \frac{T}{2}\right)}. \end{aligned} \tag{37}$$

## 2.4. Advantages of our prior setup

We consider the prior specification above as advantageous for the following reasons:

a) It provides an intuitive framework for setting priors on the contemporaneous relationship between variables on the basis of the economic theory;
b) It generalises the commonly used Normal-Wishart prior for VARs (Appendix A);
c) It enables overidentifying restrictions in recursive identification schemes and the sampling from the posterior distribution is exact;
d) There is an analytical expression for the marginal data density which facilitates model comparisons and the choice of hyperparameters;
e) One may differentiate between the lag of the same or different variables, as advocated e.g. by Litterman (1986).

The next section illustrates all the advantages through the application of the methodological framework to calculate impulse responses from a structural VAR model with priors taken from a backward-looking New Keynesian model.

## 3. Empirical application

We consider a small New Keynesian model that consists of three equations expressed in terms of output gap $z_t$, inflation $\pi_t$ and nominal interest rate $R_t$ (see Orphanides, 2003, for a more detailed description):

$$z_t = \rho_z z_{t-1} - \xi(R_{t-1} - \pi_{t-1}) + \epsilon_t^D, \tag{38a}$$

$$\pi_t = \rho_\pi \pi_{t-1} + \kappa z_t + \epsilon_t^{MU}, \tag{38b}$$

$$R_t = \rho_R R_{t-1} + \gamma \pi_t + \epsilon_t^{MP}, \tag{38c}$$

where $\epsilon_t^D$, $\epsilon_t^{MU}$ and $\epsilon_t^{MP}$ stand for the demand, mark-up and monetary shock, respectively. For convenience, the three equations could be labelled as an IS curve, a Phillips Curve and a simplified Taylor rule, respectively. We illustrate the dynamics of this model by calculating the impulse response function (IRF) from the SVAR model of the form shown in (1) with the prior given by model (38).

From Eurostat, we collect quarterly data describing the Polish economy over the period of 2004:1-2024:4. For $z_t$, $\pi_t$ and $R_t$, we use the following series: the 3-month WIBOR (quarterly average), GDP deflator (seasonally adjusted, quarter on quarter at an annualised rate) and GDP (SCA, constant prices). The output gap is calculated as a cyclical part with the Hodrick-Prescott filter (with $\lambda = 1{,}600$).

Let $y_t = [R_t \ \pi_t \ z_t]'$ so that we could write down (38) in the form of SVAR (1) with the prior centred on:

$$E(A) = \begin{bmatrix} 1 & -\gamma & 0 \\ 0 & 1 & -\kappa \\ 0 & 0 & 1 \end{bmatrix} \text{ and } E(B) = \begin{bmatrix} \rho_R & 0 & 0 & 0 \\ 0 & \rho_\pi & 0 & 0 \\ -\xi & \xi & \rho_z & 0 \end{bmatrix}. \tag{39}$$

Apart from $\gamma$ and $\kappa$, we fix the remaining parameters of the $A$ matrix at zero, which means that we impose one overidentifying restriction. As discussed in the methodological part of the paper, our setup makes it straightforward to elicit non-zero prior beliefs for contemporaneous relations. To achieve this, we set $\kappa = 0.1$ in the Phillips curve and $\gamma = 0.15$ in the Taylor rule. For the remaining parameters, we set $\xi = 0.1$, $\rho_z = 0.9$, $\rho_\pi = 0.9$ and $\rho_R = 0.9$. The values above are broadly in line with the literature on New Keynesian (e.g Orphanides, 2003).

For the hyperparameters, we choose values close to those suggested by Sims and Zha (1998) and set $\lambda_0 = 1$, $\lambda_3 = 1000$, $\lambda_4 = 1$, $\underline{v} = N + 2$, whereas for hyperparameter $\lambda_2$ that is not present in the normal-Wishart setup, we set $\lambda_2 = 0.5$. We do not fix the overall tightness hyperparameter at a specified value, but assume a hierarchical prior structure, as advocated by e.g. Giannone et al. (2015). In particular, we assume $\lambda_1 \sim \mathcal{IG}(2{,}0.1)$ so that $E(\lambda_1) = 0.1$.

Let us notice that, depending on the hyperparameters, the marginal data density is available in a closed form (see 37). Treating $\lambda_1$ as an unknown parameter, (37) can be written as $p(Y|\lambda_1)$. The marginal posterior of $\lambda_1$ is:

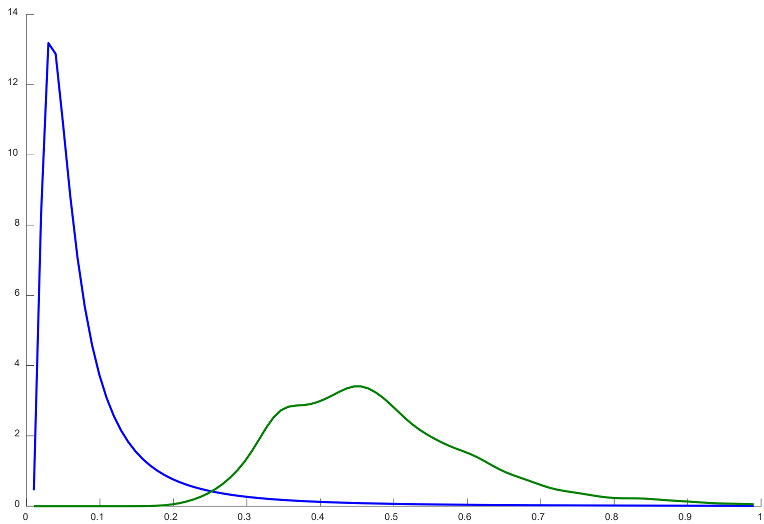
$$p(\lambda_1|Y) \propto p(\lambda_1)p(Y|\lambda_1). \tag{40}$$

As a result, the Random Walk Metropolis-Hastings (MH) algorithm, which involves drawing from posterior of $\lambda_1$ and calculating impulse responses, is as follows:

i. Set $j = -J_0$ and initialise $\lambda_1^{(j-1)} = 0.1$;
ii. Draw candidate $\lambda_1^* = \lambda_1^{(j-1)} + \delta\epsilon$, where $\delta$ is a calibrating factor and $\epsilon \sim \mathcal{N}(0{,}1)$;
iii. Calculate $\theta = \min\{1, \frac{p(\lambda_1^*)p(Y|\lambda_1^*)}{p\left(\lambda_1^{(j-1)}\right)p\left(Y|\lambda_1^{(j-1)}\right)}\}$ and draw $u$ from $\mathcal{U}(0{,}1)$, where $\mathcal{U}$ denotes the uniform distribution on $(0{,}1)$;
iv. If $\theta < u$, set $\lambda_1^{(j)} = \lambda_1^{(j-1)}$, otherwise set $\lambda_1^{(j)} = \lambda_1^*$;
v. If $j > 0$, draw $A$, $B$ and $\Omega$ from $p\left(A, B, \Omega|Y, \lambda_1^{(j)}\right)$ and compute the value of IRF;
vi. If j<J, go to (ii). Otherwise stop.

The values of $J_0 = 1{,}000$ and $J = 100{,}000$ describe the size of the burn-in sample and the number of MH draws. As a result, after running the algorithm, we obtain $J = 100{,}000$ realisations of IRF from the posterior.

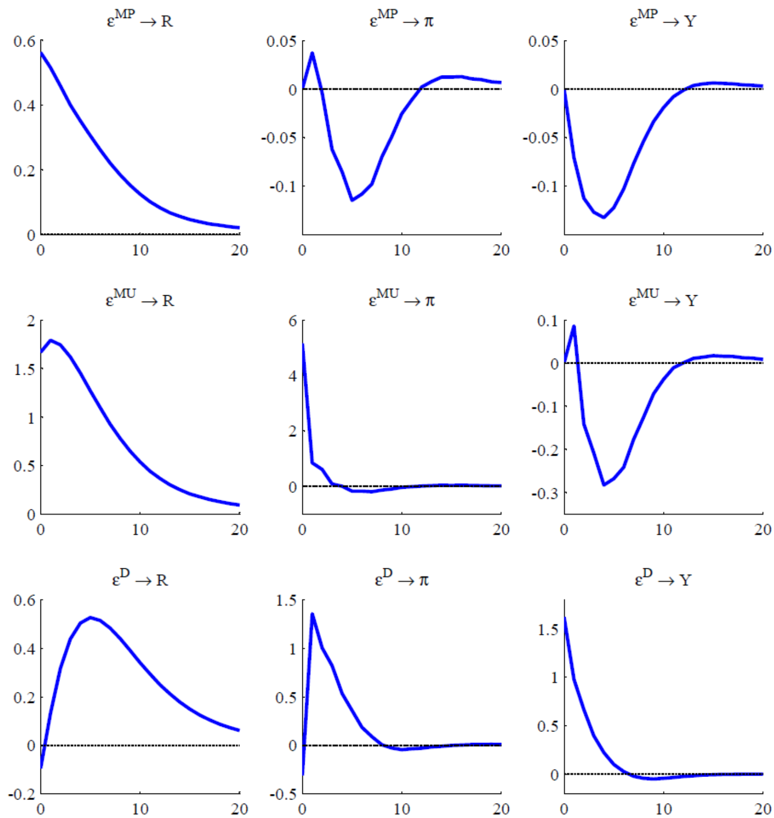
**Figure 1.** Prior and posterior density of an overall tightness hyperparameter




Note. The blue and green lines stand for prior and posterior density, respectively.
Source: authors' calculations.

Figure 1 presents the prior and posterior density of $\lambda_1$, showing that the mean of the latter (0.48) is much higher compared to the former (0.1). This means that MDD is higher when $\lambda_1$ ranges between 0.3 and 0.6, rather than if one takes the standard value of 0.1. Figure 2 outlines the median value of impulse responses for the three shocks of the model. A standardised monetary policy shock is characterised by a temporary but rather persistent increase of the nominal interest rate by about 55 basis points. The negative impact of the monetary shock on inflation and output (relative to the trend) reaches the peak about 1.5 years after the shock, with annualised inflation falling by 0.1 percentage point and output by 0.15%. The mark-up shock exerts an immediate impact on inflation of about 5 percentage points and a contemporaneous response of monetary policy, evidenced by the rise in the nominal interest rate by almost 2 percentage points. The impact on the output gap is lagged and negative, amounting to about 0.25% after one year from the occurrence of the shock. Finally, a positive demand shock raises output by 1.5% relative to the trend with an impact on inflation of about 1 percentage point in the next quarter. The overall impact on both variables eventually dies out, as the rise in the nominal interest rate has an offsetting impact. The properties of the estimated model are therefore very intuitive, stemming from (i) the model structure (ii), the VAR dynamics and also (iii) the priors of the modeler about the coefficients in the structural equations.

**Figure 2.** Impulse response functions




Note. Median of posterior draws.
Source: authors' calculations.

## 4. Conclusions

In this paper, we have proposed a Structural Bayesian recursive VAR framework that has several novel features compared to the existing methods. The prior setup that we have designed is advantageous from the econometric perspective as the MDD has an analytical form and there is no need to resort to MCMC techniques. Our prior setup is also appealing from an economic perspective: it is effective in eliciting priors on the contemporaneous relationship between variables, thus facilitating a meaningful definition of prior beliefs consistent with the economic theory. This paper opens a number of new avenues for further research. The ability of drawing from exact distributions could be exploited through a variety of applications, for example for setting up a large SVAR model or in the context of applications with different hierarchical priors. Additionally, the current framework appears particularly useful in applications where the researcher has prior beliefs on the contemporaneous coefficients of a given model. Finally, from a theoretical perspective, this methodological framework can be extended to alternative identification schemes and forward-looking models.

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# Appendix

## Prior comparison with WZ

In this appendix, we show that the commonly used Normal-Wishart prior for VAR models (Kadiyala & Karlsson, 1997) is a specific case of our prior defined in (5). Let Σ denote the error term covariance matrix of the reduced form representation corresponding to the structural model given by (1):

$$\Sigma = A^{-1}\Omega A'^{-1}. \tag{A1}$$

The prior for $\Sigma$ is of the inverted Wishart ($\mathcal{IW}$) form:

$$p(\Sigma) \propto |\Sigma|^{-\frac{1}{2}(\underline{v}+N+1)} etr\{-0.5\Sigma^{-1}\underline{Q}\}. \tag{A2}$$

It is common practice to set:

$$\underline{Q} = \left(\underline{v} - N - 1\right) \times \operatorname{diag}\left(\left(\frac{\hat{\sigma}_1}{\lambda_0}\right)^2, \left(\frac{\hat{\sigma}_2}{\lambda_0}\right)^2, \dots, \left(\frac{\hat{\sigma}_N}{\lambda_0}\right)^2\right) \tag{A3}$$

so that:

$$E(\Sigma) = \operatorname{diag}\left(\left(\frac{\hat{\sigma}_1}{\lambda_0}\right)^2, \left(\frac{\hat{\sigma}_2}{\lambda_0}\right)^2, \dots, \left(\frac{\hat{\sigma}_N}{\lambda_0}\right)^2\right). \tag{A4}$$

Below, we elicit the values of $\underline{v}_{1n}$, $\underline{v}_{2n}$, $\underline{A}_n$ and $\underline{F}_n$ for our prior setup that are consistent with the $\mathcal{IW}$ prior and $\underline{Q}$ given by (A2) and (A3).

We consider a case in which $A$ is unit upper triangular so that the correspondence between $\Sigma$ and $\{A, \Omega\}$ is one-to-one. To derive the joint prior for $\{A, \Omega\}$, we substitute the Jacobian:

$$\mathcal{J}(\Sigma \to A, \Omega) = \prod_{n=1}^{N} (\omega_n)^{n-1} \tag{A5}$$

into (A2), which yields:

$$p(A, \Omega) = \prod_{n=1}^{N} \omega_n^{-\frac{1}{2}(\underline{v}+N-2n+3)} \times \exp\{-0.5\omega_n^{-1} A_n \underline{Q} A_n'\} \tag{A6}$$

and the conditional prior for $A$

$$p(A|\Omega) \propto \left| \quad \right| \exp\{-0.5\omega_n^{-1} A_n Q A_n{}'\}. \tag{A7}$$

Let us define:

$$Q_n = S_n Q S_n', \tag{A8}$$

where selection matrices $S_n$ introduced in (5) for the upper-triangular $A$ are:

$$S_n = \left[ 0_{(N-n+1)\times(n-1)} \; I_{N-n+1} \right]. \tag{A9}$$

Given the form of $\underline{Q}$ in (A3), we can partition $\underline{Q}_n$ into:

$$\underline{Q}_n = \begin{bmatrix} \underline{q}_{nn}^* & 0 \\ 0 & \underline{Q}_n^* \end{bmatrix}, \tag{A10}$$

where $\underline{q}_{nn} = \left(\underline{v} - N - 1\right)\lambda_0^{-2}\hat{\sigma}_n^2$ and $\underline{Q}_n^* = \left(\underline{v} - N - 1\right)\lambda_0^{-2}\mathrm{diag}(\hat{\sigma}_{n+1}^2, \dots, \hat{\sigma}_N^2)$.[7] Consequently, (A7) can be written as:

$$p(A|\Omega) \propto \prod_{n=1}^{N-1} \exp\{-0.5\omega_n^{-1}\tilde{A}_n \underline{Q}_n^* \tilde{A}'_n\} = \prod_{n=1}^{N-1} \mathcal{N}\left(0, \omega_n \underline{Q}_n^{*\,-1}\right). \tag{A11}$$

It is now evident that the conditional prior given by (A11) is a specific form of the prior defined in (5b), i.e. if we set $\underline{A}_n = 0$ and $\underline{F}_n = \underline{Q}_n^{*\,-1}$. Let us notice that this choice of the prior corresponds to the value of $\underline{F}_n$ proposed in (8).

Finally, we derive the marginal prior for $\Omega$ induced by the $\mathcal{IW}$ prior (A2). Since $p(\Omega) = \int p(A, \Omega) dA$, the use of (A6) yields:

$$p(\Omega) = \prod_{n=1}^{N} \mathcal{IG}\left(\frac{1}{2}\left(\underline{v} - n + 1\right), \frac{1}{2}\underline{q}_{nn}\right), \tag{A12}$$

where the $\mathcal{IG}()$ pdf is defined in (6). It is now evident that we need to set $\underline{v}_{1n} = 0.5 \left(\underline{v} - (n-1)\right)$ and $\underline{v}_{2n} = 0.5\underline{q}_{nn}$ in the prior defined in (5a) so that it is consistent with the $\mathcal{IW}$ prior. Let us notice that for upper triangular $A$, when $(N - M_n - 1) = = (n-1)$, these are the values of $\underline{v}_{1n}$ and $\underline{v}_{1n}$ proposed in (7).

[7] Notice that $\underline{Q}_N^*$ reduces to the empty matrix.

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# Fiscal Multipliers for the United States

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**Abstract.** The aim of this study is to trace the effects of fiscal policy shocks. We calculate the level of fiscal multipliers and short-term output fiscal elasticities for the United States. We do so by estimating a Bayesian three-variate fiscal vector autoregression model that accounts for uncertain identification assumptions. The government spending multiplier is equal to 1.65 on impact and 0.53 after one year, while the tax multiplier is equal to -2.00 on impact and -0.10 after one year. The posterior output elasticity of taxes is equal to 2.20. Increasing the prior assumptions for output elasticity of taxes leads to a lower tax multiplier. The study shows that both increasing spending and decreasing taxes can stimulate the economy. However, the effects of tax decreases may be larger for the economy.

**Keywords:** fiscal multipliers, Bayesian VAR model, short-term elasticities, output elasticity of taxes

**JEL:** C11, C32, E62, E63, H50

## 1. Introduction

Fiscal policy can play a crucial role in supporting aggregate demand, particularly when policy rates are at their effective lower bound and when the economy is in recession (see Auerbach & Gorodnichenko, 2012; Ramey & Zubairy, 2018). The effectiveness of fiscal policy actions may be measured using fiscal multipliers, whose quantification is a difficult task (see Angelini et al., 2023; Čapek & Cuaresma, 2020).

Many studies which use vector autoregression (VAR) models (i.e. Angelini et al., 2023; Caldara & Kamps, 2017; Mertens & Ravn, 2014) agree that the differences in fiscal multipliers estimates stem from different assumptions concerning the level of contemporaneous elasticities. Caldara and Kamps (2017) analytically show a negative relationship between the systemic response to the output of fiscal variables[1] and the size of tax and spending multipliers. Mertens and Ravn (2014) state that the output elasticity of tax revenues is significantly greater than calculated by international organisations (by, for instance, Blanchard & Perotti, 2002). This leads them to the conclusion that tax multipliers are at the higher end of the range, such as those of Mountford and Uhlig (2009) and Romer and Romer (2010).

Building on Blanchard and Perotti (2002) and the subsequent studies, this study extends the existing literature in three ways. Firstly, we employ a slightly different

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[1] Meaning short-term output elasticities of government spending and tax revenues.

methodology estimating the size of fiscal elasticities and the associated fiscal multipliers using the flexible Bayesian SVAR methodology of Baumeister and Hamilton (2015, 2018, 2019). Secondly, our study can be viewed as an extension of Sznajderska et al. (2024), who applied this methodology to examine the effects of both monetary and fiscal policy in the United States. Unlike their study, we use a three-equation model à la Blanchard and Perotti (2002) that allows us to focus more precisely on the effects of the fiscal policy without identifying monetary and price shocks. Thirdly, our study diverges from Sznajderska et al. (2024), as it tests different prior assumptions. Specifically, we concentrate on the prior assumptions for output tax short-term elasticity, following the prominent study of Mertens and Ravn (2014).

The aim of the research described in this article is to measure the level of fiscal multipliers in the United States using the Baumeister and Hamilton (2015, 2018, 2019) approach. The two research questions are: what are the values of fiscal multipliers in a three-variate Baumeister and Hamilton fiscal VAR model? How do the results change when we set different prior assumptions for the output elasticity of taxes?

The paper is organised as follows. Section 2 briefly summarises the literature on the subject, Section 3 describes the model and data, and Section 4 presents the results. The last section summarises our conclusions.

## 2. Literature review

Fiscal VAR models are widely studied to evaluate the effects of the fiscal policy in the United States (see Angelini et al., 2023; Auerbach & Gorodnichenko, 2012; Blanchard & Perotti, 2002; Caldara & Kamps, 2017; Klein & Linnemann, 2019; Mountford & Uhlig, 2009; Sznajderska et al., 2024). A common approach in this respect is to use proxy-VAR models, as it is challenging to correctly identify unexpected fiscal shocks in traditional VAR models. This difficulty results from the fact that, for instance, any planned fiscal policy actions are often announced well before they are implemented, a phenomenon known as fiscal foresight. However, the necessity of finding the correct proxies is the main disadvantage of the proxy-VAR method. Angelini et al. (2023) show that the set of the used instruments (proxies) can crucially affect the multiplier. Additionally, the problem of potentially endogenous proxies arises. This may concern whether or not to impose an orthogonality assumption between tax shocks and total factor productivity shock (see Angelini et al., 2023) or government spending shocks and total factor productivity shock (see Ben Zeev & Pappa, 2015).

Fiscal policy faces not only implementation lags (caused by the fact that it takes time for policy changes to be put into effect), but also decision lags (resulting from the time needed for the policy to change in response to shocks). Decision lags help in identifying fiscal policy shocks. They allow for the assumption that the ongoing changes in output do not affect spending decisions. This approach was implemented by Blanchard and Perotti (2002) and followed by many others. It usually requires the identification of short-term tax elasticity of output or short-term output elasticity of taxes, which can be problematic and uncertain.

Our study is related to the fiscal VAR literature that discusses which fiscal policy actions are most effective. Mountford and Uhlig (2009), for example, find that among the three scenarios: deficit-spending, balanced budget spending expansion and deficit-financed tax cuts, the last one is the most effective, with the largest present value multiplier equal to five after five years.

The effectiveness of fiscal policy may be evaluated using fiscal multipliers. However, there is no consensus concerning the value of fiscal multipliers. Ramey (2019) shows that fiscal multipliers range in value from 0.60 to 2.00 in government spending and -5.00 to 0 in tax revenues. She also finds evidence that the range of estimates for average fiscal multipliers has been reduced considerably, particularly for government purchases.

Caldara and Kamps (2017) report the peak spending multiplier to be between 1.00 and 1.30 and the tax multiplier between 0.50 and 0.70. Mertens and Ravn (2014) find the tax multiplier around -2.00 on impact and up to -3.00 after six quarters.

It is worth noting that the results of Caldara and Kamps (2017) and Mertens and Ravn (2014) are contradictory: the first study finds that the government spending multiplier is larger than the tax multiplier and the other one concludes the opposite.

The work by Angelini et al. (2023) shows that the tax multiplier is larger than the spending multiplier. They report that the spending multiplier is in the range between 1.60 and 2.10, i.e. statistically significantly larger than 1. Their findings show, on the other hand, that the tax multiplier is between 0.70 and 3.60. They underline, however, that tax multipliers are characterised by a larger statistical uncertainty. The authors' interpretation of this result may incline policymakers with an aversion towards parameter uncertainty to assign a larger weight to the fiscal spending level than to tax revenues.

## 3. Methodology

We estimate the Bayesian fiscal VAR model for the United States. The methodology is based on the studies of Baumeister and Hamilton. A detailed description of the methodology may be found in Baumeister and Hamilton (2015, 2018, 2019) and here, we briefly summarise the approach. The model may be written in the following, short version:

$$\boldsymbol{A}y_t = \boldsymbol{B}x_{t-1} + u_t, \text{where } u_t \sim N(0, \boldsymbol{D}). \tag{1}$$

$y_t = (y_{1t}, \dots, y_{nt})'$ and is an $n \times 1$ vector of endogenous variables, $\boldsymbol{A}$ is a matrix of contemporaneous relationships, which is the main interest of this study, $x_{t-1}$ is an $(mn + 1) \times 1$ vector consisting of $m$ lags of $y_t$ and a constant, $\boldsymbol{B}$ is an $n \times (mn + 1)$ matrix of the lagged variable parameters, and $u_t$ is an $n \times 1$ vector of structural shocks. Finally, $\boldsymbol{D}$ is an $n \times n$ diagonal matrix.

We set the prior distributions for the elements of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{D}$. The diagonal elements of covariance matrix $\boldsymbol{D}$ follow inverse Gamma distribution $\Gamma(\kappa_i, \tau_i(\boldsymbol{A}))$. The rows in $\boldsymbol{B}$ follow multivariate normal distribution with the mean equal to 0. We set all the hyperparameters according to Baumeister and Hamilton (2015). We use standard hyperparameters applied in Bayesian VAR analyses: overall tightness ($\lambda_0$ = 0.2), lag decay ($\lambda_1$ = 1) and tightness around constant ($\lambda_3$ = 1,000).

The elements of $\boldsymbol{A}$ follow $t$-Student distributions, symmetric or truncated to account for sign restrictions. In our system of three variables, $\boldsymbol{A}$ can be presented as:

$$\begin{bmatrix} \mathbf{1} & -\boldsymbol{\alpha}_{gy} & -\boldsymbol{\alpha}_{gt} \\ -\boldsymbol{\alpha}_{yg} & \mathbf{1} & -\boldsymbol{\alpha}_{yt} \\ -\boldsymbol{\alpha}_{tg} & -\boldsymbol{\alpha}_{ty} & \mathbf{1} \end{bmatrix}, \tag{2}$$

where:
$\boldsymbol{\alpha}_{gy}$ is the short-term output elasticity of government spending,
$\boldsymbol{\alpha}_{gt}$ is the short-term tax elasticity of government spending,
$\boldsymbol{\alpha}_{yg}$ is the short-term government spending elasticity of the output,
$\boldsymbol{\alpha}_{yt}$ is the short-term tax elasticity of the output,
$\boldsymbol{\alpha}_{tg}$ is the short-term government spending elasticity of taxes,
$\boldsymbol{\alpha}_{ty}$ is the short-term output elasticity of taxes.

The model can be written in the following way:

$$g_t = \boldsymbol{\alpha}_{gy} y_t + \boldsymbol{\alpha}_{gt} t_t + b_1' x_{t-1} + u_t^g \ , \tag{3}$$

$$y_t = \boldsymbol{\alpha}_{yg} g_t + \boldsymbol{\alpha}_{yt} t_t + b_1' x_{t-1} + u_t^y \ , \tag{4}$$

$$t_t = \boldsymbol{\alpha}_{tg} g_t + \boldsymbol{\alpha}_{ty} y_t + b_1' x_{t-1} + u_t^t \ , \tag{5}$$

where $g_t$, $y_t$, $t_t$ are the levels of the government spending gap, output gap and tax revenues gap in quarter $t$. Therefore, each row of the $\boldsymbol{A}$ matrix may be interpreted as a different type of rule such as a spending rule, aggregate output rule and tax revenue rule.

The standard assumption in the literature, which has been adopted in this study, is to set $\boldsymbol{\alpha}_{gt}$, i.e. short-term tax elasticity of government spending, to zero (see Angelini et al., 2023; Blanchard & Perotti, 2002, Caldara & Kamps, 2017). This means that during one quarter, changes in tax revenues do not affect government spending. The remaining elements of the $\boldsymbol{A}$ matrix have the following distributions:

$$\boldsymbol{\alpha_{gy}} \sim t_3(0{,}0.4), \boldsymbol{\alpha_{yg}} \sim t_3^+(0.5{,}0.4), \boldsymbol{\alpha_{yt}} \sim t_3^-(-0.5{,}0.4),$$
$$\boldsymbol{\alpha_{tg}} \sim t_3(0{,}0.4)\ \boldsymbol{\alpha_{ty}} \sim t_3^+(2.00{,}0.4),$$

where $t_3(a,b)$ is the $t$-Student distribution with 3 degrees of freedom and location equal to $a$ and scale equal to $b$. $t_3^+(a,b)$ means that the distribution is truncated to be positive and $t_3^-(a,b)$ means that the distribution is truncated to be negative.

The density functions can be described as the following family of asymmetric $t$-Student distributions:

$$p(h) = k\sigma_h^{-1}\tilde{\phi}_{v_h}\left(\frac{h-\mu_h}{\sigma_h}\right)\Phi\left(\frac{\lambda_h h}{\sigma_h}\right), \tag{6}$$

where $k$ is a constant to make density integrate to 1. $\Phi$ denotes the cumulative distribution function for a standard N (0,1) variable, $\tilde{\phi}_{v_h}$ denotes the $t$-Student distribution with $v_h$ degrees of freedom:

$$\tilde{\phi}_{v_h}(x) = \frac{\Gamma\left({(v_h+1)}/{2}\right)}{\sqrt{v_h\pi}\,\Gamma\left({v_h}/{2}\right)}\left(1+\frac{x^2}{v_h}\right)^{-{(v_h+1)}/{2}}, \tag{7}$$

$\lambda_h$ governs the asymmetry of the distribution. When $\lambda_h = 0$, $p(h)$ is the density of a symmetric $t$-Student variable with location parameter $\mu_h$, scale parameter $\sigma_h$, degrees of freedom $v_h$, and with the $k = 2$ integrating constant.

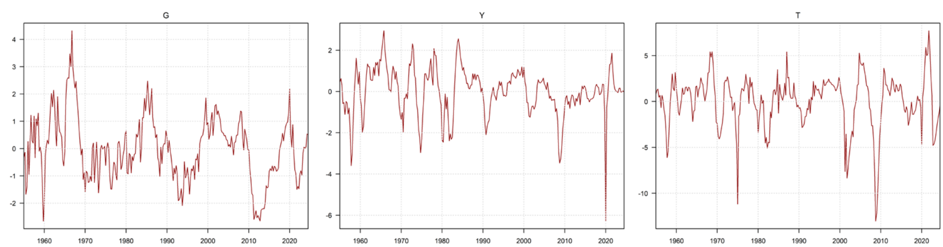
When $\lambda_h \to \infty$, $\Phi\left(\frac{\lambda_h h}{\sigma_h}\right)$ goes to 0 for any negative h and goes to 1 for any positive h, it means that when $\lambda_h \to \infty$ and $v_h = 3$, (6) becomes a Student $t_3^+(a,b)$ variable truncated to be positive. When $\lambda_h \to -\infty$, (6) becomes a Student $t_3^-(a,b)$ truncated to be negative. For further information, see Baumeister and Hamilton (2018).

We set the location parameters based on Blanchard & Perotti (2002) and Sznajderska et al. (2026), whereas the scale parameters based on Baumeister and Hamilton (2018). For instance, the prior mode for $\boldsymbol{\alpha_{ty}}$ is equal to 2.00, whereas Blanchard and Perotti (2002) set 2.08 and Favero and Giavazzi (2012) set 1.97. Additionally, we set the prior for the determinant of the $\boldsymbol{A}$ matrix, $h_1$ as done in Baumeister and Hamilton (2019). It is assumed that it follows asymmetric $t$-Student distribution $At_3$ (3.0,1.6) with parameters selected in a simulation as in Baumeister and Hamilton (2019), which gives a 93% probability of being positive.

We use the following endogenous variables in our model: nominal general government consumption and gross investment expenditures (NIPA Table 3.9.5, line 1), nominal GDP (NIPA Table 1.1.5, line 1) and nominal general government current tax receipts (NIPA Table 3.1, line 2). All series are deflated with the implicit GDP deflator (from NIPA Table 1.1.9, line 1) and *per capita*. Next, the data are logarithmised and detrended using the modified Beveridge-Nelson filter of Kamber et al. (2025). The transformations are standard in fiscal VAR literature (see Blanchard & Perotti, 2002; Caldara & Kamps, 2017).

The model is estimated on quarterly data for the United States between Q1 1955 and Q4 2024, which gives a total of 280 observations. The data are presented in Figure 1, while the descriptive statistics for the endogenous variables are shown in the Table. All variables oscillate around a zero mean. The greatest variability is observed for tax revenues. T ranges between -13.03 and 7.70.

**Figure 1.** Government spending gap (G), output gap (Y) and tax gap (T) calculated using the modified Beveridge-Nelson filter




Source: author's calculations based on data from NIPA tables.

**Table.** Descriptive statistics

| | Mean | SD | Min | Max | Skew. | Kurt. | AR(1) |
|---|---|---|---|---|---|---|---|
| G ................ | -0.0052 | 1.1826 | -2.6581 | 4.3164 | 0.2783 | 3.3958 | 0.8653 |
| Y ................ | 0.0003 | 1.1651 | -6.2858 | 2.9462 | -0.9769 | 6.1099 | 0.8082 |
| T ................ | -0.0022 | 2.8864 | -13.0316 | 7.7036 | -1.2070 | 6.1813 | 0.8306 |

Source: author's calculations.

## 4. Results

The results are presented in three steps. First, we discuss short-term elasticities, then we focus on the impulse response functions and finally, we present fiscal multipliers with a robustness check for different prior distributions of output tax elasticity. Lastly, we show the historical decomposition for real GDP.

Figure 2 depicts the prior and posterior distributions for the elements of the $\boldsymbol{A}$ matrix. The baseline prior distribution is represented using solid red lines, whereas the posterior is presented using light green histograms. Below, we discuss the posterior distributions for the five short-term elasticities.

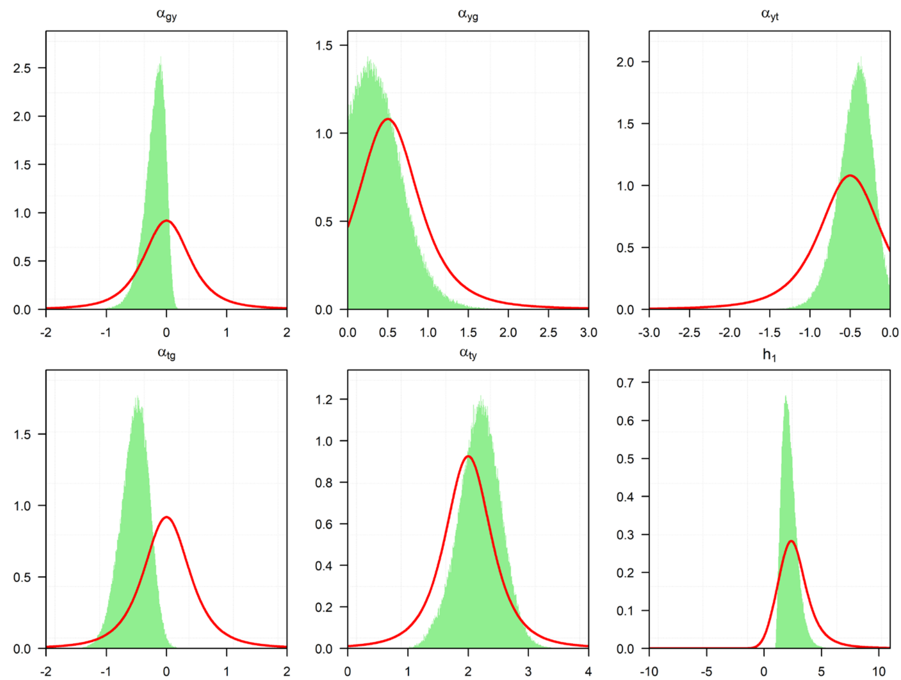
The median posterior value for $\boldsymbol{\alpha_{gy}}$, i.e. the output elasticity of government spending, amounts to merely -0.17 and the posterior 95% credible set includes 0. Our interpretation is that the value of the output does not affect the level of government spending in the same quarter. This is in line with our expectations, as spending decisions are usually made and announced earlier and implemented with a lag.

The data revise our prior beliefs about contemporaneous output elasticities. The posterior median for government spending elasticity of output $\boldsymbol{\alpha_{yg}}$ is equal to 0.39 and is lower than the assumed prior mode of 0.50. The posterior median for $\boldsymbol{\alpha_{yt}}$ is close to our prior, with its posterior median equalling -0.42. Our estimated posterior values for output elasticities are lower than the corresponding values in Blanchard and Perotti (2002).

Tax elasticity of government spending $\boldsymbol{\alpha_{tg}}$ is negative and equal to about -0.51 with a 95% posterior credible set (-0.93, -0.17), thus, the data revise our prior beliefs. The negative values for this elasticity are also reported in Caldara and Kamps (2017). The aforementioned values of tax elasticity could indicate that increases in government spending have a low negative effect on tax revenues in the current quarter.

Lastly, but most importantly, short-term output elasticity of taxes is slightly above our prior beliefs. The posterior median for $\boldsymbol{\alpha_{ty}}$ equals 2.20 and a 90% credible set (1.60, 2.75). The value is almost the same as found in a five-equation model by Sznajderska et al. (2024). It is a bit higher than in Blanchard and Perotti (2002), who assert that the output elasticity of net taxes is equal to 2.08. Additionally, it is higher than in Caldara and Kamps (2008), Favero and Giavazzi (2012), and Perotti (2008), where the value is 1.85, but it is almost the same as in Caldara and Kamps (2017), who assume its value is 2.18. Angelini et al. (2023), on the other hand, propose a range from 2.15 to 4.40. Thus, our estimates are within this range. In the sensitivity check, we test a model with a different prior assumption for $\boldsymbol{\alpha_{ty}}$. These results are shown in Figure 5 and discussed below.

**Figure 2.** Prior and posterior distributions for the ***A*** matrix




Source: author's calculations.

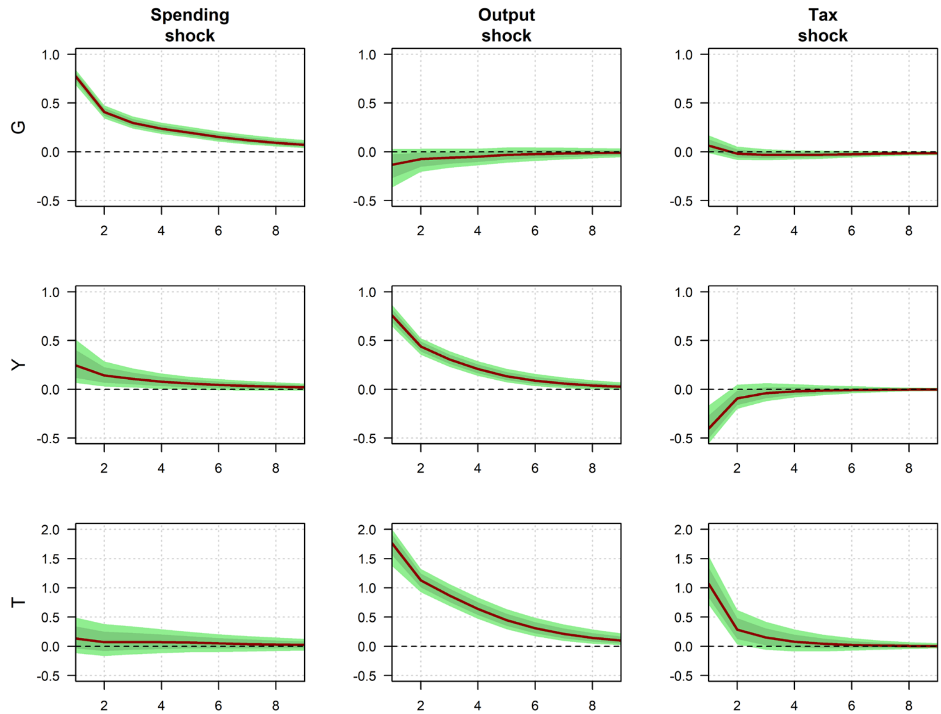
The impulse response functions from our baseline model are shown in Figure 3. The red solid lines represent the Bayesian median posterior response. The green areas denote 68% and 90% posterior credible sets. This study focuses on the results for the 68% posterior credible set. Each column in Figure 3 presents the response of endogenous variables to a spending, output and tax shock.

An unexpected increase in government spending of 0.77% leads to an increase in output, equal to 0.25% on impact. The response of the output is statistically significant for nine quarters. Moreover, tax revenues do not react to the government spending shock.

Next, an unexpected positive output shock, equal to 0.76% on impact, leads to a very large reaction of tax revenues (1.76%). This is a result of an increase in the tax base. Interestingly, after a positive output shock, we observe a decrease in government spending equal to -0.13% on impact, significant for three quarters. This could imply a slight counter-cyclicality of the fiscal policy.

The third column in Figure 3 shows the reactions to a positive tax shock, equal to 1.06% on impact. The increase in tax revenues causes a decrease in output equal to -0.40% on impact and it is statistically significant for two quarters. We observe a small increase in government spending equal to 0.06% on impact.

**Figure 3.** Impulse response functions from the baseline model




Source: author's calculations.

We follow the definition of a fiscal multiplier often found in the literature (see (8). It is the dollar response of the output to a one-dollar change in government spending or tax revenues. It is calculated as the ratio of output response at horizon $h$ to a (one-standard deviation) fiscal policy shock to the value of fiscal policy shock on impact divided by the scaling factor (cf. Angelini et al., 2023). The scaling factor is the ratio of the mean across the nominal fiscal spending or tax revenues sample (not in logs) to the mean across the level of output sample (nominal GDP, not in logs). Scaling factor $\frac{\bar{P}}{\bar{Y}}$ for both government spending and tax revenues is equal to 0.19 in the sample.

$$M_{ph} = \frac{IRF_{yh}}{IRF_{p0}} \frac{1}{\frac{\bar{P}}{\bar{Y}}}. \tag{8}$$

Figure 4 presents the median values for fiscal multipliers with 68% and 90% posterior credible sets from the baseline model. The spending multiplier is positive and statistically significant for nine quarters. The value of the median posterior of the spending multiplier is 1.65 on impact, 0.53 after one year and 0.18 after two years. The tax multiplier is negative and statistically significant for two quarters. The value of the median posterior of the tax multiplier is -2.00 on impact, -0.10 after one year and -0.02 after two years. The values of the fiscal multipliers are similar to those obtained using the five-equation model with monetary policy (see Sznajderska et al., 2024). Sznajderska et al. (2024) report the spending multiplier to equal 1.25 initially and 0.57 after a year, and the tax multiplier to equal -3.24 initially and -0.72 after a year. Thus, in absolute terms, the reported spending multiplier is slightly larger and the tax multiplier slightly lower. It is worth noting that the baseline model in Sznajderska et al. (2024) ends in Q4 2019 and does not include the COVID-19 pandemic.

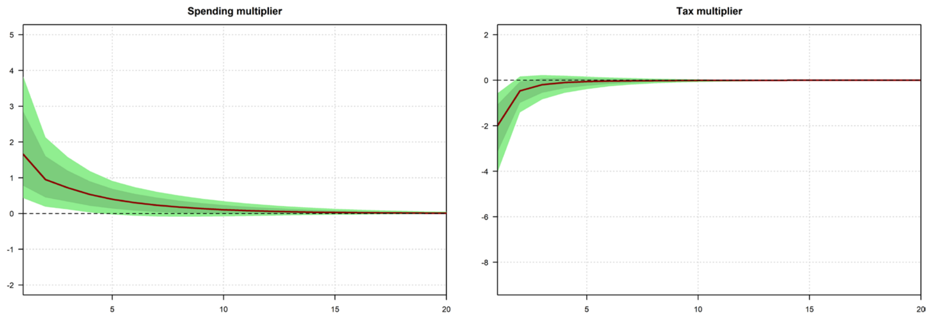
Figure 5 shows how the results change when the prior beliefs concerning output tax elasticity are changed. We assume that $\boldsymbol{\alpha_{ty}} \sim t_3^+(3.00{,}0.4)$, meaning that the prior mean is increased from 2.00 (the value used in Blanchard & Perotti, 2002; Favero & Giavazzi, 2012; Perotti, 2008) to 3.00 (the value reported in Mertens & Ravn, 2014). The priors for all other parameters of the model remain the same as in the baseline specification. The posterior median for $\boldsymbol{\alpha_{ty}}$ equals 2.79. We observe significant differences in the values of the fiscal multipliers. The spending multiplier decreases reaching 1.23 on impact, 0.44 after one year and 0.16 after two years. What is more, it is surrounded by lower uncertainty. In contrast, the posterior median for the tax multiplier decreases to -3.95 on impact, -0.44 after one year and -0.11 after two years. Thus, in absolute terms, the tax multiplier is much larger in the first quarter than the spending multiplier. We confirm the result found in Mertens and Ravn (2014) for a proxy VAR model that the higher output elasticity of the tax revenues is associated with the higher values of tax multipliers in absolute terms. Furthermore, the obtained values of the tax multipliers are similar to those found by Mertens and Ravn (2014).

Figure 6 also shows the fiscal multipliers for the Blanchard and Perotti (2002) model. The purpose of this exercise is to show that the application of the Baumeister and Hamilton method may significantly change the results. We follow Equation 7 in Čapek and Cuaresma (2020):

$$\begin{bmatrix} \mathbf{1} & 0 & 0 \\ -\boldsymbol{\alpha_{yg}} & \mathbf{1} & -\boldsymbol{\alpha_{yt}} \\ 0 & -1.85 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^y \\ \varepsilon_t^t \end{bmatrix} = \begin{bmatrix} \mathbf{1} & 0 & 0 \\ 0 & \mathbf{1} & 0 \\ \beta_{tg} & 0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^y \\ \varepsilon_t^t \end{bmatrix}. \tag{9}$$

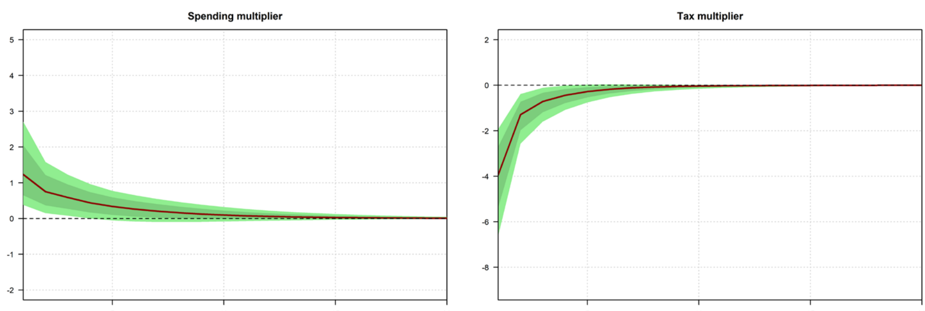
The fiscal multipliers from the Blanchard and Perotti model are lower in absolute terms than the ones obtained from the baseline model. The government spending multiplier is statistically insignificant. This is difficult to justify and highlights the superiority of the Baumeister and Hamilton method. The tax multiplier is equal to -1.20 on impact and then it steadily increases and becomes insignificant after the third quarter.

**Figure 4.** Fiscal multipliers for the baseline model



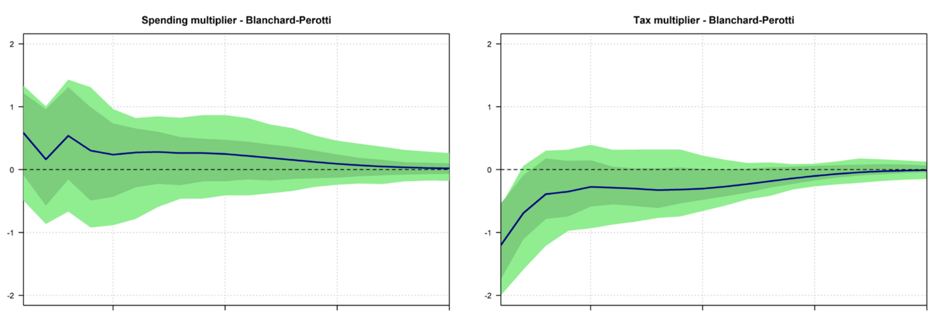

Source: author's calculations.

**Figure 5.** Fiscal multipliers for the model with higher output tax elasticity




Source: author's calculations.

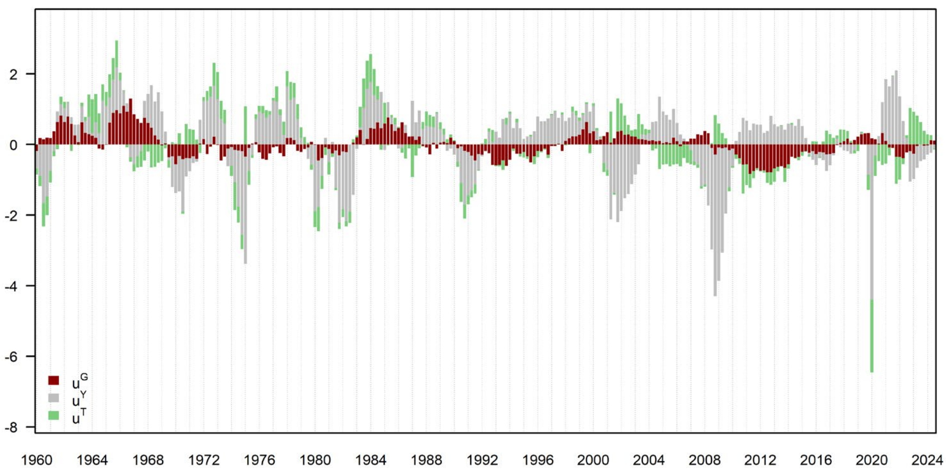
**Figure 6.** Fiscal multipliers estimated for the Blanchard and Perotti model, namely parameters $c_1$, $c_2$, $a_2$ from Table II in the paper by Blanchard and Perotti (2002)




Note. 68% and 90% bootstrap confidence intervals are presented in green.
Source: author's calculations.

Lastly, Figure 7 shows the contribution of each identified structural shock to the GDP deviation from its long-term trend according to the baseline model. A historical decomposition enables us to assess the individual impacts of government spending, output and tax shocks at specific points in time, compared to other structural shocks. The main driver of GDP deviations is the output shock (in grey in Figure 7). The contribution of government spending shocks (in red) is also substantial, with the biggest negative impact in 2011–2013 and the biggest positive impact in 1965–1966. The contribution of tax shocks (in green) is relatively smaller, with the biggest negative impact at the beginning of the COVID-19 pandemic and the biggest positive impact in Q1 1975 and Q4 2022.

**Figure 7.** Historical decomposition for GDP deviations from its long-term trend from the baseline model




Source: author's calculations.

We performed several robustness checks, such as estimating the model for the sample that ends before the COVID-19 pandemic, increasing the tightness parameter ($\lambda_0$) or increasing the scale parameters to 0.6 for all prior distributions. These changes do not affect our results and are available from the author upon request.

## 5. Discussion and conclusions

This paper is devoted to tracing the effects of fiscal policy shocks in the United States. We apply the Baumeister and Hamilton (2015, 2018, 2019) method to calculate the level of fiscal multipliers in a three-variate fiscal VAR à la Blanchard and Perotti (2002). The method allows us to apply different priors on the parameters describing the contemporaneous relations in the structural form VAR model. We incorporate our knowledge and uncertainty in the model by setting sign restrictions and determining the parameters of the prior distributions of the $\boldsymbol{A}$ matrix.

Importantly, the baseline model is estimated for quarterly data between 1955 and 2024 including the COVID-19 pandemic. However, the results are robust to shortening the sample, i.e. the pre-COVID-19 sample provides similar results.

The most important result is that the one-period spending multiplier is equal to 1.65 on impact, 0.53 after one year and 0.18 after two years, and the tax multiplier is equal to -2.00 on impact, -0.10 after one year and -0.02 after two years. The posterior output elasticity of taxes is equal to 2.20, which is in the range of the values reported in the literature. An increase in the prior mode for output elasticity of taxes results in a lower tax and spending multipliers.

The study shows that both increasing spending and decreasing taxes can stimulate the economy. However, the effects of tax decreases may be larger for the economy. This confirms the findings of Mountford and Uhlig (2009) and Mertens and Ravn (2014).

## 6. Acknowledgements

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# The Modelling Preferences and Risk national scientific conference and the International Workshop on Multiple Criteria Decision Making: a summary of the conferences' long-standing success

Tadeusz Trzaskalik,[a] Tomasz Wachowicz[b]

On 7th–8th April 2025, the University of Economics in Katowice hosted the jubilee, 15th Modelling Preferences and Risk 2025 (MPaR) national scientific conference, held jointly with the 11th International Workshop on Multiple Criteria Decision Making 2025 (IWoMCDM). Both events were organised by the Department of Operations Research of the University of Economics in Katowice, which celebrated its 25th anniversary last year, in collaboration with the INFORMS Poland Chapter. The academic supervision of the events was provided by Professors Tadeusz Trzaskalik and Tomasz Wachowicz. The conferences were held under the honorary patronage of the Rector of the University of Economics in Katowice, Professor Celina Olszak, with additional scholarly patronage provided by the Committee of Statistics and Econometrics of the Polish Academy of Sciences.

The MPaR/IWoMCDM conferences, initiated by Professor Tadeusz Trzaskalik, have now become a regular position in the calendar of the academic events devoted to the application of the decision theory in economics and management. The purpose of the conferences is to provide an interdisciplinary forum for the exchange of ideas and experiences concerning the latest developments in the mathematical description and modelling of decision-making processes, with a particular focus on the issues of risk and multiple criteria decision making. The first MPaR conference was held in 1998, followed by another edition in 1999, and since then the meetings have been organised biennially.

The 15th MPaR/IWoMCDM conference was inaugurated in the Professor Zbigniew Pawłowski Auditorium. It is worth recalling that in this very hall – then regularly filled students of the Silesian International School of Business – Professor Kazimierz Zaraś

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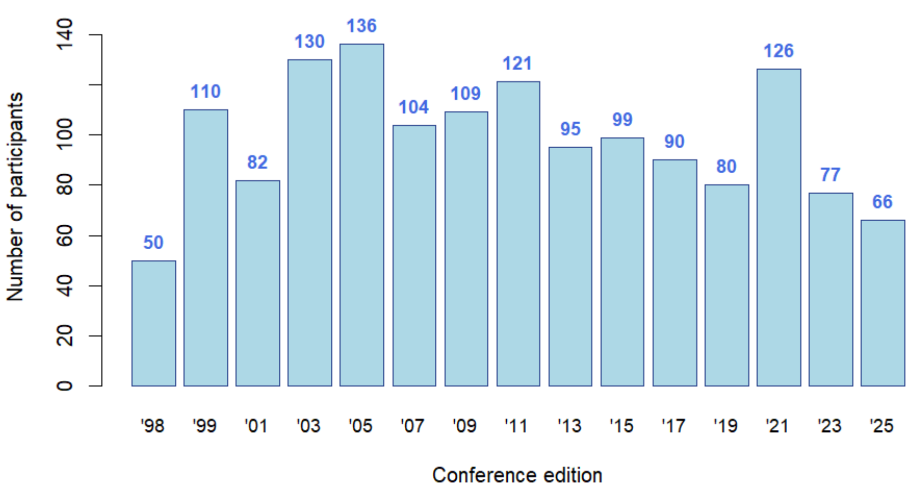
of the Université du Québec en Abitibi–Témiscamingue in Canada delivered a guest lecture in 1994 entitled *Modelling Preferences and Risk*. It focused on the application of stochastic dominance in supporting multiple criteria decision making, and the current conference series took its name directly from the title of that lecture.

The inaugural MPaR conference in 1998 was opened with a plenary lecture delivered by Professor Jean-Marc Martel of Laval University in Quebec City, Canada, entitled *Multicriterion Analysis under Uncertainty: The Approach of Outranking Synthesis*. It initiated a sequence of 29 plenary lectures delivered over the years by distinguished scholars from diverse academic centres worldwide. Since 2005, the MPaR conference has been accompanied by the IWoMCDM workshops, devoted to the theory and applications of multiple criteria decision making. These workshops continue the tradition of the large international conference, *The 4th International Conference on Multiobjective Programming and Goal Programming*, organised in 2000 by the Department of Operations Research at the Jaskółka Hotel in Ustroń. Subsequent MPaR/IWoMCDM conferences were held at this venue until 2023. This year, owing to the transformation of the hotel into a health resort, the conference took place at the University of Economics in Katowice.

The conferences have consistently attracted strong interest among researchers engaged in applying the decision theory to economics and management, both in Poland and abroad. This is demonstrated by the fact that the events have gathered a total of 1,475 participants. Figure 1 illustrates the number of participants in each edition.

**Figure 1.** Number of participants in each edition of the MPaR/IWoMCDM conference

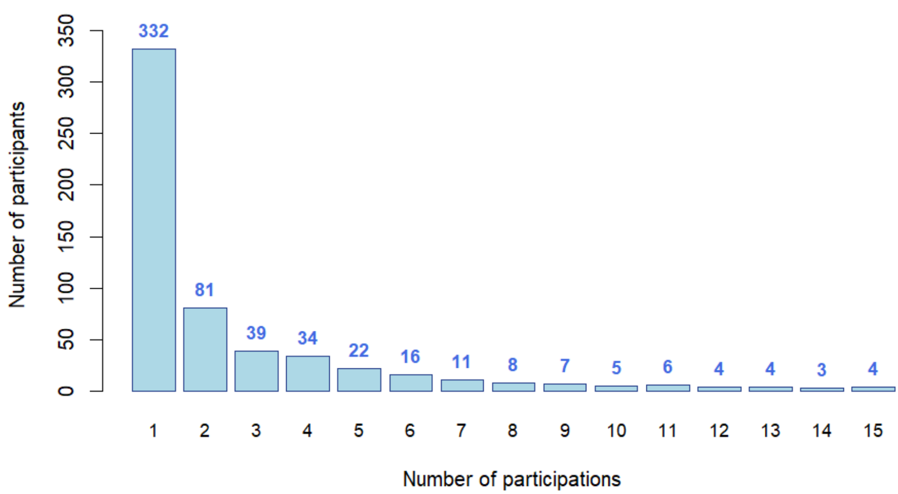



Source: authors' work.

The largest attendance – 136 participants – was recorded in 2005, when the IWoMCDM conference was organised for the first time. In 2021, the third highest number was achieved, with 126 participants. This edition was held online due to the COVID-19 pandemic, which enabled exceptionally broad participation from scholars representing every continent except Australia – something that would not have been possible in a traditional in-person format. On average, 95 participants attended each edition of the conference. It is also noteworthy that the successive editions have attracted participants from a wide range of academic centres; for instance, in 2011, 34 institutions, both Polish and international, were represented.

Equally noteworthy are the statistics concerning repeated participation. Across all editions, there have been 576 individual participants. Figure 2 presents the distribution of participation. As many as 244 individuals have taken part in the conference at least twice, attesting to their strong or very strong association with this event. Significantly, 15 of the 50 participants at the inaugural conference in 1998 were also present at this year's 15th edition.

**Figure 2.** Distribution of participation in the MPaR/IWoMCDM conferences




Source: authors' work.

In 1998, the first volume of the *Modelling Preferences and Risk* series was published by the Publisher of the University of Economics in Katowice, subsequently appearing annually and containing articles presented at the MPaR conferences. The IWoMCDM workshops, in turn, gave rise to the annual journal *Multiple Criteria Decision Making (MCDM)*, also published by the University of Economics in Katowice. The journal featured a variety of contributions, including papers presented during the successive workshops. The *Modelling Preferences and Risk* series was published in Polish between 1998 and 2022, while the *MCDM* journal has been issued in English since 2005. Both series are edited by Professor Tadeusz Trzaskalik. For many years they have attracted considerable interest from both Polish and international authors. This in part resulted from the fact that many participants in these conferences were at the beginning of their academic careers and sought opportunities for advancement, which the publication of their articles facilitated. All papers published in the successive volumes of the *Modelling Preferences and Risk* series are available online at www.mpar.ue.katowice.pl.

Articles published in these conference-related series, particularly in the *MCDM* journal, have often attracted international attention, as demonstrated by the citation data. The most frequently cited in the SCOPUS database, with 262 citations is the article titled *Multi-criteria decision making models by applying the TOPSIS method to crisp and interval data* by Professor Ewa Roszkowska.

On the occasion of the MPaR/IWoMCDM jubilee, the most active participants (outside the Department of Operations Research of the University of Economics in Katowice) were presented with diplomas. The first one was awarded to Professor Kazimierz Zaraś, whose lecture title – as mentioned earlier – gave the name to the MPaR conference. The second diploma was awarded to Professor Józef Stawicki, who not only took part in every edition of the MPaR/IWoMCDM conference, but also contributed extensively to their organisation by preparing programmes and reviewing numerous papers submitted for publication in the conference proceedings. The third diploma was presented to Professor Ewa Roszkowska, author of the most frequently cited paper delivered at the conference. Other diplomas were awarded for participation in at least ten editions of MPaR to Professors Andrzej Skulimowski, Ignacy Kaliszewski, Grażyna Trzpiot, Marcin Anholcer, Dorota Kuchta, Stefan Grzesiak, Ewa Konarzewska-Gubała, Lech Kruś, Włodzimierz Szkutnik, Zbigniew Świtalski, Krzysztof Dmytrów, Joanna Olbryś, Honorata Sosnowska, and Stanisław Wieteska. Diplomas were also prepared for the most active international participants of the IWoMCDM workshops: Professors Petr Fiala, Josef Jablonský, Jaroslav Ramík, Ralph Steuer, Leonas Ustinovičius and Lidija Zadnik-Stirn. One should also acknowledge the staff of the Department of Operations Research, who, under the leadership of Professor Tomasz Wachowicz, organised the successive meetings. These include Renata Dudzińska-Baryła, Anna Gorczyca-Goraj, Krzysztof Grzanka, Beata Humańska, Sławomir Jarek, Ewa Michalska, and Krzysztof Targiel.

This year's edition of the MPaR/IWoMCDM conference was attended by 66 participants representing 26 academic and research centres as well as companies from Poland and abroad. In total, 44 papers were presented in 14 sessions, addressing topics such as preference analysis, the search for optimal decisions, project management, risk measurement, analysis and management, and multiple criteria decision making. In this year's opening plenary session of the conference and workshop, the following lectures were delivered:

- Professor Milena Bieniek, *The Impact of Demand Uncertainty on Logistic Decision Making*;
- Professor Wojciech Sałabun, *Recent Advances in the Development of MCDA Methods and Tools*.

A regular feature of the MPaR conference is a session co-organised with the INFORMS Poland Chapter, providing a platform for practitioners' perspectives. This year two INFORMS sessions were held. On the first day of the event, Maciej Gwóźdź, CEO of the Ammega Group, delivered a lecture entitled *Decision Making and Risk Management in a Private Equity Environment Based on Ammega*. On the next day, Doctor Przemysław Juszczuk of the Institute of Computer Science, Polish Academy of Sciences, presented a lecture entitled *A System for Heterogeneous Memory Management Based on Multi-criteria Optimisation and Machine Learning*, showcasing a commercial solution developed in cooperation with the Infoklinika S.A. company. Among the additional activities accompanying the previous editions of MPaR were open meetings of the Operations Research Section of the Committee of Statistics and Econometrics of the Polish Academy of Sciences, as well as open sessions of the Commission on the Didactics of Mathematics for Economic Studies of the Polish Mathematical Society.

The MPaR/IWoMCDM conferences are also regularly accompanied by special concerts. In previous years, the participants had the opportunity to enjoy performances by the 'Silesianie' Song and Dance Ensemble of the University of Economics in Katowice, the University's Choir (on two occasions), an organ recital on a positive organ by Małgorzata Trzaskalik-Wyrwa, a quartet from the National Polish Radio Symphony Orchestra in Katowice, the distinguished harpsichordist Marcin Świątkiewicz, the soprano Edyta Nowicka, and the piano duo of Ayaka Meiwa (Japan) and Tadeusz Trzaskalik. In this year's edition, tenor Michał Ryguła performed an engaging programme of Italian and Neapolitan songs.

We would like to warmly invite all those interested in the issues of preference modelling, risk and multiple criteria decision support to participate in the next edition of the MPaR/IWoMCDM conference, which is scheduled to take place in the spring of 2027.

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