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Clustering problem in self-organising communication networks on the example of smart vehicle transportation system

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Abstract. Efficient communication in many types of dynamic networks depends critically on how nodes are clustered into subnetworks. As such networks grow and evolve rapidly, there is a need for fast and robust clustering algorithms that account for communication cost structures induced by different node partitions. In this paper, we evaluate how classic community detection algorithms (CDAs), i.e. the Louvain and Ensemble Clustering for Graphs (ECG), adapted to incorporate a flexible family of cost functions, perform relative to standard heuristic and metaheuristic approaches. Using simulations on l-nearest neighbour graphs of varying sizes, we find that especially the Louvain algorithm consistently delivers high quality solutions at a substantially lower computational cost. In contrast, metaheuristic methods fail to scale effectively, and the ECG algorithm does not provide performance improvements in our setting, despite its reported stabilising effect in traditional community detection tasks. Overall, our results indicate that classic CDAs are well suited for real time clustering in dynamic communication networks and constitute a strong basis for developing scalable communication optimisation strategies.

Keywords: transport networks, communication networks, network design, graph clustering
JEL: D85, C61, L90

1. Introduction

The efficient exchange of information and communication in large, dynamic networks crucially depends on how nodes are divided into groups. In many settings, the efficiency of communication of the entire network is determined by two counteracting forces. On the one hand, the cost of communication between nodes belonging to different clusters is typically larger than for nodes belonging to the

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same cluster. On the other hand, if an individual cluster contains a large number of nodes, then the efficiency of communication within this cluster is also hampered. An example of such a setting is a social network representing communication within a department and cross-department communication (Creswick et al., 2009), or telecommunication networks. For instance, in Transmission Control Protocol/Internet Protocol (TCP/IP) networks, subnets can be formed and the observed latency within a single subnet is lower than in cross-subnet communication (O'Malley & Peterson, 1992).

Another example that is currently gaining importance and which this paper focuses on is the clustering problem in a smart vehicle network. The emergence of Vehicular Networks (VN) of connected smart vehicles requires new communication patterns between vehicles. This standard should take into consideration the optimisation of information flow, defined according to a specific objective, e.g. reducing latency or maximising the quality of service. One of the main challenges in this area is the constantly growing size and structure of the network, which necessitates algorithms fast enough to remain practical at scale and under continuous topology change. While we focus on smart car transportation systems, this formulation can be applied in other settings such as social networks or artificial networks, as the Internet of Things (IoT).

The aim of this paper is to formulate a general, objective-function-agnostic clustering optimisation framework for weighted graphs and to evaluate the efficiency of community detection algorithms (CDA; Lancichinetti & Fortunato, 2009), such as Louvain and Ensemble Clustering for Graphs (ECG). We compare their performance in terms of solution quality and speed relative to simple greedy baselines and metaheuristics (stochastic hill climbing, simulated annealing), and how the runtime of algorithms scales with network size. The framework handles communication costs through two families of value functions (size-insensitive and size-sensitive), enabling practitioners to encode application-specific trade-offs while maintaining evaluation efficient.

Until now, research in this field has mostly been based on the game theory approach (Sun et al., 2020), while in other areas such as other types of networks, community detection and optimisation metaheuristics were successfully used, especially due to their computation time. Our paper aims to bridge this gap. Furthermore, most literature surveys in network clustering indicate a dependency of clustering methods on the objective function (Shahraki et al., 2020). In our research, we define the clustering problem using a flexible family of objective cost functions. In particular, the parameters in the objective function (weights) subject to optimisation during the clustering process can be defined and interpreted according to the specific problem and network type. In our case, weights represent (or alternatively, are correlated with) the efficiency of the network data transmission (packet loss ratio subject to minimisation). Nevertheless, since they can represent any cost of within- and between-group communication, they may be suitable for a broader class of social network problems.

Algorithms that can quickly generate good clustering under a specific objective function will become increasingly important in areas such as Intelligent Transportation System (Qureshi & Abdullah, 2013), Internet of Vehicles (Coppola & Morisio, 2016; Kaiwartya et al., 2016; Lu et al., 2014), and Vehicular ad hoc Networks (VANET; Hbaieb et al., 2021; Soares & Rodrigues, 2021). This trend is driven by developments in smart technologies which intensify the communication not only between users, but also between users and infrastructure, which leads to a significant increase in the amount of the transferred data. VANET is a system based on a network of vehicles equipped with sensors and using wireless communication (Badis & Rachedi, 2015; Paul et al., 2017). The VANET concept is an extension of Mobile ad hoc Networks (MANET), and therefore it 'inherits' their main features such as dynamic topology resulting from high node mobility and the need for scalable solutions as the number of vehicles increases. Finally, it is important to highlight that clustering is not only a crucial concept in VANET (Mukhtaruzzaman & Atiquzzaman, 2020), but also a growing area of research in the broader field regarding networks in general (Kamiński et al., 2021; Shahraki et al., 2020).

Finding the optimal way of dividing network nodes into subnets can serve different objectives:

- minimising power consumption (not applicable to VANET);
- maximising efficient resource utilisation;
- maximising the quality of service (QoS, e.g. minimising data transfer loss or latency);
- network load balancing;
- minimising costs or maximising profits.

Clustering methods can be divided with respect to the technique into:

- intelligence-based strategies;
- mobility-based strategies;
- multi-hop-based strategies.

Intelligence-based strategies comprise machine-learning methods and fuzzy logic algorithms. While the latter are characterised by a higher stability of clustering, they tend to be computationally ineffective.

Although machine-learning algorithms perform better in this respect, they also face significant problems. First of all, a k-means algorithm requires the number of clusters to be known in advance, which makes its solution suboptimal (or requires a high number of runs, which, in turn, results in loss of efficiency). Secondly, it is highly sensitive to starting points (initial centroids), so the solution is not stable. Another commonly used machine-learning algorithm is hierarchical clustering, the implementation of which is far more complex (requires a proximity matrix calculation and results in $O(N^2 \log(N))$ to $)(N^3)$ time complexity).

Optimisation techniques are used in the problem of network clustering as well, however, they specifically concentrate on energy consumption. In these approaches, the wireless sensor nodes are battery-powered, thus minimising the use of energy. The key challenge in this case is maximising the time until the first sensor runs out of energy. The best-known applications in this area are the following algorithms: LEACH 9 (Heinzelman et al., 2000), HEED (Younis & Fahmy, 2004) and FLOC (Demirbas et al., 2004). These approaches are not applicable to the problem of VANET, as it is not concerned with the minimisation of energy consumption due to lack of restrictions.

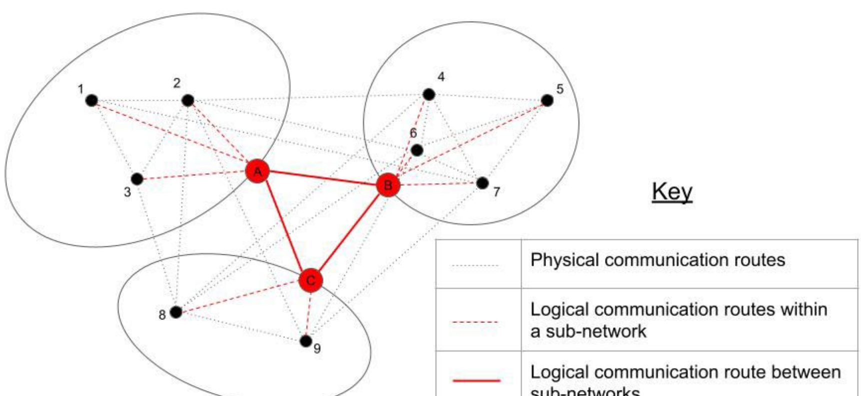
2. Model

In this section, we present the formulation optimisation model for the efficient communication within intelligent transportation networks. In the first subsection, we provide a general non-technical description of the problem, which introduces the next subsection, where we present the formal mathematical optimisation model.

2.1. Description

In our setup, we consider a weighted undirected graph, which consists of nodes and edges that connect them. We assume that the graph is connected, that is, a path between any pair of the nodes exists. The nodes of the graph represent physical cellular base networks through which vehicles are communicating with each other. The edges represent the flow of communication or information between the nodes, with the weights being the amount of communication flow in a unit of time (based on either real-time or historical data, depending on the architecture).

Figure 1. Sample network and division into clusters



Source: authors' work.

Figure 1 represents the physical network and it shows how much information needs to be transmitted between the nodes. The network and the physical connections are represented as black dots and black dotted lines on the graph. We assume that the actual communication between the nodes takes place through a logical network, which is built on top of the physical network as a logical layer (Esteves et al., 2013).

Creating the logical network involves clustering (grouping) particular nodes of the physical network into distinct subnetworks (Gerla & Kleinrock, 1977). In the example presented in Figure 1, the black nodes represent the physical network, whereas logical subnetworks are marked by black ellipses. In our example, there are three of them. The communication between the nodes in the logical subnetwork takes place through logical nodes, shown as red circles, which are located at the edges of the ellipses. The efficiency of the communication between any two nodes depends on whether they are located within the same subnetwork or not, and the overall efficiency of the network depends on the structure of the logical network, as well as on the clustering algorithm used to perform the grouping (Tai et al., 2011). Given the constant increase of nodes in the network, there is also a need to take computation efficiency into account and choose an algorithm which will be able to solve a task of increasing complexity in a feasible time.

As stated before, the aim of our study is to find an optimal clustering of nodes into logical groups (subnetworks) in order to optimise the efficiency of communication or, in other words, the cost associated with communication. To do this, we first need to propose an objective function which will measure the efficiency of the logical network, and then develop an algorithm which will find the optimal network. The objective function should reflect the following considerations regarding the efficiency of communication for nodes belonging to the same or to different subnetworks. For nodes belonging to the same cluster (subnetwork), the communication is conducted from one of the nodes to the red logical hub and then back to the other node along the logical connections portrayed by the red dotted lines. The more nodes in the cluster, the more time will be required for the communication to take place. Therefore, we assume that the efficiency of communication for such nodes will be lower when the weight and the size of the cluster is high. For nodes belonging to different logical groups, the communication is conducted from the node to the logical hub along the dotted red line, then to the logical hub of the other node along the solid red line, and finally along the dotted red line to the target node. If the logical network consists of a large number of clusters (subnetworks), the communication time between the logical hubs is long. Therefore, the efficiency of communication between the nodes belonging to different clusters is a decreasing function of the weight, size, and the total number of clusters.

The mathematical formulation of the proposed objective function that measures the efficiency of the network is described in the next subsection. The measure is consistent with the throughput of the network or with the packet delivery/loss ratio (but may be generalised to any type of cost communication, depending on the type of network). We assume that the efficiency of communication between nodes depends on the following three statistics: a) the volume of communication (weights) within clusters, b) the volume of communication between clusters, and c) the distribution of cluster sizes (in particular, the number of clusters).

2.2. Mathematical formulation

Let us consider an undirected, connected, and weighted graph $G = (V, E)$ having $n = |V|$ nodes representing physical cellular base network stations and $m = |E|$ edges representing desired information flows between stations. Let $w : E \to R_+$ be the weight function; each edge $e \in E$ has weight $w(e) \in R_+$ associated with it. The Table contains a list of mathematical symbols used throughout the document.

Let P be a partition of the set of nodes V, that is, $P = (V_1, \dots, V_k)$ for some $k \in N$ such that $V_i \subseteq V$ for $i \in [k]$, $V = V_1 \cup \dots \cup V_k$, and $V_i \cap V_j = \emptyset$ if $1 \leq i < j \leq k$. We assume that cost of communication c associated with partition P (for a given graph G and weight function w of its edges) is not affected by any external parameters, that is, $c = c(G, w, P)$. The overall objective in our study is to find the partition P which minimises the cost function $c(G, w, P)$:

$$\min_{\mathrm{P}} c(G, w, P). \tag{1}$$

We restrict our study to some natural family of functions, still very large and flexible, that covers most of the functions one needs to deal with in practice. However, we are looking for a specification of the cost function for which the recalculation of the value will not be computationally intensive given the algorithms that we use for finding the optimum. In particular, the algorithms operate in such a way that the current partition which is being considered is modified in a minor, local way, usually by changing the part for a single node. If we assumed a complex, highly nonlinear cost function, we would need to recalculate the cost function for the entire graph, which for a large graph would be a costly operation. In order to avoid this, we opted for a cost function for which we can update the objective function by performing calculations only for a part of the cost function which corresponds to the parts that are being modified in a single step. Keeping this in mind, we propose the following cost-function specification.

Let $s = (s_1, \dots, s_k)$ be the distribution of part sizes in partition P, that is, $s_i = |V_i|$ for $i = [k]$. Let $\boldsymbol{t} = (t_1, \dots, t_k)$ be the distribution of weights within parts, that is, t_i is the total weight associated with the edges in graph $G[V_i]$ induced by a set of nodes V_i. Finally, let $u = (u_{1,2}, u_{1,3}, \dots, u_{k-1,k})$ be the distribution of weights between parts, that is, u_i $(1 \leq i < j \leq k)$ is the total weight associated with the edges between part i and part j. Note that

$$W = W(w) \coloneqq \sum_{e \in E} w(e) = \sum_{i=1}^{k} t_k + \sum_{1 \leq i < j \leq k} u_{i,j}, \tag{2}$$

that is, each weight is captured by either $\boldsymbol{t}$ or $\boldsymbol{u}$. Moreover, vectors $\boldsymbol{s}$ and $\boldsymbol{t}$ have k coordinates each and vector $\boldsymbol{u}$ has $\binom{k}{2}$ coordinates. Please note that all these three vectors together have only $\frac{k(k+3)}{2} = \Theta(k^2)$ coordinates, but they capture a lot of structure of a given partition P. We will restrict ourselves to efficiency functions that only depend on these three vectors, that is, $c = c(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{u})$, where $\boldsymbol{s} = \boldsymbol{s}(\boldsymbol{G}, \boldsymbol{w}, \boldsymbol{P})$, $\boldsymbol{t} = \boldsymbol{t}(\boldsymbol{G}, \boldsymbol{w}, \boldsymbol{P})$, $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{G}, \boldsymbol{w}, \boldsymbol{P})$.

Table. Symbols used in the document

Symbol	Interpretation
G	graph, $G = (V, E)$ where V and E are the sets of nodes and edges
n	number of nodes, $n = \lvert V \rvert$
v_i	node number $i, v_i \in V$
m	number of edges, $m = \lvert E \rvert$
w	weight function of edges
e_i	edge number $i, e_i \in E$
P	partition of the set of nodes
k	number of parts in partition P
V_i	cluster (part) $i, V_i \in P$
c	efficiency function of partition, $c = c(G, w, P)$
s	distribution of part sizes of partition P
s_i	size of cluster V_i
$\boldsymbol{t}$	distribution of weights within parts
t_i	sum of the weights of edges within cluster V_i
$\boldsymbol{u}$	distribution of weights between parts
$u_{i,j}$	sum of weights of the edges between cluster V_i and V_j
a	weight of within-cluster communication in the cost function
b	weight of the number of parts in the cost function

Source: authors' work.

2.3. Objective functions

Algorithms which will be applied to the optimisation problem further in the paper treat efficiency functions as 'black boxes', i.e. can be applied to any function from that large family and give freedom of interpretation of weights in the graph, according to the specific network. However, there are some natural properties that are typically satisfied in practice. For example, the efficiency usually decreases when the number of parts, k, decreases. Moreover, for a fixed k, the efficiency is typically smaller for unbalanced partitions or, equivalently, vectors $\boldsymbol{s}$.

The functions examined in our experiments are similar to the objective function considered by Hallac et al. (2015). We also consider simpler functions that are not highly convex and therefore the value function can be updated quickly, ensuring quick run-time of our algorithms.

2.3.1. Function A: size-insensitive

We assume that the efficiency associated with communication between the parts is fixed, that is, it is proportional to the total weight of edges between the parts. On the other hand, the efficiency of communication between nodes within very small subnets is negligible, but increases with their sizes, and in the extreme case when all the nodes belong to a single network, it is a^2 times larger than the communication between the parts. Therefore, parameter a is responsible for setting the relative cost of within-cluster communication. In this objective function, the cost does not depend explicitly on the distribution of the sizes of parts, i.e. on vector $\boldsymbol{s}$:

$$\mathrm{c_A} = c_A(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{u}) = \sum_{i=1}^{k} \left(\frac{at_i}{W}\right)^2 t_i + \sum_{1 \le i < j \le k} u_{i,j}, \tag{3}$$

where W is the total weight defined in (2).

2.3.2. Function B: size-sensitive

In this objective function, we take into account the cost associated with the number of subnetworks. In order to keep the specification simple, we do not account for the exact distribution of part sizes but rather the number of parts k, where $k = |s|$. In this formulation, parameter b sets the relative cost of creating additional logical subnetworks:

$$\mathrm{c_B} = c_B(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{u}) = \sum_{i=1}^{k} \left(\frac{at_i}{W}\right)^2 t_i + \sum_{1 \le i < j \le k} u_{i,j} + \left(\frac{k}{b}\right) W, \tag{4}$$

where W is the total weight defined in (2).

3. Algorithms

In this section, we describe the graphs on which we conduct our simulation experiments as well as the algorithm in which we generate them. We concentrate on geometric graphs, since typically in our applications, nodes (base cellular stations) are associated with a geographical location and weights depend on the distance between these locations. Moreover, geometric graphs allow a convenient visualisation. Next, we present the algorithms that we compare which are responsible for finding the optimal clustering of the graphs. We decided to compare two algorithms that were originally proposed for the community detection, namely the Louvain algorithm (Blondel et al., 2008) and the Ensemble Clustering for Graphs (ECG) algorithm (Poulin & Theberge, 2018) with classic metaheuristic approaches, namely the Stochastic Hill Climbing and Simulated Annealing. We also tested a simple greedy and naive algorithm as benchmark solutions.

3.1. Graph generation algorithm

We conduct our simulation experiments on the l-nearest neighbour graph. These graphs have a variety of applications in bioinformatics, data mining, machine learning, manifold learning, clustering analysis and pattern recognition. There are two parameters of the model: n (the number of nodes in a graph) and l (the number of neighbours each node connects to). Our graphs are undirected and weighted, but let us start with a definition of a directed and unweighted auxiliary graph $D = (V, E)$, the l-nearest neighbour graph.

We first generate n nodes forming set V by placing them independently and uniformly at random from the unit square $[0,1]^2$. Then each node $v \in V$ is connected via l directed edges to nodes $u \in V$, where nodes u are the nearest neighbours of v (that is, nodes other than v itself whose distance from v is among the top l smallest ones). Clearly, with probability 1, all pairwise distances are unique, and so the neighbourhoods are well defined, but we can break ties arbitrarily if needed. Please note that in this directed graph, each node has out-degree equal to l, but in-degrees have more complex distribution. In particular, it might happen that there is an edge from v to u but not from u to v.

We assign weights to the directed edges of D in the way described below. Each directed edge uv has a weight being the Euclidean distance between u and v. Finally, we ignore directions to create an undirected and weighted graph G. Please note that if there are two edges, one from u to v and the other one from v to u, then the weights are combined (i.e., the weight of an undirected edge uv is twice the distance between u and v).

It is known that if $l < 0.3043 \ln n$, then asymptotically almost surely (a.a.s.) the graph is not connected, whereas if $l > 0.5139 \ln n$, then it is a.a.s. connected (Balister et al., 2005). In our experiments, we take $l = [\ln n|$ and resample (if needed) to get a connected graph. Let us briefly mention a few algorithmic aspects. The complexity of the trivial brute force method is clearly $O(n^2)$, but it can be easily improved. For example, an $O(n \log^{d-1} n)$ algorithm was presented in Bentley (1980), where d was the dimension of the geometric space (in our case, $d = 2$).

Clarkson (1983) presented a $O(c^d n \, log \, n)$ algorithm, and Vaidya (1989) showed a worst-case $O((c^r d)^d n \, log \, n)$ algorithm. In our scenario, with $d = 2$ and the nodes generated uniformly at random from the unit square, the graph can be generated even quicker (in linear time) by tessellating the unit square into small squares. By tracking where node v lands, one may investigate the nodes in the same small square and a few neighbouring squares to determine the neighbourhood of v.

3.2. Clustering algorithms

3.2.1. Greedy and naive algorithms

According to greedy and naive algorithms, the optimal choice is made at each step, without a broader context or the knowledge of previous as well as future steps. Due to their simplicity, these algorithms fail to find globally optimal solution in many classes of problems (e.g. the shortest path). However, in many cases, such a heuristic can approximate a global optimal solution in a feasible time by finding a local optimum.

We designed the greedy algorithm in the following way: in each step we chose to merge two clusters which yielded the largest gain in the objective function, whereas the naive one considered all the possible merging operations of two clusters in a random sequence and merged two clusters if this operation yielded an improvement to the objective function.

3.2.2. Louvain

The Louvain algorithm is a variation of a greedy algorithm. Due to the favourable ratio of the quality of its work to its speed, it is considered as one of the best algorithms for community detection (Kamiński et al., 2021). The run time of the algorithm is short, $O(n \log^2 n)$, where n is the number of nodes. Compared to other community-detection algorithms, it offers very good quality, but the results are sometimes unstable. When it was introduced in Blondel et al. (2008), the authors showed that it grossly outperformed the existing algorithms both in terms of the quality of results and speed. Furthermore, when comparing it to newer, more advanced algorithms, Louvain still performed remarkably well in both these categories (see Mothe et al., 2017). The operation of the Louvain algorithm can be divided into two steps:

1. The modularity function is optimised locally on all nodes to find communities;
2. The communities are then merged: all the edges within a community are replaced by a single weighted loop, resulting in a smaller graph.

Then, the first step is performed on the smaller graph and the algorithm operates until no further increase in the modularity function value is achieved. In our problem, we use a cost function instead of the modularity function.

3.2.3. Ensemble clustering for graphs (ECG)

Ensemble Clustering for Graphs (ECG) is another example of graph community detection algorithms, similar to the Louvain algorithm. As mentioned above, although the latter has strong advantages in terms of results and efficiency, it can produce unstable results. It is attributed to the fact that there are several possibilities resulting in the same change in modularity function value. The Ensemble Clustering for Graphs (ECG) algorithm is aimed at improving these drawbacks by using several runs of the Louvain algorithm and combining them all, in order to produce a more stable outcome.

1. The first phase of the Louvain algorithm is performed l independent times to obtain l partitions of communities;
2. A new version of $G = (V, E)$ is obtained by assigning new weights according to the following formula[1]:

$$w_{uw} = f(x) = \begin{cases} w_* + (1 + w_*)\dfrac{1}{l}\displaystyle\sum_{i=1}^{l} \alpha_i(u,v), & if\ (u,v) \in C_2(G) \\ w_*, & otherwise \end{cases}; \qquad (5)$$

3. The complete Louvain algorithm is applied to G.

Due to executing l independent runs of Louvain algorithms and ensembling them, the ECG algorithm reduces randomness stemming from the fact that the Louvain on its own might produce different solutions in one run, making the algorithm more robust.

3.2.4. Simulated annealing and stochastic hill climbing algorithm

In this subsection, two metaheuristics are presented: a stochastic hill climbing algorithm and simulated annealing. Metaheuristics are usually used when a problem is too complex to find a solution using brute force algorithm in a computationally efficient time (Hussain et al., 2019). As they are 'state of the art' tools regarding solving similar problems, it would be worth comparing graph clustering algorithms to these metaheuristics. Both of them use randomness during the search process, but while hill climbing is a local algorithm, simulated annealing is a global one.

The stochastic hill climbing is a local search optimisation algorithm that improves the current solution by randomly selecting from available improving moves in the neighborhood of the current solution rather than greedily choosing the best one. Such an approach converges more slowly than steepest ascent methods, but it has a smaller chance of being stuck in a local optimum.

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[1] The k-core of graph G, denoted as $Ck(G)$, is the maximal subgraph of G in which all the nodes have at least degree k.

Simulated annealing operates in a similar manner; however, in certain situations it is able to accept a solution which is worse regarding the objective function value (this makes the algorithm a global one). The probability of accepting a worse solution is a decreasing function of the number of steps that have been performed.

4. Numerical efficiency of clustering

4.1. Experiment design

In this section, the simulation setup is described. We test the performance of the algorithms on graph problems of various sizes understood as the number of nodes. We begin with simulating the algorithms on small problems for which we are able to find the global optimum using an exhaustive search method. We calculate the cost function values for $n = \{1, 2, 3, \dots, 12\}$ using brute force, naive, greedy, Louvain, ECG, uphill and simulated annealing algorithms. For medium-size problems where the number of nodes is equal to $n = \{20, 30, \dots, 100\}$, we make simulations using the naive, greedy, Louvain, ECG, simulated annealing and uphill algorithms. We assume that $a = 5$ and $b = 20$.

For the large-scale problems with $n = 5{,}000$, we test the performance of the Louvain and ECG algorithms using both the objective functions specified in Section 2.3. Furthermore, in order to comprehensively assess the algorithms, we conduct simulations on a grid of parameters governing both objective functions. For the objective function A, we perform simulations for parameter a taking the values from 1 to 10. For the objective function B, we conduct simulations for all the pairs of parameter values a and b, where $a \in \{1, 2, \dots, 10\}$ and $b \in \{5, 10, \dots, 50\}$. In each simulation run, we generate the optimal clustering found by the naive, Louvain and ECG algorithms.

We calculate the mean of the objective function over all simulation runs and report the relative performance of the algorithms with respect to the brute force approach (small problems) and the naive approach (medium and large problems). We also assess the stability of the algorithms by analysing the standard deviation of the objective function values found by the algorithms and the computation time of the algorithms.

We run each algorithm 30 times for each graph size and value function parameter and in each run we randomly generate a new graph. During each calculation, the network is constructed on the basis of an l-nearest neighbour graph defined in Section 3.1.

4.2. Experiment results

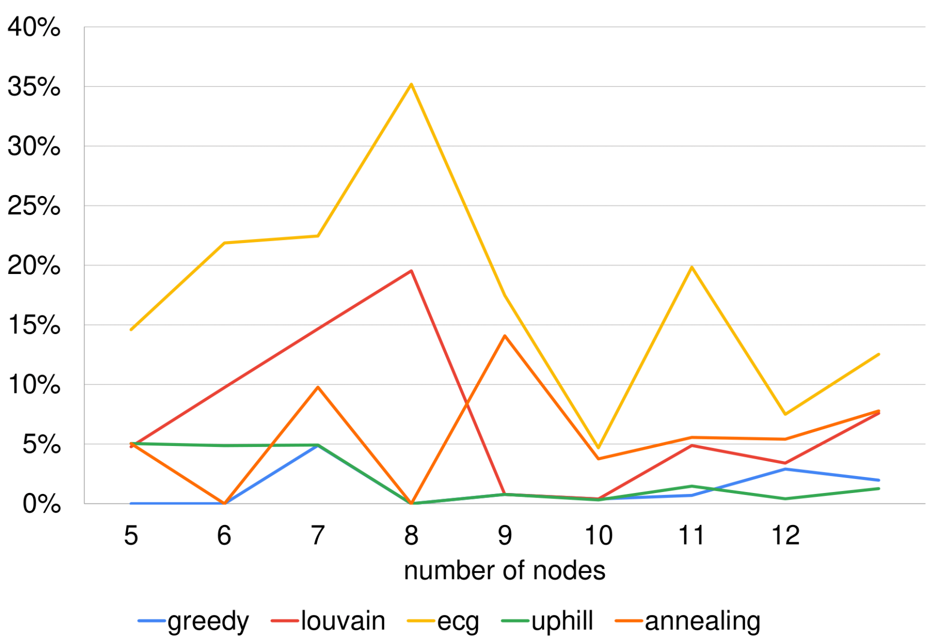
In this section, we present the results of our experiments, i.e. applying clustering algorithms to optimising VNs efficiency problem, including the computing time, as well as objective function values.

4.2.1. Small problems

In this subsection, the results of simulations for small and medium graphs are presented and discussed. We show the deviation of the objective function from the brute force (small problem sizes) and naive approach (medium problem sizes) over the simulation runs for the adopted algorithms, as well as the computation time of the simulations.

In Figure 2, we show the results for the value function in those cases where the number of nodes is small. For these simulation runs, we are able to list all the possible solutions, and therefore find the global optimal solution. Overall, we concluded that the algorithms which we used yielded good results compared to the global optimal solution provided by the brute force approach. The worst result was obtained by the ECG algorithm for the problem where the number of nodes was equal to 8. The objective function turned out 35% worse than the optimum. That algorithm also tended to provide the worst results for almost all small problem sizes, and additionally it performed worse than the naive approach. The greedy and uphill algorithms tended to yield the best results.

Figure 2. Objective function loss with respect to brute force approach for the studied algorithms for small number of nodes (in %)

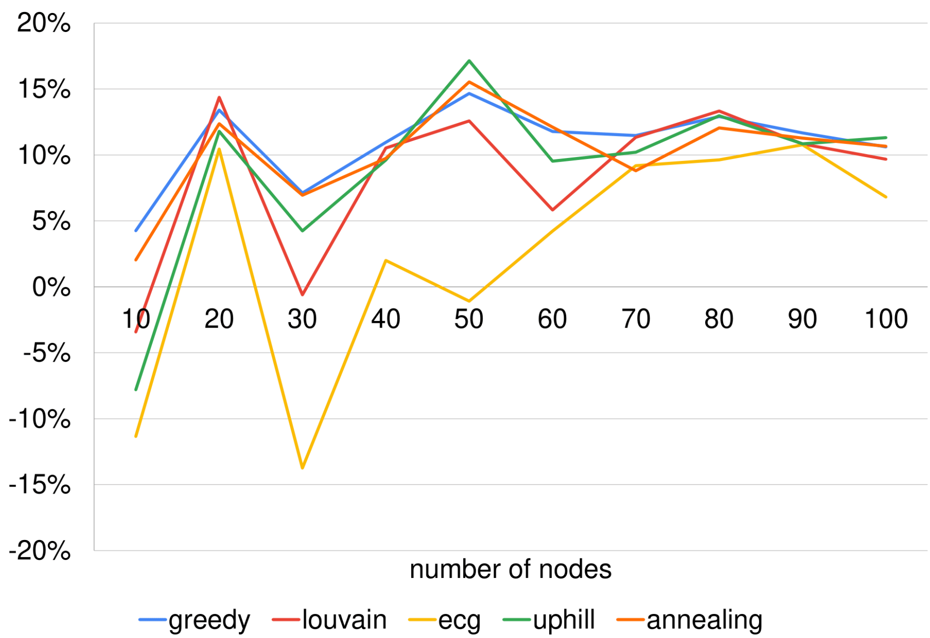


Source: authors' work based on simulation results.

4.2.2. Medium problems

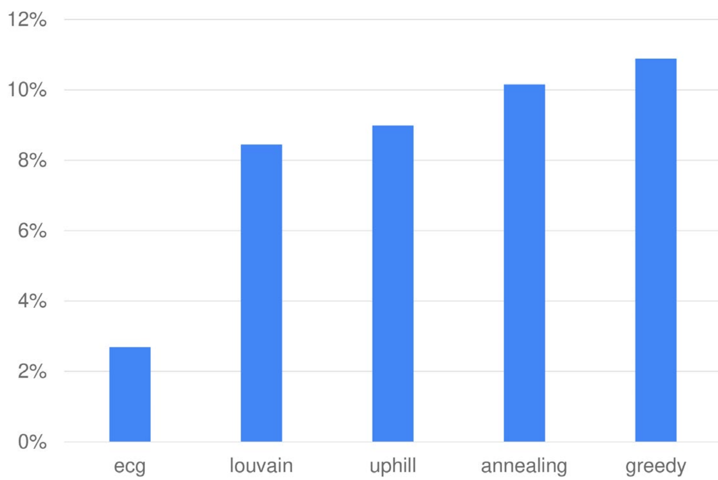
In Figure 3, we show the average resulting value function gain with respect to the naive algorithm for problems in which the number of nodes is up to 100. Comparing the results for medium problem sizes gives more accurate information on the performance of the algorithms. None of the algorithms uniformly outperforms all the others. Also, we can see that for problem sizes exceeding 50, the naive approach, which is the quickest, yields the worst result. We can observe that similarily to what was the case with small problems, the ECG algorithm is the worst-performing formula. In Figure 4, we show the average gain with respect to the naive algorithm over medium problem sizes. This figure confirms the relatively poor performance of the ECG, however the other algorithms performed similarily badly, with the Louvain showing the average gain of 8.4%. The remaining algorithms show slightly larger gains, e.g. the greedy approach with an average gain of 10.9%. The same results would be achieved if we limited this analysis to problem sizes exceeding 50 nodes.

Figure 3. Objective function gain with respect to the naive approach for the studied algorithms for a medium number of nodes (in %)



Source: authors' work based on simulation results.

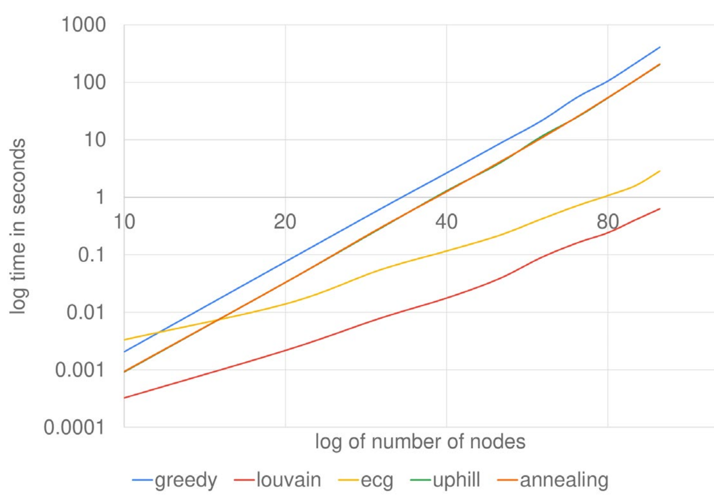
Figure 4. Average percentage gain in objective function value for problems with the number of nodes ranging from 10 to 100 for the studied algorithms



Source: authors' work based on simulation results.

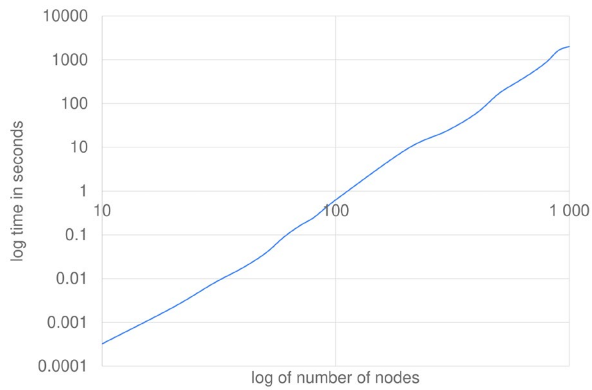
In Figure 5, we show the computation time for the algorithms, and the computation time for the Louvain algorithm for problem sizes of up to 1,000 nodes is presented in Figure 6. The computation times are shown on a log-log plot. We can see that the computation time increases exponentially with the problem size. It is also noticeable that the computation time for both the Louvain algorithm and the ECG are significantly shorter than for the remaining approaches.

Figure 5. Average computation time of the studied algorithms in a log log scale



Source: authors' work based on simulation results.

Figure 6. Average computation time of the Louvain algorithm for a large number of nodes in a log log scale

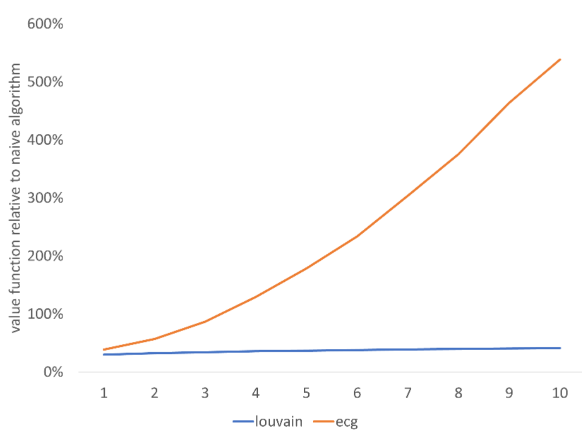


Source: authors' work based on simulation results.

4.2.3. Large problems

We show the results of our simulations in Figures 7–9. In each of the figures, we show the mean objective function value for the algorithm relative to the performance of the naive algorithm. In Figures 8 and 9, darker colours represent lower values, i.e. a better performance of the algorithm. Overall, the community detection algorithms that we used in our study, namely the Louvain and the ECG, show that they perform better than the naive approach, although this is not the case for all parameter values of the objective function. Furthermore, the Louvain algorithm usually performs better than the ECG, unlike in the case of typical community detection problems, where the Louvain algorithm is often unstable.

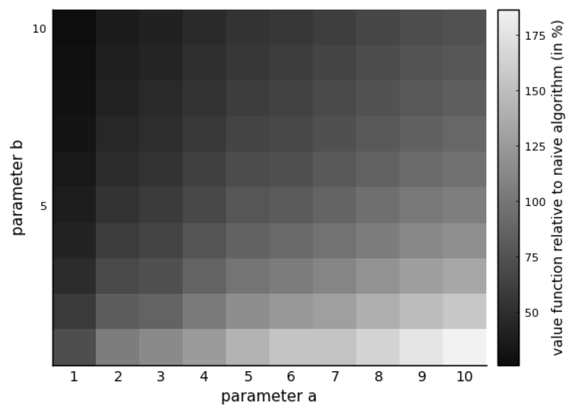
Figure 7. Value function of the Louvain and the ECG algorithms relative to the naive approach for size-insensitive objective function (in %)



Source: authors' work based on simulation results.

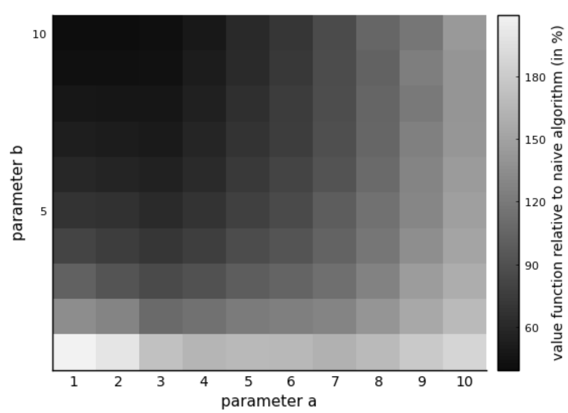
In Figure 7, we show how the two algorithms compare against the naive approach for the size-insensitive objective function, which is the one where we do not take into account the number of clusters when calculating the objective function. It is visible that the Louvain algorithm provides excellent results across all the parameter values that we simulate. When compared to the naive algorithm, it provides a mean objective value function that is between 59% and 70% better than the naive approach. For smaller values of parameter a, the algorithm provides the best results. Furthermore, the Louvain algorithm completely outperforms the ECG for all the values of a. The ECG algorithm shows satisfactory results only for very small values of a; for values larger than 3 this algorithm performs worse than the naive approach.

Figure 8. Value function of the Louvain algorithm relative to the naive approach for size-sensitive objective function (in %)



Source: authors' work based on simulation results.

Figure 9. Value function of the ECG algorithm relative to the naive approach for size-sensitive objective function (in %)



Source: authors' work based on simulation results.

In Figures 8 and 9, we present how the two algorithms compare against the naive approach for the size-sensitive objective function in which we take into account the number of clusters when calculating the objective function. Since the value function depends on two parameters, we show the results as heatmaps, where darker colours indicate better performance than that of the naive algorithm.

The performance of these two algorithms is more nuanced than in the case of value function A. First of all, none of them uniformly outperforms the naive approach. The larger the value of parameter a and the smaller the value of parameter b, the poorer their results. Secondly, their performance is of similar quality, with the Louvain algorithm being slightly better than the ECG.

4.2.4. Discussion

Our computational experiments show that classic community detection algorithms, namely the Louvain and the ECG, yield good-quality solutions for the optimal clustering problem in much shorter time than metaheuristic algorithms such as simulated annealing or hill climbing. Furthermore, the latter fail to converge to a good solution in a reasonable time frame for large problems, where the number of nodes equals 5,000. Out of the community detection algorithms, the Louvain algorithm tends to perform better than the ECG, offering solutions with a better value function across most cases that we considered.

It should be noted that some literature on a typical community detection problems indicates the improvement of the ECG algorithm's performance in relation to that of the Louvain formula and points out that the former alleviates some of the latter's problems (Poulin & Theberge, 2019). In our setup (which differs from the Louvain in that we do not use modularity function but a different objective function), no improvement in the ECG compared to the Louvain could be observed. On the other hand, in the case of community detection problems, the literature shows that the Louvain algorithm outperforms many classic ones (Mothe et al., 2017), but in our setup it was not always the case. However, it has to be mentioned that it is much faster than other algorithms that we tested. Finally, Hallac et al. (2015) consider a similar problem which could be adapted to our setting. They, however, concentrate on highly convex formulations, which could be solved by the Louvain algorithm. They develop the Alternating Direction Method of Multipliers algorithm, which solves the problem efficiently, and which could be applied to a wide array of problems.

5. Conclusions

In this article, we defined a graph clustering optimisation problem. The setup we presented could be applied to a broad range of network routing problems where the network topology changes frequently and where quick and reliable data transmission is necessary. This is the case of vehicular networks, where constantly moving cars change their location in real time, thus changing the topology of the underlying network. An algorithm for clustering such a physical network into a logical one must be computationally efficient, providing good solutions in a short time.

Our study has some limitations, the overcoming of which could become the basis for future research. We considered two specific value functions which represent the quality of the throughput of the network. While the value function specification is general and potentially covers numerous different applications, it is possible that some of them would require a different form of the objective function. However, in such a case, it is possible to simply replace the underlying objective function for the algorithms and run new simulations.

Secondly, we tested the algorithms for a particular network topology generated by the l–nearest neighbour graph. Such a topology mimics the behaviour generated by vehicular networks. This network design does not exhaust all the possible network topologies, and it is not clear how different algorithms would perform for other network types. Testing the performance of community detection algorithms for other networks could be an interesting direction for future research.

Finally, the maximum network size that we considered was 5,000 nodes, while in reality, vehicular networks can consist of tens or even hundreds of thousands of vehicles. Optimising the algorithms to work efficiently for such large networks or developing other procedures to accommodate them could be another problem worth looking into in more detail.

Acknowledgements

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Appendix

The Appendix contains the pseudocode of the algorithms that we are examining in the study.

```
Algorithm 1 Greedy algorithm
Let clustering_old be singleton clustering
Let eff_old be efficiency function for singleton clustering
while criterion_stop = true do
  for i = 1 to m do
    for j = i+1 to m do
       Merge clusters i, j into one cluster
       Calculate new efficiency function eff_new for new clustering
       if eff_new < eff_old then
          Let clustering_tmp be clustering with merged i, j
          Let eff_tmp be efficiency value for clustering with merged i, j
       end if
     end for
  end for
  Let clustering_old be  clustering_tmp
  Let ef_old be eff_tmp
end while
```

```
Algorithm 2 Naive algorithm
Let clustering_old be singleton clustering
Let eff_old be efficiency function for singleton clustering
for i = 1 to m do
     Let j be randomly picked cluster
     Merge clusters i, j into one cluster
     Calculate new efficiency function eff_new for new clustering
    if eff_new < eff_old then
       Let clustering_old be clustering with merged i, j
       Let ef_old be efficiency value for clustering with merged i, j
    end if
end for
```

Algorithm 3 Louvain algorithm

Let $clustering_{old}$ be singleton clustering
Let eff_{old} be efficiency function for singleton clustering
while $criterion_{stop} = true$ **do**
  for $i = 1$ to n **do**
    for j = neighbors(i) **do**
     $criterion_{stop} = false$
      Change the assignment of i to the cluster of j
      Calculate new efficiency function eff_{new} for new clustering
      if $eff_{new} < eff_{old}$ **then**
        Let $clustering_{old}$ be the new clustering
        Let eff_{old} be efficiency function value for the new clustering
       $criterion_{stop} = true$
      end if
    end for
  end for
  for $i = 1$ to $clusters_{number}$ **do**
    $weights_{new}$[i,i] = sum of weights(G) of edges from new cluster i
    for $j = i+1$ to $clusters_{number}$ **do**
      $weights_{new}$[i,j]=sum of weights(G) of edges from cluster i to j
      $weights_{new}$[j,i]=sum of weights(G) of edges from cluster i to j
     G = graph($clustering_{oLd}$) with $weights_{new}$
    end for
  end for
end while

Algorithm 4 Stochastic hill climbing

Let $clustering_{old}$ be singleton clustering
Let eff_{old} be efficiency function for singleton clustering
for $i = 1$ to m **do**
  Let *sampled* be the set of sampled clusters in proportion of *fraction*
  for j in *sampled* **do**
    Merge clusters i, j into one cluster
    Calculate new efficiency function eff_{new} for new clustering
    if $eff_{new} < eff_{old}$ **then**
      Let $clustering_{old}$ be clustering with merged i, j
      Let eff_{old} be efficiency value for clustering with merged i, j
    end if
  end for
end for

Algorithm 5 Simulated annealing algorithm

Let $clustering_{old}$ be singleton clustering
Let eff_{old} be efficiency function for singleton clustering
Let $temperature = initial_{temp} / iteration_{no}$
for $i = 1$ to m **do**
  Let $sampled$ be the set of sampled clusters in proportion of $fraction$
  for j in $sampled$ **do**
    Merge clusters i, j into one cluster
    Calculate new efficiency function eff_{new} for new clustering
    if $eff_{new} < eff_{old}$ **then**
      Let $clustering_{old}$ be clustering with merged i, j
      Let eff_{old} be efficiency value for clustering with merged i, j
    else
      Let $criterion = \exp(-(eff_{new} - eff_{best})/temperature)$
      if $random_{vaLue} < criterion$ **then**
        Let eff_{old} be efficiency value for clustering with merged i, j
      end if
    end if
  end for
end for

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Evaluating the performance of modified Anderson–Darling goodness-of-fit tests for normality

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Abstract. The main aim of the article is to define and practically apply the modified Anderson–Darling (MAD) goodness-of-fit tests for normality. The modifications consist in varying the formula for calculating the empirical distribution function (EDF). Additional contributions of the paper include the expansion of the EDF family with four new proposals and the creation of a family of alternative distributions, consisting of both older and newer distributions that belong to all groups of skewness and kurtosis signs thanks to their flexibility. Critical values are obtained using 106 order statistics for sample sizes of n=10,20 and at a significance level of α=0.05. Finally, the article shows the calculation of the power of the analysed tests for alternative distributions based on 10^5 values of the test statistics. Their parameters have been selected to show several similarities to the normal distribution. The effectiveness of the tests is illustrated through the analysis of real datasets.

Keywords: empirical distribution function, goodness-of-fit test, Anderson–Darling test, test power

JEL: C14, C15

1. Introduction

A variety of goodness-of-fit tests (GoFTs) have been considered and applied in many fields of science. GoFTs for normality are very popular in economics and finance, where they are used to analyse market behaviour (the distribution of rates of return, trading volume or asset prices), assess market efficiency and identify deviations from ideal market conditions, analyse stochastic processes (asset prices or changes in commodity prices). Other examples of scientific areas where the application of GoFTs is common is demography, where the fertility curve is almost normally distributed, and econometrics, where normality tests are used to check whether regression errors are normally distributed. This is particularly important for the proper evaluation of regression models, as violating the assumption of normality can lead to erroneous statistical conclusions. In hydrology, specialists are interested in estimating flood magnitudes for high return periods. In these cases, however, the interest is focused on the upper (right) tail of the distribution. Moreover, one of the most important problems in hydrology is the estimation of the quantile for a specific

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return period. Then the goodness of fit in the upper or lower (left) tail of the distribution is more important than the fit of the entire region (Ma et al., 2024).

One of the most common testing procedures available in statistical software is the Anderson-Darling (AD) test (Anderson & Darling, 1952), which belongs to a group of tests based on the empirical distribution function (EDF). Other popular EDF tests include the Kolmogorov-Smirnov (KS) test (Kolmogoroff, 1933; Smirnov, 1948), the Lilliefors (LF) test (Lilliefors, 1967), the Cramér-von Mises (CVM) test (Cramér, 1928; von Mises, 1931), the Kuiper (K) test (Kuiper, 1960), and the Watson (W) test (Watson, 1962).

Recently, many articles have been devoted to GoFTs for normality. Table 1 shows the works created in the 21st century and their authors.

Table 1. Articles devoted to normal GoFTs written in the 21st century

Article	Sample sizes	Article	Sample sizes
Bonett and Seier (2002)	**10**, **20**, …, **50**, 100	Afeez et al. (2018)	**10**, **30**, **50**, 100, …
Aliaga et al. (2003)	–	Marange and Qin (2019)	**15**, **30**, **50**, 80, …
Bontemps and Meddahi (2005)	100,250,500, …	Sulewski (2019)	**10**, **12**, …, **30**, **40**, **50**
Luceño (2006)	100	Tavakoli et al. (2019)	**5**, **6**, …, **15**, **20**, **25**, **30**, **40**, **50** …,100
Yazici and Yolacan (2007)	**20**, **30**, **40**, **50**	Mishra et al. (2019)	$\boldsymbol{n < 30, n > 30}$
Gel et al. (2007)	**20**, **50**, 100	Kellner and Celisse (2019)	**50**, 75,100,200, …
Coin (2008)	**20**, **50**, 200	Wijekularathna et al. (2020)	**5**, **10**, **20**, **30**, **50**, 75, …
Brys et al. (2008)	100,1000	Sulewski (2022a)	**10**, **14**, **20**
Gel and Gastwirth (2008)	**30**, **50**, 100	Hernandez (2021)	**5**, **10**, …, **30**
Romão et al. (2010)	**25**, **50**, 100	Khatun (2021)	**10**, **20**, **25**, **30**, **40**, **50**, 100,200,300
Razali and Wah (2011)	**20**, **30**, **50**, 100, …	Arnastauskaitė et al. (2021)	$\mathbf{2^5}, 2^6, \dots, 2^{10}$
Noughabi and Arghami (2011)	**10**, **20**, **30**, **50**	Bayoud (2021)	**10**, **20**, …, **50**, 60,80,100
Yap and Sim (2011)	**10**, **20**, **30**, **50**, 100, …	Uhm and Yi (2023)	**10**, **20**, **30**, 100,200,300
Chernobai et al. (2005)	–	Sulewski (2021)	**20**, **50**, 100
Ahmad and Khan (2015)	**10**, **20**, …, **50**, 100, …	Desgagne et al. (2023)	**20**, **50**, 100,200
Mbah and Paothong (2015)	**10**, **20**, **30**, **50**, 100, …	Uyanto (2022)	**10**, **30**, **50**, 70,100
Torabi et al. (2016)	**10**, **20**, **50**, 100, …	Ma et al. (2024)	**10**, **30**, **50**
Feuerverger (2016)	200	Giles (2024)	**10**, **25**, **50**, 100, 250 500,1000
Nosakhare and Bright (2017)	**5**, **10**, …, **50**, 100	Borrajo et al. (2024)	**50**, 100,200,500
Desgagné and Lafaye de Micheaux (2018)	**10**, **12**, …, **20**, **50**, 100, …	Terán-García and Pérez-Fernández (2024)	**25**, 900

Note. Sample sizes of $n \leq 50$ are in bold.

Source: authors' work.

The small samples that dominate in Table 1 are common in experimental economics, where the research described in the published articles was based on samples of a dozen or so people in a group. This is where strong tests can be particularly useful, as situations may happen that a hypothesis accepted in the original article is rejected when a stronger test is applied.

From a methodological perspective, this paper is situated within the literature on nonparametric goodness-of-fit testing, and in particular within the class of EDF-based tests for normality. Unlike approaches based on parametric modeling, tests based on EDF such as the KS, CVM and AD tests rely solely on the comparison between the empirical and theoretical distribution functions and therefore remain fully nonparametric under the null hypothesis.

The proposed modified Anderson-Darling (MAD) tests belong to this class and contribute to the existing literature by systematically investigating alternative definitions of the EDF within the Anderson-Darling framework. Special emphasis is placed on small sample sizes which dominate many practical applications and a substantial part of the existing literature, as shown in Table 1. In particular, the analysis focuses on small sample sizes where classical asymptotic approximations may be unreliable and where the improvements in test power are especially relevant.

Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be independent and identically distributed observations from unknown continuous cumulative distribution function (CDF) $F(x)$. We wish to know whether $F(x)$ coincides with a CDF of the normal distribution $\Phi(x)$. We are interested in testing hypothesis $H_0\colon F(x) = \Phi(x)$ against $H_1\colon F(x) \neq \Phi(x)$. The EDF is given by $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \theta(x - x_i)$, where $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$.

The δ-corrected KS test (Harter et al., 1984), further investigated by Khamis (1990, 1992, 1993), redefines the value of the EDF at the data points and compares the redefined EDF to the CDF at the data points. Let the EDF at the i-th data point be given by

$$F_\delta(x) = \frac{i - \delta}{n - 2\delta + 1}, 0 \leq \delta \leq 1. \tag{1}$$

Harter et al. (1984) selected $\delta = 0, 0.5, 1$ for study.

Bloom (1958) proposed the α, β transformation:

$$F_{\alpha,\beta}\big(x_{(i)}\big) = \frac{i - \alpha}{n - \alpha - \beta + 1}, \alpha, \beta \leq 1 \tag{2}$$

to decrease the MSE of certain statistics. Note that $F_{\delta,\delta}(x) = F_{\delta}(x)$. This transformation was used to create the GoFTs. Sulewski (2022a) used the Bloom formula to create the one-component Lilliefors GoFT with statistic

$$\overline{LF} = \underbrace{max}_{i}\left\{\left|F_{\alpha,\beta}\left(x_{(i)}\right) - \Phi\left(x_{(i)}\right)\right|\right\}. \tag{3}$$

It is commonly known that the greatest discrepancy between the theoretical and empirical distribution functions may occur at different positions in the series. The probability that this discrepancy appears at a given positional statistic r decreases as r becomes more extreme. The idea of a two-component test statistic is described in Sulewski (2021). The first component (denoted by $\overline{\overline{LF}}$) is, as in the original LF test, the absolute value of the greatest discrepancy between the sample and population distributions, while the second component (denoted by r) is the position in an ordered sample where this discrepancy is located. Both components, $\overline{\overline{LF}}$ and r, are random variables.

Simulation studies for the one- and two-component Lilliefors tests were carried out by Sulewski (2021, 2022a) for the following methods of calculating $F_{\alpha,\beta}\left(x_{(i)}\right)$ $(\alpha, \beta \leq 1)$:

1. $F_{0,1}\left(x_{(i)}\right) = \frac{i}{n}$ – occurs in the KS statistic;
2. $F_{1,0}\left(x_{(i)}\right) = \frac{i-1}{n}$ – occurs in the KS statistic;
3. $F_{\frac{1}{2},\frac{1}{2}}\left(x_{(i)}\right) = \frac{i-0.5}{n}$ – occurs in the CM statistic;
4. $F_{0,0}\left(x_{(i)}\right) = \frac{i}{n+1}$ – the mean value of i-th order statistic of the beta distribution;
5. $F_{\frac{3}{10},\frac{3}{10}}\left(x_{(i)}\right) = \frac{i-0.3}{n+0.4}$ – the median of i-th order statistic of the beta distribution;
6. $F_{\frac{3}{8},\frac{3}{8}}\left(x_{(i)}\right) = \frac{i-0.375}{n+0.25}$ – the mean value of i-th order statistic of the normal distribution;
7. $F_{\frac{127}{400},\frac{127}{400}}\left(x_{(i)}\right) = \frac{i-0.3175}{n+0.365}$ – proposed by Filliben (1975);
8. $F_{1,1}\left(x_{(i)}\right) = \frac{i-1}{n-1}$ – proposed by Harter et al. (1984).

In six of the EDF definitions listed above (except $F_{0,1}$ and $F_{1,0}$), $\alpha = \beta$.

The main aim of the article is to define and practically apply modified Anderson–Darling (MAD) goodness-of-fit tests for normality. We propose an MAD goodness-of-fit test for normality in which the classical EDF is replaced by Bloom's (α, β) family of EDF estimators. In contrast to the previous studies, Bloom's formula is applied using values of α and β that have not been previously investigated in the context of Anderson-Darling-type tests. Additional contributions of the study to the existing body of research involve the extension of the EDF family by four new EDF definitions, which are incorporated into the proposed modified Anderson-Darling framework. Moreover, rather than introducing new probability distributions, the paper constructs a structured family of alternative distributions by grouping the existing classical and modern distributions according to the signs of skewness and excess kurtosis, which allows a systematic and interpretable comparison of the test power. Finally, the power of the proposed tests is evaluated using Monte Carlo simulations based on $rep = 10^5$ replications, where each replication corresponds to a simulated value of the test statistic under a given alternative distribution. The critical values are obtained using simulated order statistics for sample sizes of $n = 10$ and $n = 20$ at the significance level of $\alpha = 0.05$.

2. Modified Anderson–Darling tests for normality

Let $z_{(i)} = \left(x_{(i)} - \bar{x}\right)/s$. Goodness-of-fit tests based on EDF form a broad and well-established class of procedures for assessing normality. Among them, the KS and CVM tests are the two fundamental approaches, differing primarily in the way the deviations between empirical distribution function F_n and the hypothesised distribution are aggregated.

The KS test belongs to the supremum class of EDF statistics and focuses on the maximum absolute deviation between F_n and the theoretical distribution. In contrast, the CVM test belongs to the quadratic EDF class and integrates squared deviations over the entire support of the distribution. The CVM statistic is given by

$$CVM = n\int_{-\infty}^{\infty}[F_n(z) - \Phi(z)]^2 d\Phi(z). \tag{4}$$

The computing formulas for the CVM statistic is given by

$$\begin{aligned} CVM &= \frac{1}{12n} + \sum_{i=1}^{n}\left[\Phi\left(z_{(i)}\right) - \frac{i-0.5}{n}\right]^2 = \\ &= \frac{n}{3} + \frac{1}{n}\sum_{i=1}^{n}(1-2i)\Phi\left(z_{(i)}\right) + \sum_{i=1}^{n}\Phi\left(z_{(i)}\right)^2. \end{aligned} \tag{5}$$

A general representation of the CVM family is given by

$$FAD = n\int_{-\infty}^{\infty}[F_n(z) - \Phi(z)]^2 \omega\left(\Phi(z)\right) d\Phi(z), \tag{6}$$

where $\omega\left(\Phi(z)\right)\colon [0,1] \to R^+$ is a weight function.

The AD statistic corresponds to weight function

$$\omega\big(\Phi(z)\big) = \{\Phi(z)[1-\Phi(z)]\}^{-1}, \tag{7}$$

placing greater emphasis on the tails of the distribution. The classical AD statistic is defined as

$$AD = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{\Phi(z)[1-\Phi(z)]}d\Phi(z). \tag{8}$$

The lower and upper tail AD statistics marked AD^L and AD^U are defined as (Sinclair et al., 1990)

$$AD^L = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{\Phi(z)}d\Phi(z)\,, AD^U = \\ = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{1-\Phi(z)}d\Phi(z). \tag{9}$$

Note that $AD = AD^L + AD^U$. The computing formulas for the AD, AD^L and AD^U statistics are as follows:

$$AD = -n - \frac{1}{n}\sum\nolimits_{i=1}^{n}\{(2i-1)ln[\Phi(z_{(i)})] + (2n-2i+1)ln[1-\Phi(z_{(i)})]\}, \tag{10}$$

$$AD^L = -\frac{3n}{2} + 2\sum\nolimits_{i=1}^{n}\Phi(z_{(i)}) - \frac{1}{n}\sum\nolimits_{i=1}^{n}(2i-1)ln[\Phi(z_{(i)})], \tag{11}$$

$$AD^U = \frac{n}{2} - 2\sum\nolimits_{i=1}^{n}\Phi(z_{(i)}) - \frac{1}{n}\sum\nolimits_{i=1}^{n}(2n-2i+1)ln[1-\Phi(z_{(i)})]. \tag{12}$$

The lower-tail and upper-tail AD statistics measure the discrepancies between the empirical and theoretical distributions that are concentrated in the lower and upper tails, respectively, rather than over the entire distribution. Such tail-focused statistics are particularly useful in applications where departures from normality occur mainly in the extremes and where small sample sizes limit the power of global goodness-of-fit tests.

Stephens (1974, 1979) proposed modifications of the AD statistics denoted as AD_1, AD_2 and given by

$$\overline{AD} = AD\left(1+\frac{4}{n}-\frac{25}{n^2}\right), \overline{\overline{AD}} = AD\left(1+\frac{0.75}{n}+\frac{2.25}{n^2}\right). \tag{13}$$

The AD test is the member of the CVM family for normality.

It is important to replace the weight function in order to raise the power of the test statistic. (7) denotes AD statistic (8) and $\omega(\Phi(z)) = 1$ corresponds to the CVM statistic (Cramér, 1928; von Mises, 1931).

Rodriguez and Viollaz (1995) considered $\omega(\Phi(z)) = [2\Phi(z) - \Phi(z)^2]^{-1}$ for the lower tail and $\omega(\Phi(z)) = [1 - \Phi(z)^2]^{-1}$ for the upper tail of the distribution. Thus, the next members of the CVM family are

$$ADR^L = n\int_{-\infty}^{\infty} \frac{[F_n(z) - \Phi(z)]^2}{2\Phi(z) - \Phi(z)^2} d\Phi(z)\,, ADR^U = \\ = n\int_{-\infty}^{\infty} \frac{[F_n(z) - \Phi(z)]^2}{1 - \Phi(z)^2} d\Phi(z)\,, \text{respectively.} \tag{14}$$

The computing formulas for the ADR_L and ADR_U statistics are as follows:

$$ADR^L = n[ln(4) - 1] + \\ + \frac{\sum_{i=1}^{n}(2i - 4n - 1)ln\left[2 - \Phi\left(z_{(i)}\right)\right] - \sum_{i=1}^{n}(2i - 1)ln\left[\Phi\left(z_{(i)}\right)\right]}{2n}, \tag{15}$$

$$ADR^U = n[ln(4) - 1] + \\ + \frac{\sum_{i=1}^{n}(2i - 2n - 1)ln\left[1 - \Phi\left(z_{(i)}\right)\right] - \sum_{i=1}^{n}(2i + 2n - 1)ln\left[1 + \Phi\left(z_{(i)}\right)\right]}{2n}. \tag{16}$$

Luceño (2006) proposed $\omega(\Phi(z)) = \Phi(z)^{-2}$ and $\omega(\Phi(z)) = [1 - \Phi(z)]^{-2}$ for the lower and upper tails of the distribution, respectively. The modified AD statistics marked as ADL^L and ADL^U are as follows:

$$ADL^L = n\int_{-\infty}^{\infty} \frac{[F_n(z) - \Phi(z)]^2}{\Phi(z)^2} d\Phi(z)\,, ADL^M = \\ = n\int_{-\infty}^{\infty} \frac{[F_n(z) - \Phi(z)]^2}{[1 - \Phi(z)]^2} d\Phi(z), \tag{17}$$

The statistic for the entire distribution is given by Luceño (2006):

$$ADL = ADL_L + ADL_U. \tag{18}$$

The computing formulas for the ADL^L, ADL^U and ADL statistics are given by Chernobai et al. (2005) and Luceño (2006):

$$ADL^L = 2\sum_{i=1}^{n} ln\left[\Phi\left(z_{(i)}\right)\right] + \frac{1}{n}\sum_{i=1}^{n}\frac{2i-1}{\Phi\left(z_{(i)}\right)}, \tag{19}$$

$$ADL^U = 2\sum_{i=1}^{n} ln\left[1-\Phi\left(z_{(i)}\right)\right] + \frac{1}{n}\sum_{i=1}^{n}\frac{2n-2i+1}{1-\Phi\left(z_{(i)}\right)}, \tag{20}$$

$$ADL = 2ln\left\{\prod_{i=1}^{n}\Phi\left(z_{(i)}\right)\left[1-\Phi\left(z_{(i)}\right)\right]\right\} + \frac{1}{n}\sum_{i=1}^{n}\left[\frac{2i-1}{\Phi\left(z_{(i)}\right)} + \frac{2n-2i+1}{1-\Phi\left(z_{(i)}\right)}\right]. \tag{21}$$

Ma et al. (2024) proposed $\omega\left(\Phi(z)\right) = \Phi(z)^{-1.5}$ and $\omega\left(\Phi(z)\right) = [1-\Phi(z)]^{-1.5}$ for the lower and upper tails of the distribution, respectively. The modified AD statistics marked as ADM^L and ADM^U are as follows:

$$\begin{gathered} ADM^L = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{\Phi(z)^{1.5}}d\Phi(z)\,, ADM^U = \\ = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{[1-\Phi(z)]^{1.5}}d\Phi(z). \end{gathered} \tag{22}$$

The computing formulas for the ADM^L and ADM^U statistics are defined as

$$ADM^L = 2\sum_{i=1}^{n}\left[2\sqrt{\Phi\left(z_{(i)}\right)} + \frac{2i-1}{n\sqrt{\Phi\left(z_{(i)}\right)}}\right] - \frac{16}{3}n, \tag{23}$$

$$ADM^U = 2\sum_{i=1}^{n}\left[2\sqrt{1-\Phi\left(z_{(i)}\right)} + \frac{2n-2i+1}{n\sqrt{1-\Phi\left(z_{(i)}\right)}}\right] - \frac{16}{3}n, \tag{24}$$

Feuerverger (2016) considered $\omega\left(\Phi(z)\right) = [1-\Phi(z)]^{-\vartheta}$ $(0 \le \vartheta < 2)$ for the upper tail of the distribution. Thus, the next member of the CVM family is as follows:

$$ADF^U(v) = n\int_{-\infty}^{\infty}\frac{[F_n(z)-\Phi(z)]^2}{[1-\Phi(z)]^{v}}d\Phi(z)\ (0 \le \vartheta \le 2). \tag{25}$$

Note that $ADF^U(1) = AD^U$, $ADF^U(2) = ADC^U$, $ADF^U(1.5) = ADM^U$, $ADF^U(0) = $ $= CVM$. The computing formula for the $ADF_U(v)$ statistic has the form of:

$$ADF^{U}(\vartheta) = C(n,v) + \\ +\frac{2}{2-\vartheta}\sum_{i=1}^{n}\left[1-\Phi\left(z_{(i)}\right)\right]^{2-v} + \frac{\sum_{i=1}^{n}(2n-2i+1)\left[1-\Phi\left(z_{(i)}\right)\right]^{1-\vartheta}}{n(v-1)}, \tag{26}$$

where $C(n,v) = \frac{n}{3-\vartheta} - \frac{2n}{2-\vartheta} + \frac{n}{1-\vartheta}$.

There is a second version of the AD test for normality – the variance-weighted KS test marked as $|AD|$, which belongs to the supremum class (Chernobai et al., 2005). The $|AD|$ statistic is given by

$$|AD| = \sqrt{n}max\left\{\underbrace{sup}_{i}\left[\frac{\frac{i}{n}-\Phi\left(z_{(i)}\right)}{\sqrt{\Phi\left(z_{(i)}\right)\left[1-\Phi\left(z_{(i)}\right)\right]}}\right], \underbrace{sup}_{i}\left[\frac{\Phi\left(z_{(i)}\right)-\frac{i-1}{n}}{\sqrt{\Phi\left(z_{(i)}\right)\left[1-\Phi\left(z_{(i)}\right)\right]}}\right]\right\}. \tag{27}$$

The lower and upper tails $|AD|$ statistics marked as $|AD_L|$ and $|AD_U|$ are defined as

$$|AD^{L}| = \sqrt{n}max\left\{\underbrace{sup}_{i}\left[\frac{\frac{i}{n}-\Phi\left(z_{(i)}\right)}{\Phi\left(z_{(i)}\right)}\right], \underbrace{sup}_{i}\left[\frac{\Phi\left(z_{(i)}\right)-\frac{i-1}{n}}{\Phi\left(z_{(i)}\right)}\right]\right\}, \tag{28}$$

$$|AD^{U}| = \sqrt{n}max\left\{\underbrace{sup}_{i}\left[\frac{\frac{i}{n}-\Phi\left(z_{(i)}\right)}{1-\Phi\left(z_{(i)}\right)}\right], \underbrace{sup}_{i}\left[\frac{\Phi\left(z_{(i)}\right)-\frac{i-1}{n}}{1-\Phi\left(z_{(i)}\right)}\right]\right\}, \tag{29}$$

All computing formulas were verified with appropriate integral formulas in Mathcad by comparing the closed-form results with a numerical integration for a range of sample sizes and order statistics.

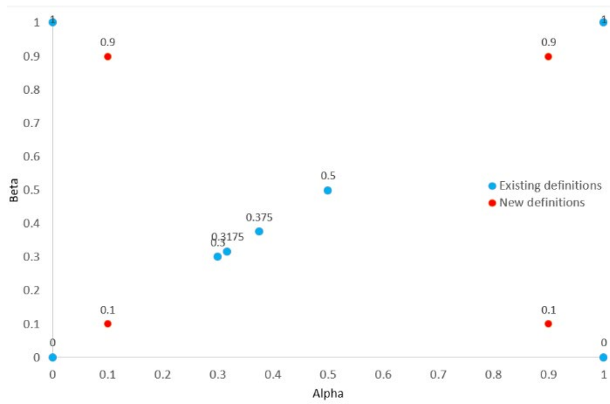
3. New modified Anderson–Darling test for normality

Before we present the MAD test, we expand the EDF family with four new proposals: $F_{0.1,0.1}$, $F_{0.9,0.1}$, $F_{0.9,0.9}$ and $F_{0.1,0.9}$, given by

$$F_{\frac{1}{10},\frac{1}{10}}\left(z_{(i)}\right) = \frac{i-0.1}{n+0.8}, F_{\frac{9}{10},\frac{1}{10}}\left(z_{(i)}\right) = \frac{i-0.9}{n}, F_{\frac{9}{10},\frac{9}{10}}\left(z_{(i)}\right) = \frac{i-0.9}{n-0.8}, F_{\frac{1}{10},\frac{9}{10}}\left(z_{(i)}\right) = \frac{i-0.1}{n}. \tag{30}$$

Thus, eight of the EDF definitions listed earlier are on the $\beta = \alpha$ line and five of them are on the $\beta = -\alpha + 1$ line (see Figure 1). The previously analysed values unevenly fill the $\beta = \alpha$ line over interval $[0, 1]$. Four values belong to the $[0.3, 0.5]$ interval. Value 0.1 represents interval $(0, 0.3)$ and the values of 0.9 represent interval $(0.5, 1)$. The new representatives of the $\beta = -\alpha + 1$ line, also located at the corners of the square, are EDFs with $\alpha = 0.9, \beta = 0.1$ and $\alpha = 0.1, \beta = 0.9$.

Figure 1. Graphical representation of EDF definitions



Source: authors' work.

The computing formula for the AD statistic (10) can be written as

$$AD = -n - 2\sum_{i=1}^{n}\left\{\frac{i-0.5}{n}ln\left[\Phi\left(z_{(i)}\right)\right] + \frac{n-i+0.5}{n}ln\left[1-\Phi\left(z_{(i)}\right)\right]\right\} \quad (31)$$

or

$$AD = -n - 2\sum_{i=1}^{n}\left\{\frac{i-0.5}{n}ln\left[\Phi\left(z_{(i)}\right)\right] + \left(1-\frac{i-0.5}{n}\right)ln\left[1-\Phi\left(z_{(i)}\right)\right]\right\}. \quad (32)$$

The AD statistic can be expressed as a functional of the EDF. The commonly used computing formulas (31)-(32) correspond to the classical choice of the EDF given by

$$A\hat{F}\left(x_{(i)}\right) = \frac{i-0.5}{n}.$$

Bloom (1958) proposed a two-parameter family of EDF estimators of the form of (2) which includes the classical EDF as a special case for $\alpha = \beta = 0.5$. Replacing $\hat{F}$ in the AD functional with $F_{\alpha,\beta}$ yields the following modified Anderson–Darling statistic:

$$AD_{\alpha,\beta} = -n - 2\sum_{i=1}^{n}\left\{F_{\alpha,\beta}\left(x_{(i)}\right)ln\left[\Phi\left(z_{(i)}\right)\right] + \left[1-F_{\alpha,\beta}\left(x_{(i)}\right)\right]ln\left[1-\Phi\left(z_{(i)}\right)\right]\right\}. \quad (33)$$

Note that $AD_{0.5,0.5} = AD$.

4. Similarity measure

Let's assume that

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \,, s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \,, z_{(i)} = \frac{\left(x_{(i)} - \bar{x}\right)}{s}, \tag{34}$$

$$m_k = \frac{1}{n}\sum\nolimits_{i=1}^{n}\left(x_{(i)} - \bar{x}\right)^k, \gamma_1 = \frac{m_3}{s^3}, \bar{\gamma}_2 = \frac{m_4}{s^4} - 3. \tag{35}$$

Note that the Malachov inequality is defined as $\bar{\gamma}_2 \geq {\gamma_1}^2 - 2$ (Malachov, 1978).

A review of the recent statistical literature shows that the values of small skewness γ_1 and excess kurtosis $\bar{\gamma}_2$ do not dominate in testing for normality. It is very interesting to see how the GoFTs will respond to samples coming from alternatives close to normal distribution.

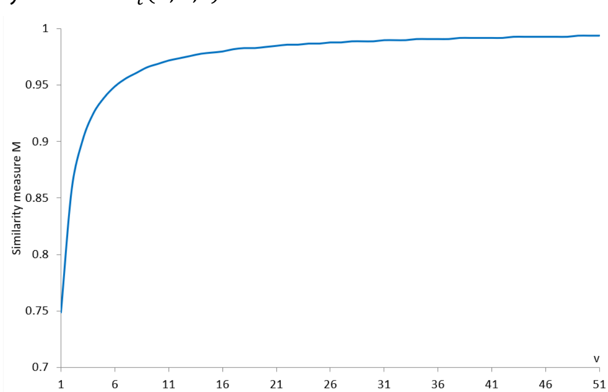
Let $f(x;\boldsymbol{\theta})$ be a PDF of the alternative with vector of parameters $\boldsymbol{\theta}$. Similarity measure M of the alternative (Alt) to the normal distribution is defined as (Sulewski, 2022a)

$$M(\boldsymbol{\theta};\mu,\sigma) = \int_{-\infty}^{\infty} min[f(x;\boldsymbol{\theta}),\phi(x;\mu,\sigma)]dx,$$

where $\phi(x;\mu,\sigma)$ is the PDF of the normal distribution. $M(\boldsymbol{\theta};\mu,\sigma)$ takes on the values of $[0,1]$. $M(\boldsymbol{\theta};\mu,\sigma) = 1$ when the PDFs are identical.

Figure 2 shows the values of similarity measure (36), when the alternative is the Student's t distribution with v degrees of freedom. Note that if $v \to +\infty$, then obviously $M_t(v;0,1) \to 1$.

Figure 2. Similarity measure $M_t(v;0,1)$ for Student's *t* distribution



Source: authors' work.

5. Alternative distributions

As mentioned before, there are numerous articles devoted to testing for normality. In these articles, many alternative distributions (alternatives) are used, including asymmetric and symmetric ones. Note that symmetric distributions with undefined γ_1 and $\bar{\gamma}_2$ are Cauchy and slash distributions (S), which are defined as

$$S = \frac{N(0,1)}{U},$$

where $N(0,1)$ denotes standard normal distribution and $U \sim Uniform(0,1)$.

According to the statistical literature, alternatives can be divided into four groups, depending on the support and shape of their densities (see e.g. Esteban et al., 2001; Torabi et al., 2016). These groups include symmetric alternatives with support $(-\infty, \infty)$, asymmetric alternatives with support $(-\infty, \infty)$, alternatives with support $(0, \infty)$ and alternatives with support $(0,1)$. Gan and Koehler (1990), Krauczi (2009) and Torabi et al. (2016) divided alternatives into five groups, namely: asymmetric short-tailed, asymmetric long-tailed, symmetric short-tailed, symmetric close to normal and symmetric long-tailed alternatives. Sulewski (2021, 2022a) divided alternatives into 12 groups (A1-F2) by their γ_1 and $\bar{\gamma}_2$ signs and bimodality. The author showed that bimodality of the alternative does not affect the choice of the EDF definition.

As stated before, one of the aims of our article is to provide a new division of alternatives. Based on the proposal presented in Sulewski and Stoltmann (2026), our idea is to divide the alternatives into nine groups according to their γ_1 and $\bar{\gamma}_2$ signs. Groups O–H are defined in Table 2.

Table 2. Groups of alternatives with signs of γ_1 and $\bar{\gamma}_2$

Group	γ_1	$\bar{\gamma}_2$
O	zero	zero
A	positive	positive
B	negative	positive
C	zero	positive
D	zero	negative
E	positive	negative
F	negative	negative
G	positive	zero
H	negative	zero

Source: authors' work.

The main criterion for selecting an alternative for the Monte Carlo simulation is that γ_1 and $\bar{\gamma}_2$ calculated for the alternative parameters belong to the O, A–H groups. This criterion is fulfilled by distributions defined in an infinite domain such as:

- the Edgeworth series (ES) with parameters γ_1 and $\bar{\gamma}_2$ as a monolithic distribution (Kendall & Stuart, 1968);

- the Pearson (P) distribution, with parameters γ_1 and $\bar{\gamma}_2$ as a monolithic distribution (Pearson, 1895);
- the normal mixture (NM) distribution with five parameters as a mixture of two normal distributions;
- the normal distribution with plasticising component (NDPC) (Sulewski, 2022b) with six parameters as a mixture of normal and non-normal distributions;
- the plasticising component mixture (PCM) with seven parameters (Sulewski, 2022a) as a mixture of two identical non-normal distributions that characterises multimodality.

In each group of alternatives, we consider four families of distributions and four values of similarity measure M (36), namely $M = 0.50, 0.75, 0.90$ and 0.95. This results in a total of 16 alternative distributions per group. An exception concerns the P distribution in group C, for which the similarity measure could not be defined for all parameter values.

The Appendix presents Tables 1A–5A with vectors of alternative parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M for the analysed alternatives. The skewness and excess kurtosis tend to zero, while the similarity measure tends to unity. Often, the mean tends to zero and the standard deviation tends to unity, while the similarity measure tends to unity. PDF formulas and PDF curves (see Figures 1A–5A) for the alternative $\boldsymbol{\theta}$ values are also provided in the Appendix. As shown in Figure 1A, the ES distribution is not suitable for simulation studies due to the negative PDF values for groups D-H even though the normalisation condition is met. Figure 4A shows interesting bimodal shapes. Figures 2A and 3A show both unimodal and bimodal shapes. Figure 1A shows unimodal shapes and Figure 5A presents very interesting multimodal shapes.

6. Power study

The use of lower and upper tail versions of the AD-type statistics is motivated by the fact that many practically relevant departures from normality are asymmetric and manifest primarily in one tail of the distribution. In such cases, global goodness-of-fit statistics may lose power, especially for small sample sizes, whereas tail-focused tests can detect localised deviations in the lower or upper tail more effectively. This approach is consistent with the earlier findings reported in Chernobai et al. (2005) and is particularly relevant for the grouped alternatives considered in this study.

The use of Bloom's formula (2) in the one-component (Sulewski, 2022a) and two-component (Sulewski, 2021) LF statistic significantly influenced the power of tests (PoTs). Can we expect a similar situation to occur for the new modified AD test?

In Hernandez (2021), a sample of the most recent comparisons (since 1990) has been used to rank 55 different normality tests. The overall winner of this analysis is the regression-based Shapiro-Wilk (SW) test of normality.

29 GoFTs were selected for the simulation studies (see Table 3). New proposals numbered 19-29 were compared with LF, CVM, SW tests (Shapiro & Wilk, 1965) numbered 1–3 and AD modifications numbered 4–18 listed in Section 1. To study the power of each of the discussed tests, critical values $cv_{0.05}$ (the type I error $\alpha = 0.05$) were calculated using $m = 10^6$ simulated order statistics. The PoTs were calculated based on $rep = 10^5$ Monte Carlo replications for each alternative distribution and each value of the similarity measure. Table 3 shows the critical values (CVs) used in the simulation study and the test sizes (TSs) for the analysed tests. The names of the new modifications are in bold. In total, the simulation design combines 29 tests, 4 families of alternative distributions, 4 values of the similarity measure and two sample sizes.

Table 3. Critical values (CV) and test sizes (TS) of the analysed GoFTs for normality

No.	GoFT	CV		TS		No.	GoFT	CV		TS	
		n=10	n=20	n=10	n=20			n=10	n=20	n=10	n=20
1	LF	0.262	0.192	0.050	0.050	16	ADL	6.931	12.685	0.050	0.051
2	CVM	0.120	0.123	0.051	0.051	17	ADM^L	0.891	1.075	0.052	0.051
3	SW	0.844	0.904	0.050	0.050	18	ADM^U	0.889	1.078	0.050	0.051
4	$\|AD^L\|$	18.200	37.830	0.051	0.050	19	$\boldsymbol{AD_{0.1}}$	0.693	0.726	0.051	0.049
5	$\|AD^U\|$	18.235	38.061	0.050	0.050	20	$\boldsymbol{AD_{1.0}}$	0.693	0.726	0.050	0.050
6	$\|AD\|$	2.697	3.632	0.051	0.051	21	$\boldsymbol{AD_{0.0}}$	1.479	1.610	0.052	0.051
7	AD^L	6.931	12.685	0.050	0.051	22	$\boldsymbol{AD_{1.1}}$	−0.281	−0.261	0.052	0.051
8	AD^U	0.354	0.374	0.051	0.051	23	$\boldsymbol{AD_{0.3.0.3}}$	1.022	1.087	0.052	0.051
9	$\underline{AD}$	0.687	0.721	0.052	0.051	24	$\boldsymbol{AD_{3/8.3/8}}$	0.900	0.952	0.052	0.051
10	$\overline{AD}$	0.790	0.821	0.052	0.051	25	$\boldsymbol{AD_{127\,127}}$	0.994	1.056	0.052	0.051
11	$\overline{\overline{AD}}$	0.754	0.753	0.052	0.051	26	$\boldsymbol{AD_{0.1.0.1}}$	1.333	1.439	0.052	0.051
12	ADR^L	0.216	0.226	0.052	0.051	27	$\boldsymbol{AD_{0.9.0.9}}$	−0.070	−0.056	0.052	0.051
13	ADR^U	0.216	0.227	0.052	0.051	28	$\boldsymbol{AD_{0.9.0.1}}$	0.691	0.724	0.051	0.050
14	ADL^L	3.510	6.256	0.051	0.051	29	$\boldsymbol{AD_{0.1.0.9}}$	0.691	0.725	0.051	0.049
15	ADL^U	3.515	6.289	0.051	0.051						

Source: authors' work.

The complete simulation results with power values fill a table with 29 columns (29 tests) and 32 rows (4 alternatives, 2 sample sizes, 4 values of a single similarity measure). Presenting such large tables is difficult due to the size of the article. Therefore, the conclusions apply to the full results, and only the most interesting results are shown in Tables 6–13. Alternatives are indexed, i.e. the larger the index, the more the distribution resembles a normal distribution (e.g. index 4 denotes the similarity measure of 0.95). The highest values in the row are in bold.

Of course, it is expected that the power of the GoFTs will increase as the sample size increases and decrease as the value of similarity measure (36) increases. The following analysed tests do not meet these basic assumptions: $|AD^L|$ (groups A, D, E, F, G), $|AD^U|$ (B, D, E, F, H), $|AD|$ (D, E, F, G, H), AD^L (D, E, F, H), ADL^L (A, D, E, F, G), ADL^U (B, D, E, F, H), ADM^L for (A, D, E, G), ADM^U (D F, H) and ADM (D). Powers of the $\overline{AD}$ and $\overline{\overline{AD}}$ tests are the same for all the analysed cases.

The average PoT is the highest for group B of the alternatives, followed by group A. This means that these GoFTs best detect samples from asymmetric distributions with positive excess kurtosis. The most problematic case, as might be expected, is detecting samples from symmetric distributions. The GoFTs best detect samples from the P distribution and the worst from the NM distribution.

The power of the modified AD tests for normality with $\alpha = \beta$ is very similar for all groups of alternatives. Tables 4 and 5 show exemplary results for groups A and H at significance level $\alpha = 0.05$. It is noteworthy that one test dominates for all the analysed alternatives and similarity measures. The $AD_{1,0}$ test is recommended for groups A, E and G. The $AD_{0,1}$ test is recommended for groups B, F and H. The $AD_{1,1}$ and $AD_{0,0}$ tests are recommended for groups C and D, respectively.

Table 4. The power of MAD tests for group A of alternatives (Alts)

Alt	n	MAD test										
		19	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
P_1	10	0.764	**0.844**	0.817	0.814	0.816	0.816	0.816	0.817	0.814	0.839	0.775
P_2		0.231	**0.340**	0.291	0.291	0.291	0.291	0.291	0.291	0.291	0.331	0.243
P_3		0.082	**0.131**	0.106	0.107	0.106	0.106	0.106	0.106	0.107	0.126	0.087
P_4		0.064	**0.085**	0.073	0.074	0.073	0.073	0.073	0.073	0.074	0.082	0.066
P_1	20	0.990	**0.995**	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.995	0.990
P_2		0.535	**0.650**	0.599	0.599	0.599	0.599	0.599	0.599	0.599	0.642	0.549
P_3		0.146	**0.213**	0.179	0.183	0.180	0.181	0.180	0.180	0.183	0.208	0.153
P_4		0.080	**0.112**	0.097	0.099	0.097	0.098	0.097	0.097	0.099	0.109	0.084
NM_1	10	0.078	**0.126**	0.102	0.104	0.103	0.103	0.103	0.102	0.103	0.122	0.084
NM_2		0.074	**0.123**	0.097	0.098	0.098	0.098	0.098	0.097	0.098	0.117	0.078
NM_3		0.053	**0.074**	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.070	0.055
NM_4		0.049	**0.054**	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.054	0.050
NM_1	20	0.138	**0.207**	0.169	0.172	0.170	0.170	0.170	0.169	0.172	0.198	0.142
NM_2		0.133	**0.199**	0.167	0.169	0.168	0.168	0.168	0.167	0.169	0.195	0.141
NM_3		0.065	**0.092**	0.079	0.080	0.079	0.079	0.079	0.079	0.080	0.091	0.068
NM_4		0.051	0.057	0.055	0.056	0.055	0.055	0.055	0.055	0.056	**0.059**	0.052
$NDPC_1$	10	0.286	**0.381**	0.335	0.338	0.336	0.336	0.336	0.335	0.338	0.372	0.295
$NDPC_2$		0.103	**0.152**	0.130	0.129	0.130	0.130	0.130	0.130	0.129	0.148	0.108
$NDPC_3$		0.053	**0.059**	0.056	0.057	0.056	0.056	0.056	0.056	0.057	0.059	0.054
$NDPC_4$		0.052	**0.052**	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.052
$NDPC_1$	20	0.615	**0.697**	0.661	0.666	0.662	0.663	0.662	0.662	0.665	0.694	0.626
$NDPC_2$		0.198	**0.255**	0.230	0.229	0.230	0.229	0.229	0.230	0.229	0.254	0.206
$NDPC_3$		0.055	**0.063**	0.059	0.060	0.059	0.059	0.059	0.059	0.060	0.063	0.057
$NDPC_4$		0.052	**0.052**	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.051
PCM_1	10	0.559	**0.607**	0.594	0.588	0.592	0.592	0.592	0.594	0.588	0.605	0.567
PCM_2		0.162	**0.261**	0.217	0.218	0.217	0.217	0.217	0.217	0.218	0.252	0.173
PCM_3		0.054	**0.076**	0.065	0.066	0.065	0.065	0.065	0.065	0.066	0.074	0.056
PCM_4		0.051	**0.056**	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.055	0.051
PCM_1	20	0.879	**0.909**	0.896	0.894	0.895	0.895	0.895	0.896	0.894	0.908	0.883
PCM_2		0.412	**0.531**	0.481	0.481	0.481	0.481	0.481	0.481	0.482	0.522	0.426
PCM_3		0.065	**0.096**	0.082	0.083	0.082	0.082	0.082	0.082	0.083	0.094	0.068
PCM_4		0.052	**0.059**	0.055	0.056	0.056	0.056	0.056	0.056	0.056	0.058	0.052

Note. The highest test power is in bold.

Source: authors' work.

Table 5. The power of MAD tests for group H of alternatives (Alts)

Alt	n	MAD test										
		19	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
P_1	10	**0.797**	0.724	0.769	0.762	0.767	0.767	0.767	0.768	0.763	0.735	0.792
P_2		**0.270**	0.175	0.228	0.225	0.227	0.227	0.227	0.228	0.225	0.186	0.262
P_3		**0.096**	0.056	0.078	0.077	0.078	0.078	0.078	0.078	0.078	0.060	0.091
P_4		**0.068**	0.047	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.049	0.066
P_1	20	**0.987**	0.980	0.986	0.985	0.985	0.985	0.985	0.985	0.985	0.981	0.987
P_2		**0.557**	0.446	0.507	0.501	0.505	0.505	0.505	0.506	0.501	0.459	0.547
P_3		**0.154**	0.099	0.126	0.124	0.126	0.125	0.125	0.126	0.125	0.105	0.149
P_4		**0.083**	0.054	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.057	0.080
NM_1	10	**0.099**	0.060	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.065	0.097
NM_2		**0.079**	0.051	0.066	0.066	0.066	0.066	0.066	0.066	0.066	0.054	0.077
NM_3		**0.054**	0.048	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.048	0.054
NM_4		**0.052**	0.049	0.050	0.051	0.050	0.051	0.050	0.050	0.051	0.050	0.052
NM_1	20	**0.156**	0.102	0.128	0.127	0.128	0.128	0.128	0.128	0.127	0.106	0.148
NM_2		**0.110**	0.071	0.091	0.091	0.092	0.091	0.092	0.092	0.091	0.075	0.106
NM_3		**0.056**	0.049	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.049	0.056
NM_4		**0.053**	0.050	0.051	0.050	0.051	0.051	0.051	0.051	0.050	0.049	0.052
$NDPC_1$	10	**0.142**	0.087	0.117	0.117	0.117	0.117	0.117	0.117	0.117	0.093	0.137
$NDPC_2$		**0.084**	0.053	0.067	0.066	0.066	0.066	0.066	0.066	0.066	0.056	0.081
$NDPC_3$		**0.053**	0.048	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.048	0.052
$NDPC_4$		**0.052**	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051
$NDPC_1$	20	**0.253**	0.179	0.218	0.218	0.218	0.218	0.218	0.218	0.218	0.187	0.246
$NDPC_2$		**0.120**	0.075	0.098	0.097	0.098	0.097	0.098	0.098	0.098	0.080	0.115
$NDPC_3$		**0.055**	0.049	0.053	0.052	0.052	0.052	0.052	0.052	0.052	0.050	0.054
$NDPC_4$		**0.052**	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.051	0.052
PCM_1	10	**0.068**	0.048	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.050	0.066
PCM_2		**0.057**	0.047	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.048	0.056
PCM_3		**0.063**	0.050	0.057	0.057	0.057	0.057	0.057	0.057	0.057	0.051	0.062
PCM_4		**0.061**	0.050	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.051	0.060
PCM_1	20	**0.083**	0.057	0.069	0.068	0.068	0.068	0.068	0.069	0.068	0.059	0.080
PCM_2		**0.060**	0.047	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.049	0.059
PCM_3		**0.074**	0.058	0.066	0.066	0.066	0.066	0.066	0.066	0.066	0.060	0.073
PCM_4		**0.066**	0.052	0.059	0.059	0.059	0.059	0.059	0.059	0.059	0.054	0.064

Note. The highest test power is in bold.

Source: authors' work.

Table 6. The power of GoFTs for group A of alternatives (Alts)

Alt	n	GoFT									
		2	3	5	8	12	15	18	**20**	**28**	**27**
P_1	10	0.787	0.847	0.513	0.681	**0.856**	0.602	0.651	0.844	0.839	0.814
P_2		0.271	0.316	0.302	0.250	0.300	**0.322**	0.301	0.340	0.331	0.291
P_3		0.102	0.111	0.165	0.108	0.100	**0.167**	0.145	0.131	0.126	0.107
P_4		0.071	0.076	0.108	0.076	0.069	**0.108**	0.095	0.085	0.082	0.074
P_1	20	0.987	**0.997**	0.689	0.960	0.997	0.824	0.927	0.995	0.995	0.993
P_2		0.545	**0.671**	0.450	0.502	0.620	0.512	0.549	0.650	0.642	0.599
P_3		0.164	0.209	0.252	0.184	0.162	**0.269**	0.256	0.213	0.208	0.183
P_4		0.091	0.108	0.153	0.103	0.086	**0.159**	0.147	0.112	0.109	0.099
NM_1	10	0.100	0.106	0.160	0.106	0.095	**0.164**	0.142	0.126	0.122	0.103
NM_2		0.095	0.100	0.146	0.098	0.093	**0.150**	0.130	0.123	0.117	0.098
NM_3		0.061	0.062	0.090	0.062	0.061	**0.090**	0.078	0.074	0.070	0.062
NM_4		0.052	0.052	**0.060**	0.053	0.051	0.060	0.058	0.054	0.054	0.052
NM_1	20	0.158	0.185	0.224	0.180	0.147	**0.249**	0.246	0.207	0.198	0.172
NM_2		0.160	0.178	0.189	0.169	0.152	0.214	**0.218**	0.199	0.195	0.169
NM_3		0.077	0.083	0.108	0.082	0.074	**0.113**	0.108	0.092	0.091	0.080
NM_4		0.055	0.057	0.071	0.057	0.053	**0.072**	0.067	0.057	0.059	0.056
$NDPC_1$	10	0.338	0.318	0.292	0.349	0.302	0.344	0.368	**0.380**	0.372	0.338
$NDPC_2$		**0.135**	0.121	0.119	0.120	0.130	0.130	0.128	**0.152**	0.148	0.129
$NDPC_3$		0.057	0.054	0.062	0.056	0.057	**0.063**	0.061	0.060	0.059	0.057
$NDPC_4$		0.052	0.052	0.054	0.052	0.052	**0.054**	0.053	0.053	0.053	0.053
$NDPC_1$	20	0.661	0.608	0.349	**0.700**	0.584	0.504	0.674	0.700	0.694	0.665
$NDPC_2$		**0.245**	0.186	0.116	0.207	0.227	0.138	0.180	0.258	0.254	0.229
$NDPC_3$		0.062	0.054	0.059	0.060	0.059	0.062	**0.062**	0.064	0.063	0.060
$NDPC_4$		0.051	0.053	0.054	0.051	0.051	**0.056**	0.054	0.052	0.052	0.052
PCM_1	10	0.580	0.594	0.424	0.572	0.581	0.492	0.551	0.607	0.605	0.588
PCM_2		0.202	0.227	0.232	0.214	0.200	**0.258**	0.257	0.261	0.252	0.218
PCM_3		0.064	0.067	**0.099**	0.067	0.062	0.099	0.085	0.076	0.074	0.066
PCM_4		0.054	0.054	0.063	0.055	0.053	**0.063**	0.060	0.056	0.055	0.054
PCM_1	20	0.871	**0.896**	0.711	0.866	0.883	0.772	0.838	0.909	0.908	0.894
PCM_2		0.436	0.495	0.258	0.483	0.428	0.366	**0.496**	0.531	0.522	0.482
PCM_3		0.077	0.093	0.134	0.085	0.076	**0.139**	0.126	0.096	0.094	0.083
PCM_4		0.055	0.058	0.074	0.057	0.054	**0.075**	0.069	0.059	0.058	0.056

Source: authors' work.

Table 7. The power of GoFTs for group B of alternatives (Alts)

Alt	n	GoFT									
		2	3	4	8	13	14	17	19	**29**	**22**
P_1	10	0.785	0.846	0.510	**0.869**	0.855	0.600	0.645	0.846	0.841	0.811
P_2		0.270	0.315	0.302	0.306	0.300	**0.321**	0.297	0.340	0.331	0.290
P_3		0.101	0.112	0.166	0.098	0.100	**0.168**	0.144	0.130	0.125	0.107
P_4		0.071	0.074	**0.107**	0.069	0.070	**0.107**	0.094	0.083	0.081	0.074
P_1	20	0.986	0.997	0.691	**0.997**	0.996	0.826	0.927	0.995	0.995	0.993
P_2		0.547	**0.674**	0.454	0.635	0.621	0.516	0.553	0.650	0.640	0.602
P_3		0.166	0.207	0.251	0.159	0.162	**0.266**	0.254	0.215	0.209	0.183
P_4		0.091	0.108	0.154	0.083	0.085	**0.160**	0.147	0.110	0.107	0.099
NM_1	10	0.130	0.138	0.191	0.122	0.125	**0.197**	0.176	0.161	0.156	0.137
NM_2		0.102	0.105	0.135	0.102	0.103	**0.140**	0.124	0.125	0.123	0.105
NM_3		0.078	0.078	0.091	0.081	0.081	**0.094**	0.085	0.093	0.091	0.079
NM_4		0.055	0.056	0.077	0.054	0.055	**0.077**	0.068	0.063	0.062	0.056
NM_1	20	0.228	0.258	0.284	0.204	0.212	0.318	**0.323**	0.279	0.271	0.247
NM_2		0.175	0.174	0.159	0.168	0.170	0.180	**0.193**	0.206	0.200	0.178
NM_3		0.113	0.109	0.097	**0.117**	0.117	0.106	0.110	0.132	0.127	0.113
NM_4		0.063	0.065	0.092	0.059	0.059	**0.096**	0.090	0.073	0.072	0.064
$NDPC_1$	10	0.652	0.660	0.451	0.660	**0.664**	0.536	0.582	0.701	0.694	0.660
$NDPC_2$		0.353	0.366	0.372	0.328	0.337	**0.414**	0.408	0.408	0.401	0.366
$NDPC_3$		0.129	0.145	0.206	0.121	0.124	**0.211**	0.187	0.164	0.159	0.138
$NDPC_4$		0.051	0.050	0.062	0.051	0.051	**0.062**	0.057	0.054	0.054	0.051
$NDPC_1$	20	0.954	0.950	0.626	0.955	0.958	0.778	0.900	0.968	0.967	0.961
$NDPC_2$		0.660	0.692	0.561	0.613	0.631	0.672	**0.736**	0.715	0.709	0.687
$NDPC_3$		0.219	0.283	0.331	0.200	0.208	**0.365**	0.357	0.286	0.279	0.250
$NDPC_4$		0.052	0.056	0.069	0.053	0.053	**0.070**	0.066	0.059	0.058	0.054
PCM_1	10	0.214	0.224	0.263	0.197	0.203	**0.283**	0.269	0.260	0.253	0.224
PCM_2		**0.189**	0.177	0.165	**0.189**	**0.190**	0.181	0.177	0.217	0.212	0.186
PCM_3		0.057	0.059	**0.069**	0.058	0.058	**0.069**	0.065	0.061	0.061	0.058
PCM_4		0.052	0.054	**0.062**	0.053	0.053	**0.061**	0.059	0.056	0.056	0.054
PCM_1	20	0.422	0.442	0.367	0.377	0.393	0.447	**0.503**	0.486	0.478	0.447
PCM_2		**0.373**	0.307	0.186	0.357	0.364	0.225	0.283	0.388	0.382	0.359
PCM_3		0.064	0.078	**0.096**	0.064	0.064	**0.096**	0.090	0.073	0.072	0.069
PCM_4		0.056	0.063	**0.079**	0.055	0.055	0.077	0.074	0.062	0.061	0.058

Source: authors' work.

Table 8. The power of GoFTs for group C of alternatives (Alts)

Alt	n	GoFT									
		2	6	7	15	16	**22**	**27**	9	10	11
P_1	10	0.115	0.147	**0.148**	0.115	**0.148**	0.122	0.121	0.120	0.120	0.120
P_2		0.096	**0.122**	0.122	0.100	**0.122**	0.101	0.101	0.100	0.100	0.100
P_3		0.077	**0.097**	0.096	0.084	0.096	0.080	0.080	0.079	0.079	0.079
P_4		0.063	**0.072**	0.072	0.066	**0.072**	0.064	0.064	0.063	0.063	0.063
P_1	20	0.176	0.248	**0.253**	0.175	**0.253**	0.194	0.193	0.191	0.191	0.191
P_2		0.134	0.199	**0.203**	0.147	**0.203**	0.149	0.149	0.146	0.146	0.146
P_3		0.096	0.146	**0.148**	0.115	**0.148**	0.106	0.105	0.104	0.104	0.104
P_4		0.067	0.095	**0.095**	0.083	**0.095**	0.072	0.072	0.071	0.071	0.071
NM_1	10	**0.214**	0.165	0.174	0.140	0.174	0.202	0.202	0.200	0.200	0.200
NM_2		0.130	0.129	0.131	0.110	0.131	0.129	0.128	0.127	0.127	0.127
NM_3		0.064	**0.076**	0.076	0.071	**0.076**	0.065	0.065	0.065	0.065	0.065
NM_4		0.058	**0.064**	0.063	0.060	**0.063**	0.059	0.059	0.059	0.059	0.059
NM_1	20	**0.391**	0.204	0.225	0.178	0.225	0.361	0.361	0.357	0.357	0.357
NM_2		**0.214**	0.160	0.170	0.141	0.170	0.207	0.206	0.204	0.204	0.204
NM_3		0.072	0.101	**0.102**	0.087	**0.102**	0.077	0.076	0.076	0.076	0.076
NM_4		0.059	0.075	**0.076**	0.071	**0.076**	0.061	0.061	0.061	0.061	0.061
$NDPC_1$	10	**0.179**	0.125	0.133	0.115	0.133	0.169	0.169	0.168	0.168	0.168
$NDPC_2$		0.060	0.067	**0.067**	0.063	**0.067**	0.062	0.062	0.061	0.061	0.061
$NDPC_3$		0.052	**0.054**	0.054	0.052	**0.054**	0.052	0.052	0.052	0.052	0.052
$NDPC_4$		0.051	0.053	**0.053**	0.051	**0.053**	0.051	0.051	0.051	0.051	0.051
$NDPC_1$	20	**0.323**	0.098	0.116	0.108	0.116	0.297	0.297	0.294	0.294	0.294
$NDPC_2$		0.063	0.079	**0.080**	0.074	**0.080**	0.065	0.065	0.065	0.065	0.065
$NDPC_3$		0.054	0.059	**0.059**	0.057	**0.059**	0.054	0.054	0.054	0.054	0.054
$NDPC_4$		0.050	**0.051**	**0.051**	**0.052**	**0.051**	0.050	0.050	0.050	0.050	0.050
PCM_1	10	0.082	0.095	**0.095**	0.084	**0.095**	0.085	0.084	0.083	0.083	0.083
PCM_2		**0.107**	0.095	0.097	0.087	0.097	0.102	0.102	0.101	0.101	0.101
PCM_3		0.077	**0.078**	**0.078**	0.073	**0.078**	0.075	0.075	0.074	0.074	0.074
PCM_4		0.059	**0.067**	0.067	0.064	**0.067**	0.060	0.060	0.060	0.060	0.060
PCM_1	20	0.107	0.108	0.112	0.102	**0.112**	0.111	0.111	0.110	0.110	0.110
PCM_2		**0.160**	0.104	0.109	0.099	0.109	0.146	0.145	0.144	0.144	0.144
PCM_3		**0.100**	0.084	0.086	0.081	0.086	0.095	0.095	0.094	0.094	0.094
PCM_4		0.061	0.086	**0.087**	0.077	**0.087**	0.065	0.065	0.065	0.065	0.065

Source: authors' work.

Table 9. The power of GoFTs for group D of alternatives (Alts)

Alt	n	GoFT									
		1	2	3	12	13	18	**21**	8	**26**	**23**
P_1	10	0.365	0.524	**0.667**	0.562	0.564	0.461	0.608	0.559	0.606	0.602
P_2		0.081	0.104	**0.127**	0.112	0.113	0.074	0.121	0.113	0.120	0.118
P_3		0.045	0.045	0.043	0.045	0.046	0.033	0.046	0.046	0.046	0.045
P_4		0.042	0.041	0.039	0.040	0.041	0.035	0.041	0.041	0.041	0.040
P_1	20	0.716	0.890	**0.980**	0.925	0.925	0.843	0.953	0.921	0.952	0.951
P_2		0.148	0.229	**0.352**	0.266	0.263	0.147	0.295	0.264	0.294	0.290
P_3		0.053	0.059	0.060	0.062	0.062	0.027	**0.065**	0.062	0.065	0.064
P_4		**0.045**	0.044	0.038	0.043	0.044	0.024	0.045	0.044	0.044	0.044
NM_1	10	0.163	0.194	0.179	0.173	**0.201**	0.123	0.199	0.198	0.197	0.194
NM_2		0.141	0.164	0.212	0.113	**0.266**	**0.304**	0.201	0.298	0.200	0.198
NM_3		**0.048**	0.048	0.045	0.047	**0.048**	0.039	0.048	0.048	0.048	0.047
NM_4		**0.048**	0.048	0.047	0.048	**0.048**	0.047	0.048	0.048	0.048	0.048
NM_1	20	0.364	0.481	0.418	0.418	**0.493**	0.256	0.486	0.485	0.484	0.479
NM_2		0.323	0.308	0.478	0.205	**0.556**	**0.598**	0.440	0.597	0.438	0.435
NM_3		0.059	0.062	0.055	0.060	0.061	0.032	0.063	0.061	0.062	0.062
NM_4		0.049	0.049	0.046	0.048	0.048	0.045	0.048	0.048	0.048	0.048
$NDPC_1$	10	0.099	0.110	0.101	0.106	0.108	0.064	0.112	0.105	0.111	0.110
$NDPC_2$		**0.053**	**0.053**	0.049	**0.053**	**0.053**	0.040	**0.053**	0.052	0.053	0.052
$NDPC_3$		**0.047**	0.046	0.045	0.046	**0.047**	0.045	0.046	0.047	0.046	0.046
$NDPC_4$		0.049	0.050	0.049	0.049	**0.050**	0.049	0.049	0.050	0.049	0.049
$NDPC_1$	20	0.188	0.242	0.203	0.228	0.227	0.092	0.243	0.217	0.241	0.238
$NDPC_2$		0.071	**0.071**	0.058	0.067	0.067	0.033	0.069	0.065	0.069	0.068
$NDPC_3$		**0.047**	0.046	0.040	0.045	0.045	0.038	0.044	0.045	0.044	0.044
$NDPC_4$		**0.049**	0.049	0.047	0.048	0.047	0.048	0.048	0.048	0.048	0.048
PCM_1	10	**0.046**	0.046	0.043	0.045	0.045	0.043	0.045	0.046	0.045	0.045
PCM_2		0.056	0.056	0.054	0.055	0.055	0.046	0.056	0.055	0.056	0.055
PCM_3		**0.048**	0.047	0.046	0.047	0.047	0.044	0.048	0.048	0.047	0.047
PCM_4		0.046	0.046	0.046	0.046	0.046	0.045	0.046	0.046	0.046	0.046
PCM_1	20	**0.047**	0.044	0.039	0.044	0.044	0.035	0.043	0.043	0.043	0.043
PCM_2		**0.068**	0.068	0.063	0.066	0.066	0.046	0.067	0.065	0.067	0.066
PCM_3		**0.050**	0.048	0.045	0.048	0.047	0.040	0.048	0.047	0.048	0.048
PCM_4		**0.049**	0.048	0.045	0.047	0.047	0.041	0.047	0.047	0.047	0.047

Source: authors' work.

Table 10. The power of GoFTs for group E of alternatives (Alts)

Alt	n	GoFT									
		1	2	3	5	8	12	13	15	**23**	18
P_1	10	0.577	0.698	0.775	0.240	0.566	**0.791**	0.616	0.345	0.737	0.471
P_2		0.149	0.188	0.224	0.132	0.144	**0.236**	0.159	0.147	0.205	0.148
P_3		0.065	0.069	0.074	0.096	0.067	**0.072**	0.068	**0.098**	0.071	0.085
P_4		0.052	0.053	0.054	0.074	0.054	0.052	0.054	**0.075**	0.053	0.065
P_1	20	0.902	0.961	**0.990**	0.166	0.900	0.988	0.930	0.369	0.980	0.760
P_2		0.297	0.402	**0.531**	0.106	0.293	0.529	0.330	0.148	0.461	0.231
P_3		0.096	0.106	**0.128**	0.109	0.098	0.121	0.101	0.119	0.115	0.120
P_4		0.060	0.062	0.066	0.083	0.063	0.062	**0.064**	**0.087**	**0.064**	0.082
NM_1	10	0.086	0.093	**0.094**	0.107	0.088	0.092	0.091	**0.113**	**0.094**	0.105
NM_2		0.063	0.064	**0.065**	0.089	0.065	0.064	**0.065**	**0.091**	**0.065**	0.081
NM_3		0.052	0.052	0.051	**0.066**	0.053	0.051	0.053	0.066	0.052	**0.060**
NM_4		0.051	0.051	0.050	0.053	0.051	0.050	0.051	**0.054**	0.051	0.053
NM_1	20	0.141	0.162	0.158	0.097	0.152	0.161	0.157	0.117	**0.165**	0.148
NM_2		0.080	0.086	0.084	0.083	0.086	0.082	0.087	0.093	0.087	**0.099**
NM_3		0.054	0.056	0.053	0.054	0.058	0.054	0.058	0.060	0.056	**0.065**
NM_4		0.051	0.050	0.051	0.056	0.050	0.051	0.050	**0.056**	0.050	0.054
$NDPC_1$	10	0.594	**0.659**	0.604	0.157	0.567	0.653	0.602	0.273	0.648	0.430
$NDPC_2$		0.072	**0.075**	0.073	0.063	0.070	**0.076**	0.073	0.068	**0.075**	0.071
$NDPC_3$		0.058	0.058	0.055	0.050	0.056	**0.058**	0.057	0.053	0.057	0.055
$NDPC_4$		**0.048**	0.047	0.045	0.047	0.046	0.047	0.046	0.047	0.046	0.047
$NDPC_1$	20	0.940	**0.974**	0.940	0.080	0.940	0.969	0.956	0.225	0.969	0.750
$NDPC_2$		0.113	0.129	0.114	0.031	0.117	0.128	0.122	0.041	0.129	0.078
$NDPC_3$		0.073	**0.078**	0.065	0.029	0.073	0.074	0.075	0.033	0.076	0.052
$NDPC_4$		**0.051**	0.051	0.046	0.038	0.048	0.051	0.049	0.039	0.050	0.043
PCM_1	10	0.104	0.112	0.117	0.058	0.088	**0.131**	0.095	0.064	0.116	0.074
PCM_2		0.066	0.069	0.065	0.042	0.062	**0.070**	0.064	0.045	0.068	0.054
PCM_3		0.051	0.052	0.050	0.049	0.053	0.052	0.053	0.051	0.052	**0.054**
PCM_4		0.049	0.048	0.046	0.040	0.047	0.049	0.047	0.041	0.048	0.044
PCM_1	20	0.208	0.226	0.256	0.040	0.154	**0.294**	0.174	0.046	0.245	0.085
PCM_2		0.102	0.114	0.097	0.026	0.094	**0.116**	0.100	0.028	0.112	0.051
PCM_3		0.062	0.068	0.064	0.022	**0.075**	0.064	0.074	0.027	0.070	0.052
PCM_4		**0.053**	0.051	0.045	0.027	0.048	0.051	0.049	0.028	0.050	0.035

Source: authors' work.

Table 11. The power of GoFTs for group F of alternatives (Alts)

Alt	n	GoFT									
		1	2	3	4	8	12	14	19	**29**	13
P_1	10	0.580	0.699	0.777	0.238	**0.813**	0.615	0.344	0.762	0.758	0.794
P_2		0.151	0.191	0.227	0.134	**0.252**	0.160	0.150	0.243	0.237	0.239
P_3		0.066	0.069	0.073	0.097	0.074	0.067	**0.099**	0.089	0.086	0.073
P_4		0.052	0.053	0.053	0.076	0.053	0.053	**0.076**	0.062	0.061	0.053
P_1	20	0.901	0.961	0.990	0.166	**0.991**	0.929	0.372	0.982	0.982	0.988
P_2		0.299	0.403	0.533	0.108	**0.557**	0.334	0.150	0.501	0.493	0.530
P_3		0.095	0.105	**0.127**	0.109	0.123	0.101	0.119	0.137	0.132	0.119
P_4		0.060	0.061	0.064	0.083	0.060	0.063	**0.086**	0.073	0.070	0.061
NM_1	10	0.219	0.256	0.241	0.106	0.250	0.240	0.141	0.291	0.283	0.254
NM_2		0.064	0.067	0.070	0.057	**0.080**	0.061	0.058	0.079	0.077	0.077
NM_3		0.050	0.050	0.049	0.055	0.049	0.049	**0.055**	0.052	0.052	0.049
NM_4		0.049	0.049	0.049	**0.051**	0.049	0.048	0.051	0.050	0.049	0.050
NM_1	20	0.477	0.592	0.533	0.049	0.557	0.560	0.097	0.616	0.609	0.572
NM_2		0.091	0.102	0.111	0.053	**0.139**	0.082	0.054	0.120	0.120	0.131
NM_3		0.051	0.050	0.049	0.055	0.048	0.050	**0.056**	0.055	0.051	0.048
NM_4		0.050	0.049	0.050	0.051	0.049	0.050	**0.051**	0.050	0.049	0.049
$NDPC_1$	10	0.199	**0.217**	0.192	0.098	0.207	0.197	0.125	0.240	0.234	0.211
$NDPC_2$		0.064	0.064	0.061	0.057	**0.065**	0.061	0.061	0.075	0.073	0.065
$NDPC_3$		0.049	0.048	0.046	0.051	0.046	0.048	**0.052**	0.050	0.049	0.047
$NDPC_4$		0.048	0.047	0.046	0.047	**0.048**	0.048	0.048	0.049	0.049	0.047
$NDPC_1$	20	0.437	**0.499**	0.394	0.051	0.449	0.453	0.088	0.501	0.495	0.464
$NDPC_2$		0.093	**0.100**	0.082	0.036	0.093	0.092	0.042	0.109	0.106	0.095
$NDPC_3$		0.048	0.045	0.044	**0.050**	0.045	0.046	0.050	0.048	0.047	0.044
$NDPC_4$		**0.050**	0.048	0.046	0.044	0.048	0.047	0.044	0.050	0.049	0.048
PCM_1	10	0.100	0.118	0.130	0.057	0.110	**0.133**	0.068	0.144	0.141	0.113
PCM_2		0.055	0.055	0.052	0.050	**0.059**	0.052	0.050	0.057	0.056	0.058
PCM_3		**0.047**	0.045	0.043	0.038	0.044	0.044	0.038	0.045	0.045	0.044
PCM_4		0.047	0.047	0.046	0.038	0.046	0.047	0.038	0.047	0.047	0.046
PCM_1	20	0.196	0.271	0.341	0.013	0.244	**0.343**	0.030	0.340	0.336	0.255
PCM_2		0.066	0.069	0.066	0.054	**0.087**	0.058	0.055	0.070	0.069	0.083
PCM_3		**0.046**	0.044	0.040	0.029	0.043	0.044	0.029	0.043	0.043	0.043
PCM_4		**0.052**	0.051	0.046	0.031	0.049	0.050	0.031	0.049	0.049	0.049

Source: authors' work.

Table 12. The power of GoFTs for group G of alternatives (Alts)

Alt	n	GoFT									
		3	5	12	15	20	**28**	**21**	**26**	**23**	**25**
P_1	10	0.799	0.302	**0.818**	0.410	0.795	0.790	0.768	0.767	0.766	0.766
P_2		0.249	0.177	**0.256**	0.195	0.269	0.261	0.228	0.228	0.227	0.227
P_3		0.080	0.110	0.077	**0.112**	0.097	0.093	0.077	0.077	0.077	0.077
P_4		0.057	0.082	0.055	**0.083**	0.069	0.067	0.057	0.057	0.057	0.057
P_1	20	**0.992**	0.268	0.992	0.485	0.987	0.987	0.985	0.984	0.984	0.984
P_2		**0.585**	0.191	0.568	0.245	0.555	0.547	0.507	0.507	0.505	0.505
P_3		0.144	0.136	0.129	**0.150**	0.153	0.148	0.127	0.127	0.127	0.127
P_4		0.074	0.102	0.067	**0.106**	0.084	0.082	0.070	0.070	0.070	0.070
NM_1	10	0.065	0.093	0.063	**0.095**	0.081	0.076	0.064	0.064	0.064	0.064
NM_2		0.051	**0.066**	0.051	**0.065**	0.056	0.056	0.052	0.052	0.051	0.051
NM_3		0.051	0.061	0.050	**0.061**	0.054	0.054	0.051	0.051	0.051	0.051
NM_4		0.050	**0.056**	0.049	**0.055**	0.052	0.052	0.050	0.050	0.050	0.050
NM_1	20	0.094	0.104	0.088	**0.112**	0.112	0.107	0.092	0.092	0.092	0.092
NM_2		0.056	0.070	0.055	**0.072**	0.061	0.062	0.056	0.056	0.056	0.056
NM_3		0.053	0.064	0.052	**0.065**	0.057	0.056	0.052	0.052	0.052	0.052
NM_4		0.051	0.057	0.051	**0.057**	0.052	0.052	0.050	0.050	0.050	0.050
$NDPC_1$	10	0.125	0.111	**0.134**	0.117	0.148	0.144	0.124	0.124	0.124	0.124
$NDPC_2$		0.092	0.068	0.093	0.069	0.101	0.100	0.096	0.096	0.094	0.094
$NDPC_3$		0.067	0.091	0.067	**0.094**	0.083	0.080	0.069	0.069	0.069	0.069
$NDPC_4$		0.051	0.049	0.051	0.049	0.052	0.052	0.051	0.051	0.051	0.051
$NDPC_1$	20	0.247	0.122	**0.267**	0.138	0.269	0.263	0.240	0.240	0.239	0.239
$NDPC_2$		0.166	0.113	**0.175**	0.112	0.179	0.178	0.173	0.172	0.171	0.171
$NDPC_3$		0.088	0.088	0.089	0.099	0.112	0.109	0.094	0.094	0.094	0.094
$NDPC_4$		**0.057**	0.056	0.054	0.055	0.054	0.055	0.055	0.055	0.055	0.055
PCM_1	10	0.194	0.150	0.204	0.166	0.233	0.227	0.205	0.205	0.205	0.205
PCM_2		0.127	0.096	0.132	0.098	0.142	0.140	0.133	0.132	0.130	0.130
PCM_3		0.061	0.091	0.059	**0.092**	0.075	0.072	0.060	0.060	0.060	0.060
PCM_4		0.061	0.087	0.058	**0.089**	0.071	0.069	0.061	0.061	0.061	0.061
PCM_1	20	0.340	0.135	0.393	0.189	0.447	0.441	0.410	0.409	0.409	0.409
PCM_2		0.261	0.162	**0.273**	0.171	0.281	0.277	0.266	0.265	0.263	0.263
PCM_3		0.086	0.110	0.077	**0.118**	0.099	0.096	0.078	0.079	0.078	0.078
PCM_4		0.077	0.096	0.071	**0.105**	0.090	0.087	0.075	0.075	0.075	0.075

Source: authors' work.

Table 13. The power of GoFTs for group H of alternatives (Alts)

Alt	n	GoFT									
		4	8	14	17	19	2	**29**	3	13	21
P_1	10	0.303	**0.838**	0.412	0.513	0.797	0.732	0.792	0.801	0.819	0.769
P_2		0.178	**0.267**	0.194	0.185	0.270	0.209	0.262	0.250	0.255	0.228
P_3		0.111	0.079	**0.112**	0.096	0.096	0.075	0.091	0.081	0.079	0.078
P_4		0.082	0.055	**0.083**	0.072	0.068	0.056	0.066	0.058	0.055	0.057
P_1	20	0.267	**0.994**	0.486	0.801	0.987	0.972	0.987	0.993	0.992	0.986
P_2		0.192	**0.589**	0.246	0.316	0.557	0.445	0.547	0.584	0.565	0.507
P_3		0.137	0.130	**0.148**	0.144	0.154	0.116	0.149	0.143	0.127	0.126
P_4		0.102	0.066	**0.106**	0.098	0.083	0.068	0.080	0.075	0.066	0.070
NM_1	10	0.107	0.081	**0.111**	0.097	0.099	0.080	0.097	0.081	0.081	0.081
NM_2		0.096	0.065	**0.098**	0.085	0.079	0.064	0.077	0.066	0.064	0.066
NM_3		0.059	0.050	**0.059**	0.056	0.054	0.051	0.054	0.050	0.050	0.051
NM_4		0.055	0.051	**0.056**	0.053	0.052	0.051	0.052	0.050	0.050	0.050
NM_1	20	0.119	0.125	0.132	**0.139**	0.156	0.124	0.148	0.131	0.125	0.128
NM_2		0.102	0.084	0.112	**0.113**	0.110	0.089	0.106	0.092	0.085	0.091
NM_3		0.063	0.051	**0.065**	0.062	0.056	0.052	0.056	0.053	0.051	0.052
NM_4		0.057	0.049	**0.057**	0.056	0.053	0.050	0.052	0.051	0.049	0.051
$NDPC_1$	10	0.139	0.111	**0.148**	0.136	0.142	0.117	0.137	0.114	0.113	0.117
$NDPC_2$		0.102	0.067	**0.102**	0.087	0.084	0.064	0.081	0.068	0.066	0.067
$NDPC_3$		**0.057**	0.050	0.057	0.053	0.053	0.049	0.052	0.050	0.050	0.050
$NDPC_4$		0.051	0.051	0.051	**0.052**	0.052	0.051	0.051	0.050	0.051	0.051
$NDPC_1$	20	0.137	0.193	0.170	0.214	**0.253**	0.216	0.246	0.207	0.200	0.218
$NDPC_2$		0.122	0.098	**0.133**	0.129	0.120	0.088	0.115	0.111	0.096	0.098
$NDPC_3$		**0.067**	0.051	0.067	0.062	0.055	0.052	0.054	0.054	0.051	0.053
$NDPC_4$		**0.051**	0.050	0.051	0.051	0.052	0.050	0.052	0.050	0.050	0.050
PCM_1	10	0.082	0.056	**0.082**	0.071	0.068	0.056	0.066	0.057	0.056	0.057
PCM_2		**0.065**	0.052	0.065	0.060	0.057	0.052	0.056	0.053	0.052	0.053
PCM_3		0.065	0.059	**0.066**	0.061	0.063	0.058	0.062	0.056	0.058	0.057
PCM_4		0.067	0.054	**0.068**	0.062	0.061	0.054	0.060	0.054	0.054	0.054
PCM_1	20	0.092	0.065	**0.097**	0.091	0.083	0.067	0.080	0.070	0.066	0.069
PCM_2		0.071	0.053	**0.072**	0.068	0.060	0.054	0.059	0.056	0.053	0.055
PCM_3		0.064	0.066	0.067	0.067	**0.074**	**0.068**	0.073	0.063	0.067	0.066
PCM_4		0.062	0.057	0.066	**0.066**	0.066	0.060	0.064	0.055	0.058	0.059

Source: authors' work.

Tables 6–13 present the power of the top ten tests for groups A-H of the alternatives. The highest values in each row are in bold. The GoFTs that stand out for the groups of alternatives based on the sum of powers are as follows: $AD_{1,0}, AD_{0.9,0.1}, ADM^U$ (group A), $AD_{0,1}, AD_{0.1,0.9}, ADM^L$ (group B), $CVM, AD_{1,1}, AD_{0.9,0.9}$ (group C), $AD^U, ADR^U, AD_{0,0}$ (group D), $AD_{1,0}, AD_{0.9,0.1}, ADR^L$ (group E), $AD_{0,1}, AD_{0.1,0.9}, AD^U$ (group F), $AD_{1,0}, AD_{0.9,0.1}, ADR^L$ (group G) and $AD_{0,1}, AD_{0.1,0.9}$, SW (group H).

The sum of the powers for the groups of alternatives marked with indices 1 and 2 is the highest for $AD_{1,0}, AD_{0.9,0.1}, SW$ (group A), $AD_{0,1}, AD_{0.1,0.9}, SW$ (group B),

$CVM, AD_{1,1}, AD_{0.9,0.9}$ (group C), AD^U, ADR^U, SW (group D), $AD_{1,0}, AD_{0.9,0.1}, ADR^L$ (groups E, G), $AD_{0,1}, AD_{0.1,0.9}, AD^U$ (groups F, H) tests.

The sum of the powers for the groups of alternatives marked with indices 3 and 4 is the highest for ADL^U, $|AD^U|, ADM^U$ (groups A, G), ADL^L, $|AD^L|, ADM^L$ (groups B, H), $AD^L, ADL, |AD|$ (group C), $AD_{0,0}, LF, AD_{0.1,0.1}$ (group D), $AD_{1,0}, AD_{0.9,0.1}, AD_{0,0}$ (group E), $AD_{0,1}, AD_{0.1,0.9}, ADL^L$ (group F) tests.

The modified AD tests with test statistics (33), for the given alternative and similarity measure, achieves the highest empirical power in 28%, 22%, 0%, 34%, 53%, 31%, 41%, 19% of the considered cases related to groups A–H, respectively.

The observed dominance of particular modified AD tests within specific skewness-kurtosis regimes can be explained by the manner in which Bloom's EDF parameters (α, β) control the relative weighting of discrepancies in the lower and upper tails of the distribution. Tests with $\alpha > \beta$ place greater emphasis on the upper tail, which increases their sensitivity to right-skewed alternatives or distributions with heavy upper tails. This explains the strong performance of tests such as $AD_{1,0}$ and $AD_{0.9,0.1}$ for groups characterised by positive skewness (groups A and G).

Conversely, tests where $\beta > \alpha$ emphasise deviations in the lower tail and therefore exhibit a higher power for left-skewed alternatives, as observed for $AD_{0,1}$ and $AD_{0.1,0.9}$ in groups B, F and H. For symmetric alternatives, the dominance of tests with $\alpha \approx \beta$ reflects the fact that departures from normality occur simultaneously in both tails. In such cases, balanced tail-weighting strategies are more effective, which explains the strong performance of such tests as $AD_{1,1}$, $AD_{0.9,0.9}$, and the classical CVM test.

7. Real data examples

In this section, we present the application of the analysed tests in real datasets to illustrate their potentiality. For the real data examples, the distribution of each test statistic under the null hypothesis of normality was obtained through a Monte Carlo simulation using the same critical values as those computed in the power study for the corresponding sample size. The details related to examples I–VII are presented in Table 14.

Table 14. Real data examples with sources, sample size, skewness and excess kurtosis values

Ex.	Description	Source	n	γ_1	$\bar{\gamma}_2$
I	Socio-economic data (percentage of males involved in agriculture as occupation) for 47 French-speaking provinces of Switzerland.	R package swiss[2]	47	−0.331	−0.793
II	Measurements of the diameter of timber in 31 felled black cherry trees. The diameter is measured at 4 ft 6 in above the ground.	trees[1]	31	0.526	−0.556
III	The effect of vitamin C on tooth length in guinea pigs. Each animal received one of the three different doses of vitamin C (0.5, 1, and 2 mg/day) by one of the two alternative delivery methods, i.e. orange juice or ascorbic acid.	ToothGrowth[1]	60	−0.146	−0.976
IV	Data extracted from the Motor Trend US magazine showing fuel consumption for 32 automobiles.	mtcars[1]	32	0.640	−0.200
V	Lawyers' ratings of state judges in the US Superior Court (sound written rulings).	USJudgeRatings[1]	43	−0.699	0.030
VI	Arrests per 100,000 residents for rape in each of the 50 US states in 1973.	USArrests[4]	50	0.777	0.202
VII	Average height for American women aged 30–39.	women[1]	15	0	−1.211

Source: authors' work.

When fitting the normal distribution to the data, we calculate the p-values for the analysed GoFTs based on 10^5 statistic values (see Table 15). The lowest p-values for all the analysed tests are in bold. The lowest p-values for the MAD tests are underlined. Non-normality is the most pronounced by the $AD_{0,1}$ (examples I, III, V) and $AD_{1,0}$ tests (examples II, IV). The lowest p-values for the MAD tests are observed for the $AD_{1,0}$ and $AD_{0,0}$tests (examples VI, VII). The obtained results are consistent with the simulation results, according to which, if the real data are negatively skewed, the $AD_{0,1}$ test is powerful, and if the real data are positively skewed, the $AD_{1,0}$ is powerful, and if the real data are symmetric, the $AD_{0,0}$ test is powerful.

Table 15. The p-values for the GoFTs related to examples I–VII

No.	GoFT	I	II	III	IV	V	VI	VII
1	LF	0.233	0.110	0.167	0.208	0.324	0.331	0.996
2	CVM	0.210	0.043	0.092	0.155	0.164	0.119	0.946
3	SW	0.190	0.089	0.107	0.123	0.096	**0.025**	0.727
4	$\|AD^L\|$	0.489	0.499	0.495	0.497	0.165	0.495	0.513
5	$\|AD^U\|$	0.490	0.240	0.489	0.279	0.490	0.122	**0.512**
6	$\|AD\|$	0.328	0.456	0.544	0.246	0.365	0.150	0.987
7	AD^L	0.424	0.279	0.372	0.228	0.181	0.102	0.863
8	AD^U	0.215	0.072	0.083	0.104	0.146	0.093	0.904
9	AD	0.196	0.046	0.087	0.123	0.122	0.074	0.925
10	$\overline{AD}$	0.196	0.046	0.087	0.123	0.122	0.074	0.925
11	$\overline{\overline{AD}}$	0.196	0.046	0.087	0.123	0.122	0.074	0.925
12	ADR^L	0.194	0.040	0.103	0.164	0.125	0.079	0.919
13	ADR^U	0.213	0.062	0.084	0.110	0.147	0.094	0.918
14	ADL^L	0.437	0.208	0.293	0.291	0.111	0.150	0.781
15	ADL^U	0.302	0.271	0.337	0.138	0.229	0.062	0.779
16	ADL	0.424	0.279	0.372	0.228	0.181	0.102	0.863
17	ADM^L	0.268	0.100	0.183	0.225	<u>**0.095**</u>	0.086	0.851
18	ADM^U	0.259	0.152	0.179	0.103	0.171	0.045	0.848
19	$AD_{0,1}$	<u>0.173</u>	0.058	<u>0.082</u>	0.160	<u>0.095</u>	0.101	0.920
20	$AD_{1,0}$	0.225	<u>0.038</u>	0.093	<u>0.097</u>	0.161	<u>0.055</u>	0.920
21	$AD_{0,0}$	0.192	0.045	0.084	0.123	0.123	0.074	<u>0.916</u>
22	$AD_{1,1}$	0.201	0.048	0.090	0.123	0.122	0.073	0.933
23	$AD_{0.3,0.3}$	0.195	0.046	0.086	0.123	0.122	0.074	0.922
24	$AD_{3/8,3/8}$	0.195	0.046	0.086	0.123	0.122	0.074	0.923
25	$AD_{\frac{127}{400},\frac{127}{400}}$	0.195	0.046	0.086	0.123	0.122	0.074	0.922
26	$AD_{0.1,0.1}$	0.193	0.046	0.084	0.123	0.123	0.074	0.918
27	$AD_{0.9,0.9}$	0.200	0.047	0.089	0.123	0.122	0.073	0.932
28	$AD_{0.9,0.1}$	0.219	0.039	0.091	0.102	0.152	0.058	0.921
29	$AD_{0.1,0.9}$	0.177	0.055	0.083	0.151	0.100	0.094	0.922

Source: authors' work.

8. Conclusions

The aims of the research presented in this article were fully achieved: the family of MAD tests was expanded, four new formulas for the EDF were proposed, a flexible family of alternatives was created, consisting of older and newer distributions, and the powers of 29 tests were compared focusing on the similarity measure of the alternative to the normal distribution.

Similarly to the modified L tests (Sulewski, 2021, 2022a), the parametrisation of the EDF based on Bloom's formula also influenced the power of the MAD test.

Tests $|AD^L|$ (groups A, D, E, F, G), $|AD^U|$ (B, D, E, F, H), $|AD|$ (D, E, F, G, H), AD^L (D, E, F, H), ADL^L (A, D, E, F, G), ADL^U (B, D, E, F, H), ADM^L (A, D, E, G), ADM^U (D F, H) and ADM (D) do not meet the basic assumptions, i.e. the power increases as the sample size increases and decreases as the similarity measure (36) increases.

The power of the $\overline{AD}$ and $\overline{\overline{AD}}$ tests is the same for all the analysed cases.

The tests best detect samples from asymmetric distributions with positive excess kurtosis.

The power of the $AD_{\alpha,\beta}$ $(\alpha = \beta;\ \alpha, \beta \leq 1)$ tests is very similar for all groups of alternatives. It is noteworthy that the power of the $AD_{\alpha,\beta}$ $(\alpha \neq \beta)$ tests for the given parameter values dominate in all of the analysed alternatives and similarity measures. The $AD_{1,0}$ test is recommended for positively skewed alternatives and the $AD_{0,1}$ test for negatively skewed alternatives. The $AD_{1,1}$ and $AD_{0,0}$ tests are recommended for symmetric alternatives.

The $AD_{\alpha,\beta}$ test for the given alternative and similarity measure achieves high power in 53% of the cases in group E, 41% of the cases in group G, 34% of the cases in group D, 31% of the cases in group F and in 28% of the cases in group A. A result of less than 25% applies to groups B, C and H.

The analysis of real datasets has led to the conclusion that the MAD test is indeed effective.

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Appendix

Edgeworth series distribution

The PDF of the Edgeworth series (ES) with parameters γ_1 and $\bar{\gamma}_2$ is given by

$$f_{ES}(x;\gamma_{1,},\bar{\gamma}_2)=\phi(x;0{,}1)\left(1+\frac{1}{3!}\gamma_{1,}(x^3-3x)+\frac{1}{4!}\bar{\gamma}_2(x^4-6x^2+3)\right)\ (x\in R),$$

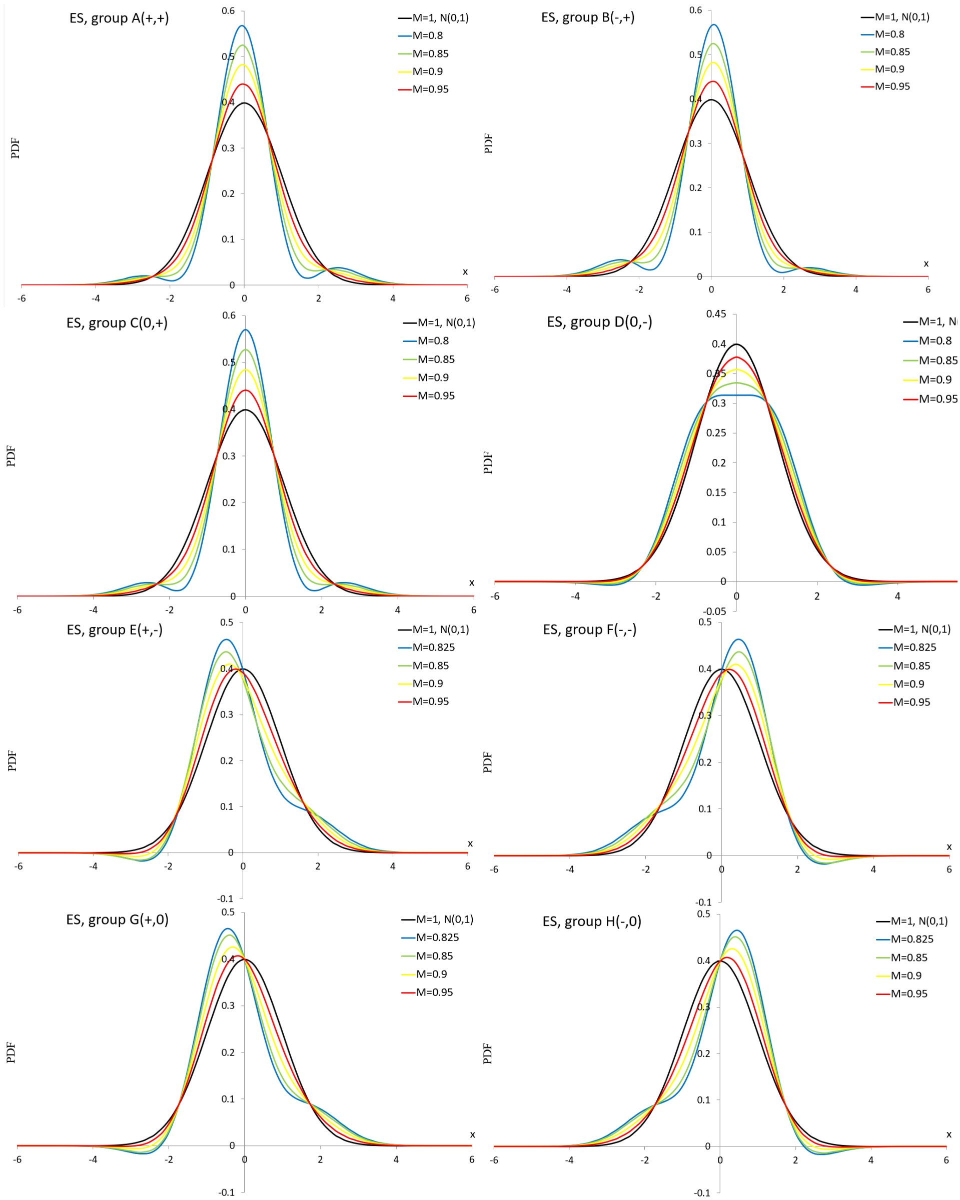
where $\gamma_{1,}\in R,\bar{\gamma}_2\geq -2$.

Table 1A. Vectors of the ES parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M. Groups O, A–H

Group	$\boldsymbol{\theta}=(\gamma_{1,},\bar{\gamma}_2)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta};\mu,\sigma)$
O	(0,0)	0	1	0	0	$M(\boldsymbol{\theta};0{,}1)=1$
A	0.4,3.33	0	1	0.4	3.33	$M(\boldsymbol{\theta};0{,}1)=0.8$
	0.3,2.499	0	1	0.3	2.499	$M(\boldsymbol{\theta};0{,}1)=0.85$
	0.2,1.666	0	1	0.2	1.666	$M(\boldsymbol{\theta};0{,}1)=0.9$
	0.1,0.833	0	1	0.1	0.833	$M(\boldsymbol{\theta};0{,}1)=0.95$
B	–0.4,3.33	0	1	–0.4	3.33	$M(\boldsymbol{\theta};0{,}1)=0.8$
	–0.3,2.499	0	1	–0.3	2.499	$M(\boldsymbol{\theta};0{,}1)=0.85$
	–0.2,1.666	0	1	–0.2	1.666	$M(\boldsymbol{\theta};0{,}1)=0.9$
	–0.1,0.833	0	1	–0.1	0.833	$M(\boldsymbol{\theta};0{,}1)=0.95$
C	0,3.428	0	1	0	3.428	$M(\boldsymbol{\theta};0{,}1)=0.8$
	0,2.571	0	1	0	2.571	$M(\boldsymbol{\theta};0{,}1)=0.85$
	0,1.71	0	1	0	1.71	$M(\boldsymbol{\theta};0{,}1)=0.9$
	0,0.85	0	1	0	0.85	$M(\boldsymbol{\theta};0{,}1)=0.95$
D	0,–3.428	0	1	0	–3.428	$M(\boldsymbol{\theta};0{,}1)=0.8$
	0,–2.571	0	1	0	–2.571	$M(\boldsymbol{\theta};0{,}1)=0.85$
	0,–1.71	0	1	0	–1.71	$M(\boldsymbol{\theta};0{,}1)=0.9$
	0,–0.85	0	1	0	–0.85	$M(\boldsymbol{\theta};0{,}1)=0.95$
E	1.39,–0.067	0	1	1.39	–0.067	$M(\boldsymbol{\theta};0{,}1)=0.825$
	1.175,–0.46	0	1	1.175	–0.46	$M(\boldsymbol{\theta};0{,}1)=0.85$
	0.775,–0.408	0	1	0.775	–0.408	$M(\boldsymbol{\theta};0{,}1)=0.9$
	0.39,–0.15	0	1	0.39	–0.15	$M(\boldsymbol{\theta};0{,}1)=0.95$
F	–1.39,–0.067	0	1	–1.39	–0.067	$M(\boldsymbol{\theta};0{,}1)=0.825$
	–1.175,–0.46	0	1	–1.175	–0.46	$M(\boldsymbol{\theta};0{,}1)=0.85$
	–0.775,–0.408	0	1	–0.775	–0.408	$M(\boldsymbol{\theta};0{,}1)=0.9$
	–0.39,–0.15	0	1	–0.39	–0.15	$M(\boldsymbol{\theta};0{,}1)=0.95$
G	1.391,0	0	1	1.391	0	$M(\boldsymbol{\theta};0{,}1)=0.825$
	1.19,0	0	1	1.19	0	$M(\boldsymbol{\theta};0{,}1)=0.85$
	0.795,0	0	1	0.795	0	$M(\boldsymbol{\theta};0{,}1)=0.9$
	0.4,0	0	1	0.4	0	$M(\boldsymbol{\theta};0{,}1)=0.95$
H	–1.391,0	0	1	–1.391	0	$M(\boldsymbol{\theta};0{,}1)=0.825$
	–1.19,0	0	1	–1.19	0	$M(\boldsymbol{\theta};0{,}1)=0.85$
	–0.795,0	0	1	–0.795	0	$M(\boldsymbol{\theta};0{,}1)=0.9$
	–0.4,0	0	1	–0.4	0	$M(\boldsymbol{\theta};0{,}1)=0.95$

Source: authors' work.

Figure 1A. PDF curves of the ES distribution for parameter values presented in Table 1A



Source: authors' work.

Pearson distribution

Let $a = \frac{2\bar{\gamma}_2 - 3\gamma_1^2}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}, b = \frac{|\gamma_1|(\bar{\gamma}_2 + 6)}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}, c = \frac{4\bar{\gamma}_2 - 3\gamma_1^2 + 12}{10\bar{\gamma}_2 - 5\gamma_1^2 + 12}, \Delta = b^2 - 4ac$, then the PDF of the Pearson (P) distribution is given by

$$f_P(x;\gamma_{1,},\bar{\gamma}_2) = \begin{cases} \dfrac{exp\left[\dfrac{2ab-b}{a(2ax+b)}\right]}{C_1(2ax+b)^{1/a}} & \Delta = 0 \\ \dfrac{exp\left[\dfrac{b-2ab}{a\sqrt{4ac-b^2}} tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)\right]}{C_2(ax^2+bx+c)^{1/(2a)}} & \Delta < 0 \\ \dfrac{\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right)^{\frac{b-2ab}{2a\sqrt{b^2-4ac}}}}{C_3(ax^2+bx+c)^{1/(2a)}} & \Delta > 0 \end{cases}$$

where $x \in R, \gamma_{1,} \in R, \bar{\gamma}_2 \geq -2$ and C_1, C_2, C_3 are normalising constants defined as

$$C_1 = \int_{-\infty}^{\infty} \frac{exp\left[\dfrac{2ab-b}{a(2ax+b)}\right]}{(2ax+b)^{\frac{1}{a}}} dx,$$

$$C_2 = \int_{-\infty}^{\infty} \frac{exp\left[\dfrac{b-2ab}{a\sqrt{4ac-b^2}} tan^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right)\right]}{(ax^2+bx+c)^{\frac{1}{2a}}} dx,$$

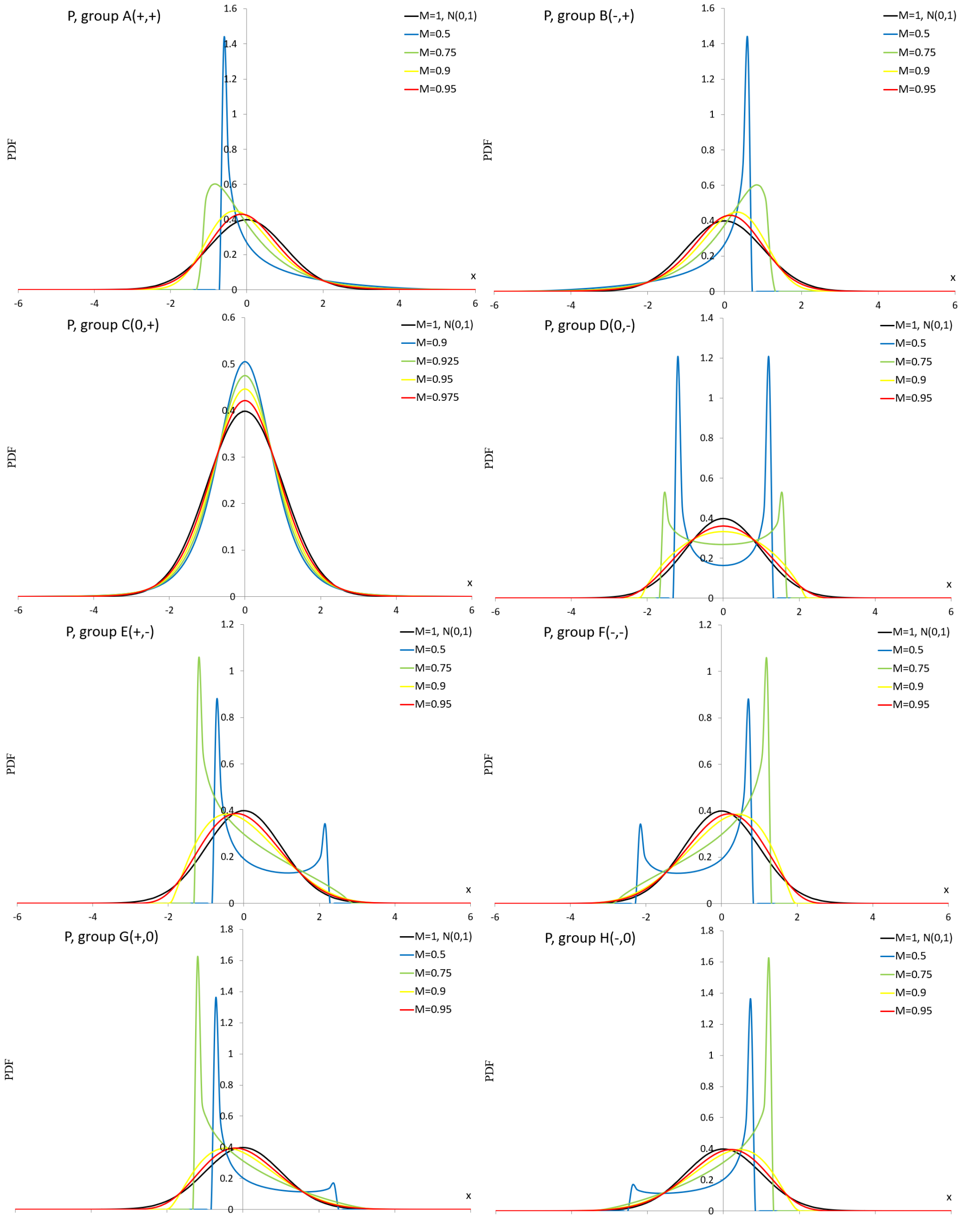
$$C_3 = \int_{-\infty}^{\infty} \frac{\left(\dfrac{2ax+b-\sqrt{\Delta}}{2ax+b+\sqrt{\Delta}}\right)^{\frac{b-2ab}{2a\sqrt{\Delta}}}}{C_8(ax^2+bx+c)^{1/(2a)}} dx.$$

Table 2A. Vectors of the P parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M. Groups O, A-H

Group	$\boldsymbol{\theta} = (\gamma_1, \bar{\gamma}_2)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta}; \mu, \sigma)$
O	(0,0)	0	1	0	0	$M(\boldsymbol{\theta}; 0,1) = 1$
A	(2.04,4.1)	0	1	2.04	4.1	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(1.62,3.845)	0	1	1.62	3.845	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(0.9,2)	0	1	0.9	2	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(0.4,0.94)	0	1	0.4	0.94	$M(\boldsymbol{\theta}; 0,1) = 0.95$
B	(–2.04,4.1)	0	1	–2.04	4.1	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(–1.62,3.845)	0	1	–1.62	3.845	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(–0.9,2)	0	1	–0.9	2	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(–0.4,0.94)	0	1	–0.4	0.94	$M(\boldsymbol{\theta}; 0,1) = 0.95$
C	(0,11.2)	0	1	0	11.2	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(0,3.65)	0	1	0	3.65	$M(\boldsymbol{\theta}; 0,1) = 0.925$
	(0,1.521)	0	1	0	1.521	$M(\boldsymbol{\theta}; 0,1) = 0.95$
	(0,0.55)	0	1	0	0.55	$M(\boldsymbol{\theta}; 0,1) = 0.975$
D	(0,–1.695)	0	1	0	–1.695	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(0,–1.315)	0	1	0	–1.315	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(0,–0.89)	0	1	0	–0.89	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(0,–0.588)	0	1	0	–0.588	$M(\boldsymbol{\theta}; 0,1) = 0.95$
E	(0.985,–0.5)	0	1	0.985	–0.5	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(0.715,–0.475)	0	1	0.715	–0.475	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(0.515,–0.2)	0	1	0.515	–0.2	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(0.315,–0.16)	0	1	0.315	–0.16	$M(\boldsymbol{\theta}; 0,1) = 0.95$
F	(–0.985,–0.5)	0	1	–0.985	–0.5	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(–0.715,–0.475)	0	1	–0.715	–0.475	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(–0.515,–0.2)	0	1	–0.515	–0.2	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(–0.315,–0.16)	0	1	–0.315	–0.16	$M(\boldsymbol{\theta}; 0,1) = 0.95$
G	(1.164,0)	0	1	1.164	0	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(0.879,0)	0	1	0.879	0	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(0.578,0)	0	1	0.578	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(0.354,0)	0	1	0.354	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$
H	(–1.164,0)	0	1	–1.164	0	$M(\boldsymbol{\theta}; 0,1) = 0.5$
	(–0.879,0)	0	1	–0.879	0	$M(\boldsymbol{\theta}; 0,1) = 0.75$
	(–0.578,0)	0	1	–0.578	0	$M(\boldsymbol{\theta}; 0,1) = 0.9$
	(–0.354,0)	0	1	–0.354	0	$M(\boldsymbol{\theta}; 0,1) = 0.95$

Source: authors' work.

Figure 2A. PDF curves of the P distribution for the parameter values presented in Table 2A



Source: authors' work.

Normal mixture distribution

The PDF of the normal mixture (NM) distribution is given by

$$f_{NM}(x;\boldsymbol{\theta}) = \omega\phi(x;\mu_1,\sigma_1) + (1-\omega)\phi(x;\mu_2,\sigma_2)\ (x \in R),$$

where $\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,\omega)$ and $\mu_1,\mu_2 \in R, \sigma_1,\sigma_2 > 0, \omega \in [0,1]$.
Special cases of the NM distribution are:

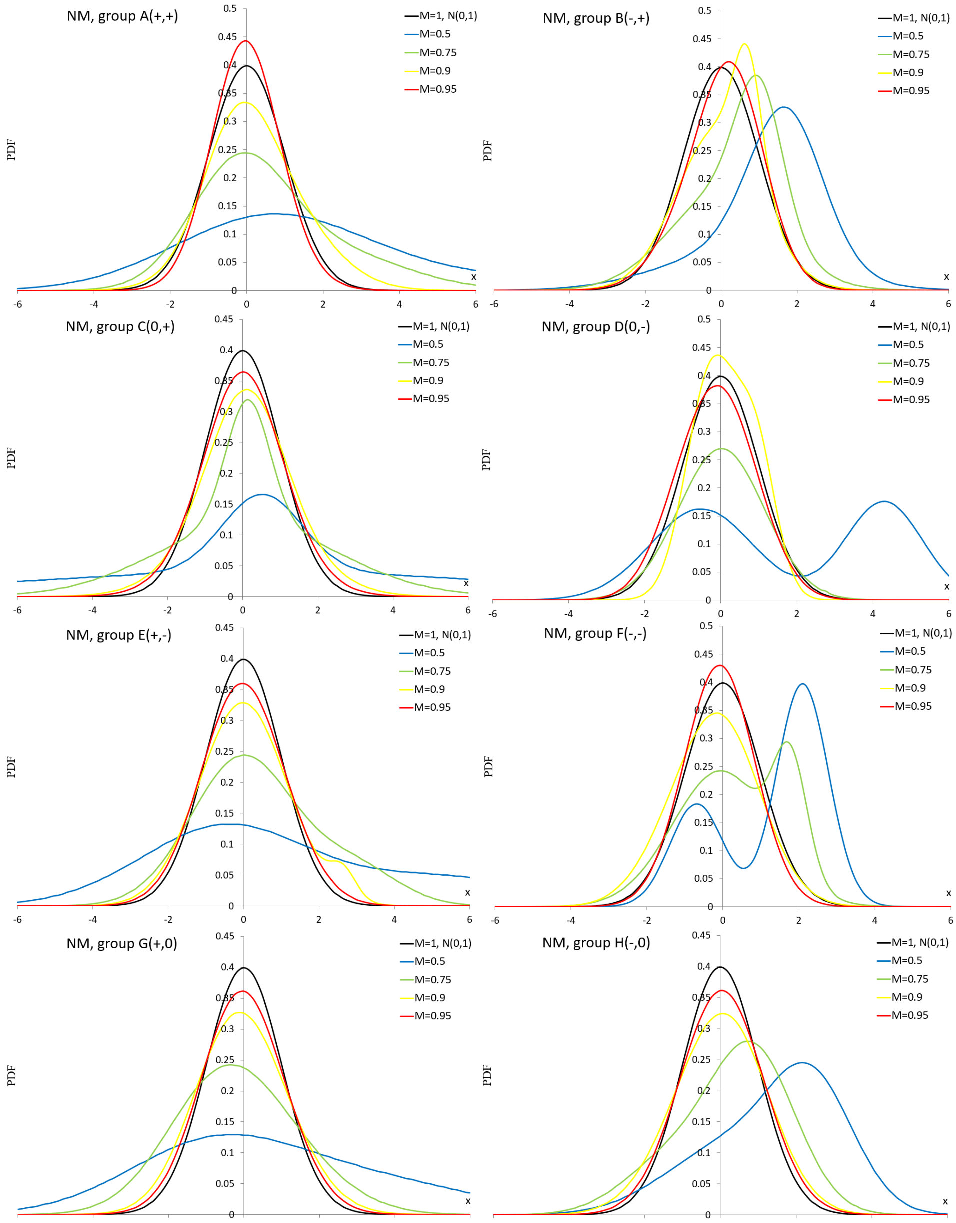
- normal $N(\mu_1,\sigma_1)$ for $\omega = 1$, $N(\mu_2,\sigma_2)$ for $\omega = 0$;
- location contaminated normal (LCN) $f_{LCM}(x;\mu_1,\omega) = f_{NM}(x;\mu_1,1,0,1,\omega)$;
- scale contaminated normal (SCN) $f_{SCN}(x;\ \sigma_1,\omega) = f_{NM}(x;0,\sigma_1,0,1,\omega)$.

Table 3A. Vectors of the NM parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,\omega)$	μ_a	σ_a	γ_1	$\bar{\gamma}_2$	$M(\boldsymbol{\theta};\mu,\sigma)$
O	$(\mu_1,\sigma_1,\mu_2,\sigma_2,1)$	0	1	0	0	$M(\boldsymbol{\theta};\mu_1,\sigma_1) = 1$
	$(\mu_1,\sigma_1,\mu_2,\sigma_2,0)$	0	1	0	0	$M(\boldsymbol{\theta};\mu_2,\sigma_2) = 1$
A	0.572,2.472,5.614,3.454,0.787	1.646	3.408	0.685	0.755	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.215,1.254,1.979,1.99,0.639	0.577	1.883	0.645	0.502	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.497,1.376,−0.268,0.884,0.612	0.2	1.265	0.287	0.249	$M(\boldsymbol{\theta};0,1) = 0.9$
	−0.098,0.857,0.31,1.007,0.767	−0.003	0.911	0.09	0.099	$M(\boldsymbol{\theta};0,1) = 0.95$
B	0.502,2.019,1.708,0.953,0.36	1.274	1.544	−0.748	1.502	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.06,1.437,1.004,0.609,0.634	0.406	1.285	−0.5	0.499	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.709,0.368,−0.072,1.115,0.193	0.079	1.06	−0.301	0.15	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.32,0.855,−0.873,0.923,0.782	0.06	1	−0.238	0.1	$M(\boldsymbol{\theta};0,1) = 0.95$
C	0.519,6.599,0.519,1.058,0.665	0.519	5.416	0	1.398	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.137,0.581,0.137,2.391,0.294	0.137	2.034	0	1.054	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.1,0.988,0.1,1.543,0.532	0.1	1.278	0	0.554	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.007,0.942,0.007,1.299,0.494	0.007	1.137	0	0.289	$M(\boldsymbol{\theta};0,1) = 0.95$
D	−0.511,1.353,4.293,1.021,0.551	1.645	2.681	0	−1.28	$M(\boldsymbol{\theta};0,1) = 0.5$
	2.707,0.013,0.017,1.125,0.238	0.657	1.509	0	−1.001	$M(\boldsymbol{\theta};0,1) = 0.75$
	1.243,0.621,−0.39,0.811,0.347	0.111	1.09	0	−0.63	$M(\boldsymbol{\theta};0,1) = 0.9$
	−1.112,0.794,0.023,0.974,0.13	0	0.897	0	−0.329	$M(\boldsymbol{\theta};0,1) = 0.95$
E	−0.475,2.22,5.318,2.427,0.721	1.141	3.457	0.5	−0.204	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.019,1.369,2.979,1.15,0.829	0.494	1.748	0.339	−0.1	$M(\boldsymbol{\theta};0,1) = 0.75$
	2.635,0.35,−0.015,1.166,0.038	0.086	1.253	0.137	−0.075	$M(\boldsymbol{\theta};0,1) = 0.9$
	1.091,0.969,−0.111,1.056,0.1	0.009	1.108	0.05	−0.01	$M(\boldsymbol{\theta};0,1) = 0.95$
F	−0.692,0.705,2.1,0.679,0.324	1.195	1.476	−0.542	−0.852	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.055,1.277,1.781,0.443,0.775	0.358	1.377	−0.3	−0.5	$M(\boldsymbol{\theta};0,1) = 0.75$
	−0.09,1.08,−1.581,0.92,0.9	−0.239	1.155	−0.071	−0.042	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.386,0.845,−0.145,0.918,0.1	−0.092	0.925	−0.01	−0.011	$M(\boldsymbol{\theta};0,1) = 0.95$
G	2.686,3.099,−0.964,2.217,0.471	0.755	3.232	0.4	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.56,1.465,1.411,1.45,0.8	−0.166	1.661	0.151	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	−0.286,1.114,0.984,1.105,0.801	−0.033	1.222	0.101	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	−0.1,1.063,1.261,0.94,0.936	−0.013	1.106	0.053	0	$M(\boldsymbol{\theta};0,1) = 0.95$
H	2.425,1.101,0.272,1.693,0.526	1.404	1.775	−0.499	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.864,1.125,−1.339,1.241,0.735	0.28	1.511	−0.386	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.429,1.078,−0.364,1.228,0.434	−0.02	1.23	−0.1	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.108,1.088,−0.524,1.073,0.879	0.032	1.106	−0.01	0	$M(\boldsymbol{\theta};0,1) = 0.95$

Source: authors' work.

Figure 3A. PDF curves of the NM distribution for parameter values presented in Table 3A



Source: authors' work.

Normal distribution with plasticising component

The PDF of the normal distribution with plasticising component (NDPC) is given by

$$f_{NDPC}(x;\boldsymbol{\theta}) = \omega\phi(x;\mu_1,\sigma_1) + (1-\omega)\frac{c_2}{\sigma_2}\left|\frac{x-\mu_2}{\sigma_2}\right|^{c_2-1}\phi\left(\left|\frac{x-\mu_2}{\sigma_2}\right|^{c_2};0,1\right)\ (x \in R),$$

where $\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,c_2,\omega)$ and $\mu_1,\mu_2 \in R, \sigma_1,\sigma_2 > 0, c_2 \geq 1, \omega \in [0,1]$.
Special cases of the NDPC distribution are:

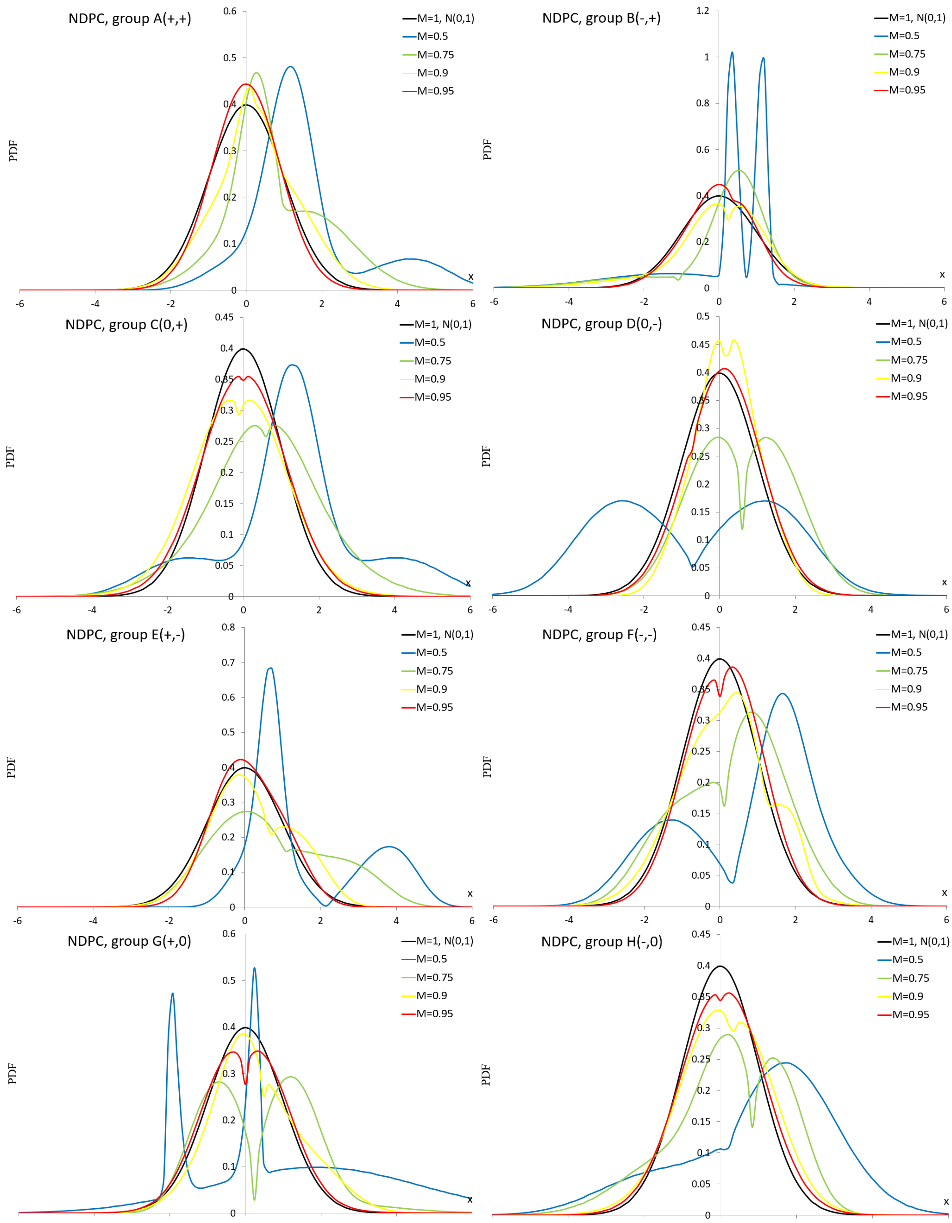
- $N(\mu_1,\sigma_1)$ for $\omega = 1$ and $N(\mu_2,\sigma_2)$ for $c_2 = 1, \omega = 0$;
- plasticising component (PC) $f_{PC}(x;\mu_2,\sigma_2,c_2)$ for $\omega = 0$.

Table 4A. Vectors of the NDPC parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1,\sigma_1,\mu_2,\sigma_2,c_2,\omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta};\mu,\sigma)$
O	$\mu_1,\sigma_1,\mu_2,\sigma_2,c_2,1$	0	1	0	0	$M(\boldsymbol{\theta};\mu_1,\sigma_1) = 1$
	$\mu_1,\sigma_1,\mu_2,\sigma_2,1,0$	0	1	0	0	$M(\boldsymbol{\theta};\mu_2,\sigma_2) = 1$
A	1.194,0.601,2.186,2.592,2,0.666	1.526	1.5	1.002	1.001	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.265,0.415,0.996,1.541,1.16,0.313	0.767	1.288	0.426	0.152	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.173,0.358,0.289,1.268,1.132,0.198	0.266	1.104	0.056	0.071	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.047,1.02,–0.014,0.872,1,0.214	–0.001	0.906	0.012	0.06	$M(\boldsymbol{\theta};0,1) = 0.95$
B	–1.321,1.842,0.741,0.459,2.56,0.287	0.15	1.4	–1.764	3.3	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.539,0.632,–1.078,2.061,1.174,0.741	0.12	1.34	–1.499	2.986	$M(\boldsymbol{\theta};0,1) = 0.75$
	–0.966,1.824,0.259,0.889,1.1,0.26	–0.059	1.305	–0.899	1.999	$M(\boldsymbol{\theta};0,1) = 0.9$
	–0.099,0.938,0.399,0.646,1.204,0.831	–0.015	0.911	–0.125	0.036	$M(\boldsymbol{\theta};0,1) = 0.95$
C	1.308,0.656,1.308,3.261,2,0.613	1.308	1.884	0	0.504	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.571,1.023,0.571,1.962,1.15,0.505	0.571	1.508	0	0.325	$M(\boldsymbol{\theta};0,1) = 0.75$
	–0.097,1.332,–0.097,1.058,1.1,0.614	–0.097	1.223	0	0.101	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.003,1.135,0.003,0.95,1.05,0.874	0.003	1.112	0	0.026	$M(\boldsymbol{\theta};0,1) = 0.95$
D	–0.692,2.203,–0.692,2.544,1.759,0.25	–0.692	2.265	0	–1	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.323,1.312,0.605,1.335,1.2,0.01	0.602	1.266	0	–0.587	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.179,0.494,0.179,1.163,1.426,0.443	0.179	0.862	0	–0.202	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.195,0.96,–0.719,0.858,1.109,0.918	0.12	0.983	0	–0.05	$M(\boldsymbol{\theta};0,1) = 0.95$
E	0.675,0.284,2.122,1.968,2.104,0.374	1.581	1.565	0.749	–0.849	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.423,1.032,1.058,2.077,1.815,0.494	0.744	1.544	0.311	–0.667	$M(\boldsymbol{\theta};0,1) = 0.75$
	–0.134,0.993,0.671,1.211,1.479,0.583	0.202	1.115	0.115	–0.4	$M(\boldsymbol{\theta};0,1) = 0.9$
	1.081,0.621,–0.216,0.755,1,0.24	0.095	0.912	0.1	–0.298	$M(\boldsymbol{\theta};0,1) = 0.95$
F	1.609,0.59,0.322,2.194,1.609,0.309	0.72	1.784	–0.491	–0.728	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.617,0.737,0.129,1.752,1.465,0.332	0.291	1.395	–0.239	–0.526	$M(\boldsymbol{\theta};0,1) = 0.75$
	–0.046,1.156,1.261,0.799,1.87,0.876	0.116	1.191	–0.1	–0.2	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.155,0.882,0.019,1.184,1.175,0.581	0.098	0.995	–0.05	–0.188	$M(\boldsymbol{\theta};0,1) = 0.95$
G	1.88,2.736,–0.848,1.122,6.437,0.679	1.005	2.656	0.524	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	2.419,1.56,0.237,1.384,1.476,0.074	0.398	1.409	0.35	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.055,0.702,0.474,1.586,1.328,0.473	0.276	1.191	0.31	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.212,1.443,–0.012,1.057,1.088,0.1	0.01	1.079	0.05	0	$M(\boldsymbol{\theta};0,1) = 0.95$
H	1.642,1.247,0.202,2.681,1.428,0.554	1	2.018	–0.594	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	–1.246,1.326,0.858,1.103,1.242,0.313	0.2	1.496	–0.5	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	–0.115,1.286,0.306,1.091,1.093,0.465	0.11	1.189	–0.1	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.084,0.949,0.03,1.214,1.047,0.423	0.053	1.098	–0.016	0	$M(\boldsymbol{\theta};0,1) = 0.95$

Source: authors' work.

Figure 4A. PDF curves of the NDPC for parameter values presented in Table 4A



Source: authors' work.

Plasticising component mixture distribution

The PDF of the plasticising component mixture distribution (PCM) is given by

$$f_{PCM}(x;\boldsymbol{\theta}) = \omega f_{PC}(x;\mu_1,\sigma_1,c_1) + (1-\omega) f_{PC}(x;\mu_2,\sigma_2,c_2)\ (x \in R),$$

where $f_{PC}(x;\mu,\sigma,c) = \frac{c}{\sigma}\left|\frac{x-\mu}{\sigma}\right|^{c-1}\phi\left(\left|\frac{x-\mu}{\sigma}\right|^{c};0,1\right)\ (x \in R)$
and $\boldsymbol{\theta} = (\mu_1,\sigma_1,c_1,\mu_2,\sigma_2,c_2,\omega)$, $\mu_1,\mu_2 \in R, \sigma_1,\sigma_2 > 0, c_1,c_2 \geq 1, \omega \in [0,1]$.
Special cases of the PCM distribution are:

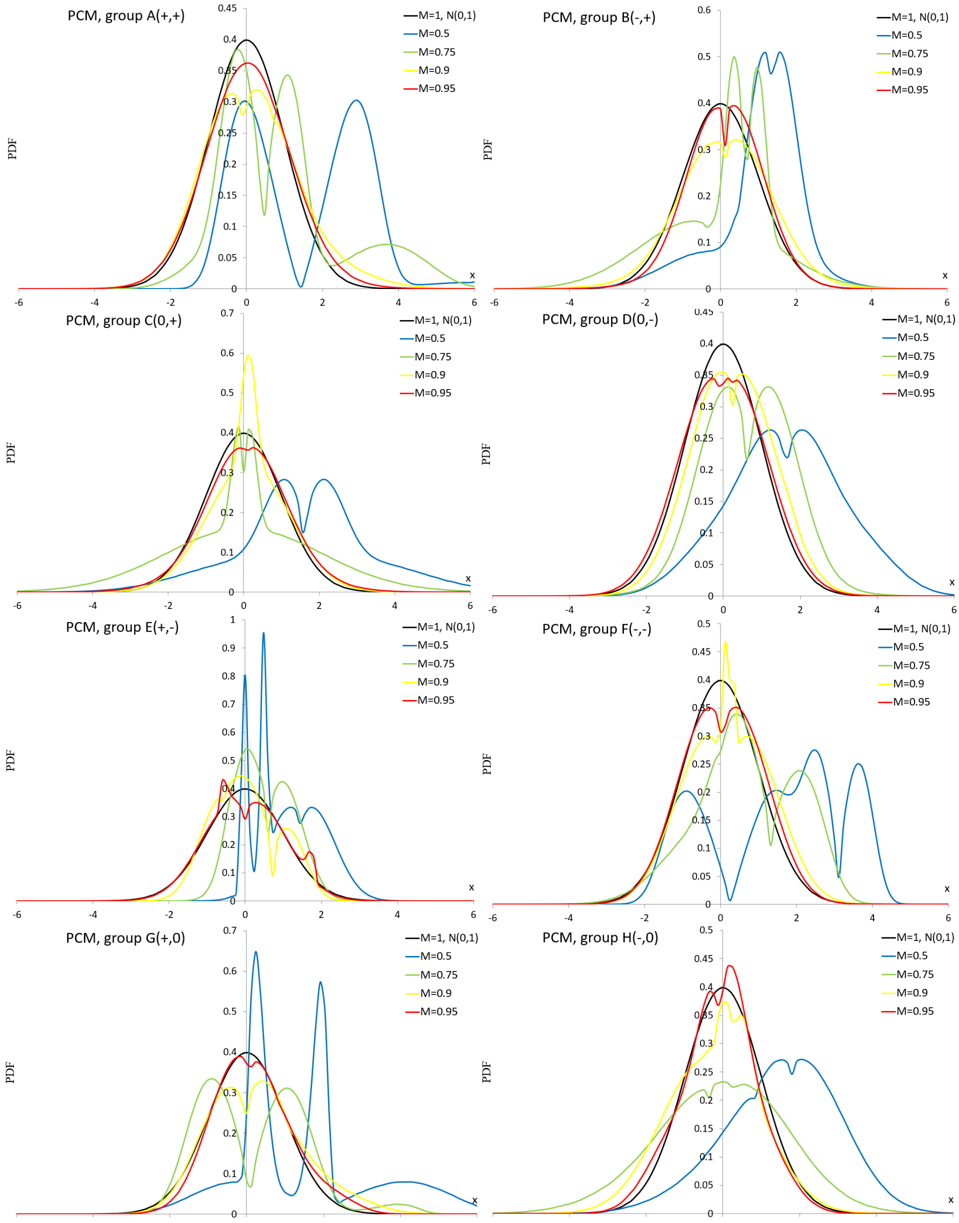
- $N(\mu_1,\sigma_1)$ for $c_1 = 1, \omega = 1$; $N(\mu_2,\sigma_2)$ for $c_2 = 1, \omega = 0$;
- plasticising component $PC(\mu_1,\sigma_1,c_1), PC(\mu_2,\sigma_2,c_2)$ for $\omega = 1, \omega = 0$, respectively.

Table 5A. Vectors of the PCM parameter $\boldsymbol{\theta}$, mean μ_a, standard deviation σ_a, skewness γ_1, excess kurtosis $\bar{\gamma}_2$ and similarity measure M. Groups O, A–H

Group	$\boldsymbol{\theta} = (\mu_1,\sigma_1,c_1,\mu_2,\sigma_2,c_2,\omega)$	μ_a	σ_a	$\gamma_{1,}$	$\bar{\gamma}_2$	$M(\boldsymbol{\theta};\mu,\sigma)$
O	$\mu_1,\sigma_1,1,\mu_2,\sigma_2,c_2,1$	0	1	0	0	$M(\boldsymbol{\theta};\mu_1,\sigma_1) = 1$
	$\mu_1,\sigma_1,c_1,\mu_2,\sigma_2,1,0$	0	1	0	0	$M(\boldsymbol{\theta};\mu_2,\sigma_2) = 1$
A	1.415,1.684,2.194,11.252,5.474,2.331,0.9	2.399	3.622	2.647	7.663	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.444,0.899,1.602,1.653,2.506,1.876,0.64	0.879	1.604	0.913	0.412	$M(\boldsymbol{\theta};0,1) = 0.75$
	−0.076,1.056,1.1,0.701,1.646,1.095,0.71	0.149	1.268	0.374	0.374	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.026,1.078,1.001,0.701,1.646,1.174,0.95	0.06	1.117	0.099	0.148	$M(\boldsymbol{\theta};0,1) = 0.95$
B	1.366,0.572,1.11,0.502,1.669,1.253,0.658	1.071	1.099	−0.978	1.565	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.67,0.425,1.576,−0.323,1.696,1.05,0.349	0.024	1.444	−0.569	0.606	$M(\boldsymbol{\theta};0,1) = 0.75$
	−0.204,2.209,1.205,0.133,1.139,1.05,0.076	0.107	1.224	−0.122	0.457	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.121,0.936,1.05,−0.17,1.917,1.411,0.95	0.106	0.982	−0.1	0.204	$M(\boldsymbol{\theta};0,1) = 0.95$
C	1.597,2.518,1.263,1.596,0.856,1.285,0.526	1.597	1.797	0	0.601	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.012,0.274,1.256,0.012,2.046,1.01,0.183	0.012	1.846	0	0.598	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.127,1.089,1.01,0.127,0.183,1.01,0.863	0.127	1.01	0	0.401	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.075,0.973,1.01,0.075,1.964,1.362,0.867	0.075	1.119	0	0.387	$M(\boldsymbol{\theta};0,1) = 0.95$
D	1.631,0.893,1.05,1.632,2.104,1.554,0.498	1.632	1.488	0	−0.268	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.639,1.576,1.167,0.64,1.085,1.199,0.163	0.64	1.12	0	−0.251	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.666,1.123,4.041,0.233,1.069,1.05,0.01	0.237	1.052	0	−0.198	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.225,1.087,1.05,−0.067,1.094,1.05,0.233	0.001	1.081	0	−0.18	$M(\boldsymbol{\theta};0,1) = 0.95$
E	1.472,0.782,1.11,0.236,0.291,3.203,0.692	1.091	0.861	0.38	−0.8	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.196,0.341,1.064,0.613,0.758,1.204,0.153	0.489	0.734	0.201	−0.7	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.722,0.703,1.304,−0.57,0.598,1.05,0.455	0.018	0.893	0.179	−0.617	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.584,1.171,9.804,−0.016,1.024,1.076,0.05	0.014	1.013	0.028	−0.351	$M(\boldsymbol{\theta};0,1) = 0.95$
F	0.261,1.419,1.909,3.099,0.744,1.567,0.57	1.481	1.757	−0.3	−1.107	$M(\boldsymbol{\theta};0,1) = 0.5$
	0.037,1.295,1.076,1.316,1.171,1.654,0.485	0.696	1.326	−0.204	−0.4	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.201,0.121,1.573,0.184,1.177,1.161,0.066	0.185	1.087	−0.003	−0.331	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.049,1.063,1.088,1.392,0.511,1.05,0.99	0.062	1.038	−0.008	−0.328	$M(\boldsymbol{\theta};0,1) = 0.95$
G	1.088,0.894,3.782,1.969,2.71,1.792,0.55	1.484	1.793	0.6	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	1.515,2.553,3.55,0.07,1.328,1.619,0.07	0.171	1.359	0.501	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	−0.034,1.072,1.159,1.146,1.51,1.301,0.756	0.254	1.238	0.401	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	0.825,1.615,1.868,0.067,0.934,1.05,0.141	0.174	1.044	0.336	0	$M(\boldsymbol{\theta};0,1) = 0.95$
H	0.816,1.867,1.24,1.787,1.272,1.05,0.278	1.517	1.475	−0.302	0	$M(\boldsymbol{\theta};0,1) = 0.5$
	−0.364,1.889,1.057,0.29,1.413,1.05,0.527	−0.055	1.682	−0.154	0	$M(\boldsymbol{\theta};0,1) = 0.75$
	0.286,0.405,1.27,−0.263,1.261,1.05,0.112	−0.202	1.188	−0.128	0	$M(\boldsymbol{\theta};0,1) = 0.9$
	−0.153,1.344,1.349,−0.024,0.539,1.05,0.565	−0.097	1	−0.12	0	$M(\boldsymbol{\theta};0,1) = 0.95$

Source: authors' work.

Figure 5A. PDF curves of the PCM distribution for parameter values presented in Table 5A



Source: authors' work.

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Demand for ESG data. Evidence from the Polish banking sector

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Abstract. Growing ESG (Environmental, Social and Governance) awareness leads to rising stakeholder expectations and regulatory pressure toward a sustainability transition. The process of sustainable development impacts different organizations and institutions, including banks which, due to their essential role as intermediaries in the financial system, channel funds into economic activity. Stakeholder expectations, regulatory pressure to report non-financial information and the need for a shift towards sustainable finance results in an increasing demand for ESG disclosure.

The aim of this article is to identify the growing importance of ESG data and the changing demands for such information at the company level for banks operating in Poland. We present an overview of the sustainable banking literature which argues that financial institutions progress from defensive compliance to proactive integration of ESG into risk management, pricing and product development. This evolution increases the demand for decision-useful, comparable and assured ESG data from corporate clients. Drawing upon the regulatory changes, we outline the concept of sustainable finance and discuss the banks' role in increasing the transparency of companies. We confront theory with practice by presenting the results of a survey conducted in 2024 and 2025 based on a sample of banks operating in Poland regarding their demands for company ESG data and the sources these data derive from.

Keywords: ESG data, banking sector, sustainable finance

JEL: M0, M1

1. Introduction

The growing importance of stakeholders combined with the urgency to address the challenges associated with the widely recognized global sustainability necessitate research on sustainability and Environmental, Social and Governance (ESG) standards. Today, sustainability has become a prevailing course of development in business practice. Sustainability aims at improving the well-being of various stakeholders addressing global concerns related to social inequality, human rights crisis, climate change and ineffective governance structure. Stakeholders pressure companies to balance their financial and non-financial goals as well as to incorporate sustainability strategies (Chelli et al., 2018; Tsang et al., 2023). These actions are

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operationalized into the ESG metrics on performance and disclosure standards and are expected to broaden the assessment of business activity and measure the progress towards a sustainable transition (Cho et al., 2012; Stacchezzini et al., 2016).

A firm's motivation for incorporating ESG practices in business strategies (Tsang et al., 2023) is driven by several reasons. Firstly, sustainability is perceived as a means to strengthen the competitive advantage of a company (Ioannou & Serafeim, 2019) and allows it to distinguish itself from its rivals in the market (Goyal et al., 2013). Superior ESG performance becomes of strategic importance for company operations; it can improve firm reputation and image (Zhang et al., 2022), gain customer loyalty and increase revenues. Higher ESG performance is expected to improve corporate financial performance and firm value (Paolone et al., 2022). Secondly, improving ESG performance is also seen as a rational company response to institutional pressures from regulators who introduce legislations on ESG disclosure and taxonomy for financing policy, investors searching for lower reputational risk (Yu et al., 2018) and stakeholders who expect practical solutions toward sustainability.

The existing literature offers studies on various aspects of sustainability transition, including the development of ESG disclosure and performance (Hassan & Romilly, 2018), its company- and country-level determinants (Gao, 2011) and the process of its institutionalization. Moreover, these studies investigate the impact that the introduction of sustainable solutions has on organizational change (Lozano et al., 2016) and financial performance (Alareeni & Hamdan, 2020; Saygili et al., 2022).

The effective implementation of sustainability at the company level requires institutional support, characterized by coercive regulatory requirements and enforcement pressure from the financial system. Economic activity requires financing, thus banks, investors and other institutions in the financial system play a crucial role in shaping the directions for business development with regard to sectors, products, services and compliance requirements. In other words, capital providers, with their capacity to channel funds across the economy, are able to translate stakeholders' expectations toward sustainability into lending and risk policies that drive sustainable development (Jeucken & Bouma, 2017). Therefore, banks may diffuse sustainability practice (Aracil et al., 2021; Yip & Bocken, 2018) and determine not only the development of particular industries but also shape the company practices to comply with ESG standards on emission, water use or human rights. As ESG indicators translate into criteria for obtaining credits, companies that fail to meet the bank's guidelines may face restricted access to financing, thereby limiting their development (Lu, 2024).

In this context, we identify the research gap regarding the insufficient understanding of the ESG data required by the financial system, particularly by banks which allocate funds amongst different corporate clients in the process of stimulating sustainable development (Carè, 2018; Menicucci & Paolucci, 2023; Nykvist & Maltais, 2022). The existing studies indicate that the role of ESG data reported by companies and used by banks is far from being overestimated, especially given the extensive shortcomings of the reported information (Rajawat & Mahajan, 2025). While the existing literature offers extensive evidence on the evolution of ESG metrics and its impact on the performance and financing of companies, scholars indicate several limitations of the prior research focusing on ESG data (Chaidali & Jones, 2017). Moreover, the lack of empirical consensus in research on ESG is attributed to certain methodological shortcomings, including inconsistent terminology and nomenclature, the absence of standardized data, the use of different measures for corporate ESG performance and material and immaterial ESG issues tend not be treated separately (Whelan et al., 2021). As a result, scholars and practitioners face difficulties comparing ESG performance across periods, firms and sectors. Furthermore, these limitations prevent reaching a consistent consensus on the ESG disclosure practices in relation to their determinants, entailed organizational change and financial performance (Platonova et al., 2018; Xie et al., 2019).

In this article, we analyze the importance of ESG data which companies provide to the market participants and stakeholders and which are essential from the perspective of the banking sector (Ling et al., 2025), while acknowledging the shortcomings of sustainability metrics and the measurement methodology. Financial institutions screen their clients for sustainability progress, potential controversies and incompliance, which complement the metrics on ESG performance (Elamer & Boulhaga, 2024). They integrate ESG variables to the construction of rating models, sector scorecards and portfolio dashboards, which increases the demand for standardized, machine-readable and assured corporate data. Since, as discussed above, ESG issues affect market, operational and reputational risks and determine company performance, banks need comparable and forward-looking indicators to understand the potential risks and prospects of their corporate clients (Carè, 2018). Well-structured, comparable, reliable and material information enables banks to calibrate their pricing, set collateral haircuts and establish appropriate covenants and portfolio limits. ESG data is also driven by consulting agencies, which produce ratings depicting potential risks and the financial sector, which evaluates investment and credit risks based on the sustainability performance of a company (Escrig-Olmedo et al., 2010).

The aim of this article is to identify the current and emerging practices of ESG data analytics regarding the demand for this type of data and access to particular topics and metrics. In particular, we investigate how banks operating in the Polish market formulate their ESG data requirements for their corporate clients and what procedures they employ to obtain these data.

2. Disclosure of company ESG data

2.1. The importance of ESG data at the company level

ESG data disclosure is currently one of the most dynamically developing themes in the area of management studies. It remains, however, a complex and evolving issue. The existing literature and business practices distinguish between disclosure (Rupley et al., 2012), reporting (Michelon et al., 2015) and accounting (Norman & MacDonald, 2004). Different topical areas such as ESG (Bernardi & Stark, 2018), sustainability (Chelli et al., 2018), social and environmental, climate-related or carbon (Pellegrino & Lodhia, 2012), non-financial, corporate social responsibility (Chauvey et al., 2015), triple bottom line (Norman & MacDonald, 2004), integrated reporting (Frias-Aceituno et al., 2014) and disclosure of corporate controversies (DasGupta, 2022) are also accounted for. Bearing in mind these differences in terminology, ESG reporting generally refers to the communication of social, environmental and governance performance (Bernardi & Stark, 2018), targeted at particular stakeholder groups and society at large, and is often referred to as financial reporting (Frias-Aceituno et al., 2014).

The growing interest in ESG information at the company level results from several factors. ESG disclosure complements traditional financial reporting, which provides a broad picture of company performance, including its impact on the society and the environment. Companies are motivated to report on their social and environmental policies and activities in response to the expectations of a variety of stakeholders: regulators, non-governmental organizations, social and environmental activist groups, customers, communities, investors, and others (Chuah et al., 2020). Thus, companies pressured to disclose information about their non-financial performance are believed to have a more balanced approach and incorporate stakeholder expectations into their strategies.

Scholars assume that this increased transparency should be perceived as the strategic direction of a firm's development, affecting the overall management of the company, its culture and leadership style. For instance, studying the concept of integrated reporting, de Villiers et al. (2017) indicate that it is an element of a different philosophy which focuses on disclosing future value creation linked to the firm's strategy, business model and the six forms of capital, i.e. financial, manufactured, intellectual, human, social and natural capital. ESG disclosure is also likely to potentially change how organizations perceive their social investment activities towards value creation and how they link them to strategy (Bose et al., 2022). Furthermore, Gond and Moon (2011) find that ESG reporting is employed across organizations as an adaptation or learning tool, as it can be relevant not only for strategy implementation (diagnostically for compliance verification), but also for strategy making (interactively for development assessment). In conclusion, the strategic approach to ESG disclosure contributes to the integration of the stakeholder's perspective into corporate strategy, thereby facilitating sustainability transition.

Disclosure also offers a number of financial benefits such as higher future cash flows (Qiu et al., 2016), higher performance (Xie et al., 2019) and profitability (Chen et al., 2018; Li et al., 2018). Greater transparency is associated with lower costs of equity capital (Baboukardos & Rimmel, 2016) and a general premium in financial markets, e.g. higher net earnings of the company (Berthelot et al., 2012). Moreover, ESG disclosure is crucial for market analysts, as it increases the accuracy of company forecasts (Bernardi & Stark, 2018).

Finally, ESG transparency serves as a tool of communication which companies use to gain legitimacy amongst constituencies and respond to the institutional environment (Baldini et al., 2018). Odriozola and Baraibar-Diez (2017) demonstrate that standardized and externally verified CSR reporting is likely to enhance company image. The aforementioned authors argue that CSR disclosure is a tool used to create and maintain the support of stakeholders, which is viewed as key to organizational success and survival.

2.2. ESG data within sustainable finance

Information on a company's sustainability performance is crucial for the stakeholders. While scholars emphasize the role of customers, suppliers, employees and communities in determining company reputation, studies also recognize the importance of ESG data to the participants of the financial system (Iatridis, 2013; Martin & Moser, 2016), such as investors and banks. Amid the changing stakeholder expectations and growing regulatory pressure, banks are driven to integrate ESG-related factors into their operations.

The discussion on sustainability disclosure and ESG data used by banks and investors rests on the premise of their central role in allocating capital, directing financial flows and aligning incentives for corporate behavior (Carè, 2018, pp. 39–64). As financial intermediaries, banks evaluate risks, price future uncertainties and exert influence over the accessibility of funding across industries and companies (Aracil et al., 2021; Yip & Bocken, 2018). Thus, banks act not only as passive recipients of ESG information but also as active transmission channels of sustainability norms, standards and regulatory expectations in the economy (Weber, 2017). In his seminal publications, Jeucken (2001) highlights that banks are the key players in sustainable development (Kumar & Prakash, 2019), balancing internal responsibility (governance, culture, ethics) with external responsibility (impact of financing). This dual approach ensures that banks manage risks while directing capital toward sustainable goals, essential for long-term viability and positive social impact (Tumewang et al., 2025). Banks undergo the transition towards sustainability under internal (direct) impacts and external (indirect) impacts. Direct impacts involve the bank's internal operations, including office energy use and paper waste, whereas indirect impacts are related to the bank's actions concerning lending and investment portfolios.

According to the theoretical foundations of sustainable banking in the process of ESG integration, banks evolve in three main stages, moving from defensive reputation management to preventive risk reduction and ultimately to proactive strategic engagement (Bouma et al., 2017; Jeucken, 2001). In the defensive stage, banks respond to social criticism and reputational exposure, relying on sustainability statements and public ESG ratings. In their evolution, banks progress to the preventive stage, in which they account for ESG factors and incorporate them into their internal risk assessment, sector policies and screening mechanisms. In the final, proactive stage, banks integrate sustainability principles into their core business mode by designing their products accordingly, linking credit pricing to ESG performance and shaping market expectations for transparent, reliable ESG data.

Thus, ESG data provide essential inputs for risk management translating into credit risks through regulatory constraints, technological shifts, physical climate impacts, supply chain disruptions and litigation exposure. For creditors, ESG disclosure provides additional information which allow information asymmetry to be lowered and to effectively understand potential risks and benefits of investing or granting loans to particular companies. ESG disclosure offers information which may potentially damage the reputation of a company or have detrimental effects on its valuation and return on investment for shareholders (Ling et al., 2025). This trend has been strengthened by EU regulations (Ottenstein et al., 2022), specifically by the Non-Financial Reporting Directive 2014/95/EU (NFRD), which came into effect in 2017. It outlined various company ESG performance requirements and policies. The legislation that followed was the Corporate Sustainability Reporting Directive (CSRD), which mandates companies to report according to the European Sustainability Reporting Standards (ESRS). In addition, the EU enacted regulations that directly address reporting by financial institutions, including the Sustainable Finance Disclosure Regulation (SFDR) and the Taxonomy Regulation, which translate into data requests for the ESRS topic-specific metrics for double materiality assessments, taxonomy metrics on revenue, Capex and Opex. Based on SFDR frameworks, financial institutions categorize their portfolio which they need to disclose to their stakeholders. As a result, banks require ESG information from their corporate clients on such issues as greenhouse gas emissions, transition plans, biodiversity-related impacts and workforce indicators and integrate them to bank-level metrics.

2.3. ESG analytics

The existing literature predominantly provides analyses results of voluntary disclosure relating to organizational and institutional determinants, organizational change, and economic and sustainability performance. However, scholars have been emphasizing the limitations of the voluntary approach (Kim & Lyon, 2011). The lack of convergence between different reporting regimes, coupled with the flexibility of standards and the deficit of assurance (Stolowy & Paugam, 2018) undermine the comparability of companies across sectors. Moreover, it hinders the evolution of the disclosure practices, making it difficult to distinguish between progress and regress in quality and scope (Aluchna, 2024). Other shortcomings include: a relatively low number of reporting companies, managerial discretion as to which topics to present in the report (leading to what is called cherry picking), no continuity in reporting particular standards (especially if a company's performance has declined), the insufficient use of metrics and techniques related to impression management, selective disclosure and decoupling. Therefore, theorists, practitioners and regulators stress the need to introduce mandatory legal requirements combined with their effective enforcement (Braam et al., 2016). Research on the determinants of sustainability reporting provides evidence that the corporate practice is strongly influenced by coercive mechanisms (García-Sánchez et al., 2016). While following reporting standards does not automatically enhance the quality of the information communicated to stakeholders, it increases the comparability and credibility of the ESG reports.

3. Research design and results

3.1. Study goal and questions

The aim of our study is to identify and understand the topics of ESG performance, which banks demand from the companies they do business with. Specifically, we focus on the ESG data whose publication becomes mandatory pursuant to the regulations on sustainable reporting and sustainable finance. The additional driver that motivates banks to obtain ESG data from companies is the banks' obligation to disclose their credit portfolios in relation to environmental (mostly CO_2 emissions),

social and governance impact by their customers. In order to achieve the goals of the study, we formulate three research questions:
RQ1: Which of the ESG data do the banks require now and which will it require in 2026 according to the provisions of the regulation?
RQ2: What sources of ESG data are used by banks to obtain the necessary information?
RQ3: What kind of support in obtaining ESG data do banks require from their stakeholders?

In order to provide answers to the formulated questions, we conducted a survey among the banks operating in the Polish market. The survey took place in 2024 and 2025 with the aim to identify the demands for ESG by banks in 2026 and beyond in the context of the European Union regulation. In particular, an online questionnaire was sent to ESG or sustainability managers of all banks in Poland. The questionnaire was divided into two parts: the first part was devoted to data/information/parameters/ESG information of bank customers, while the second part related to the expected support by banks to gain access to company ESG data. The overview of the survey structure is presented in Table 1.

Table 1. Survey structure and questions

<table>
<tr><th colspan="2">PART I Data / information / parameters / ESG information of bank customers</th></tr>
<tr><th colspan="2">Environmental</th></tr>
<tr><td>1. Data on greenhouse gas emissions/CO_2 equivalent to scope 1;
2. Data on greenhouse gas emissions/CO_2 equivalent to scope 2;
3. Data on greenhouse gas emissions/CO_2 equivalent to scope 3;
4. Greenhouse gas emissions on FTE;
5. Greenhouse gas emissions per unit of product produced;
6. The energy mix of the subject;
7. Energy consumption per unit of area / unit of production;
8. Energy Performance Certificate of the building (Energy Performance Certificate);
9. Information about the identified physical risks;
10. Environmental management policies/procedures;
11. Environmental management system standards and certificates (e.g. ISO 14001);
12. Biodiversity policy;
13. Water consumption;
14. Waste management;
15. Information on circularity of the business model;
16. Complaints and controversies in the area of environmental impact;
17. Environmental penalties;
18. EU Taxonomy Compliance Data;
19. Other?;</td><td>Options to answer:

1. Are the customer's ESG parameters needed by the bank in the current period?;
2. Does the bank currently have access to customer ESG parameters?;
3 What source of a given ESG parameter is used by the bank?
Optional:
a) Client's ESG/non-financial report is publicly available
b) Information obtained directly from the client in the form of a survey/interview
c) Market sources / ESG information bases, if so, which ones?...
d) Other sources, what kind?...;
4. Will the indicated data be needed by the bank in 2026 (according to the bank's current knowledge and predictions)?.</td></tr>
</table>

Table 1. Survey structure and questions (cont.)

PART I Data / information / parameters / ESG information of bank customers (cont.)	
Social	
20. Human resources management policy/procedure; 21. Number of employees/FTE; 22. Number of employees/FTE, broken down by form of employment; 23. Employment structure by gender; 24. Employment structure by age; 25. Employment structure by nationality; 26. Occupational health and safety policy/procedure; 27. Number of accidents; 28. Accident severity; 29. Anti-discrimination policy/procedure; 30. Number of cases of discrimination; 31. Information on the whistleblowing system; 32. Number and type of irregularities reported; 33. Number of training hours per employee/FTE; 34. Gender pay gap in base salary; 35. Total gender pay gap (base + variable + bonuses + other components); 36. The ratio of the salary of the highest paid employee to the median salary; 37. Complaints and controversies in the employee-related matters; 38. Penalties in the employee area; 39. Other?;	Options to answer: 1. Are the customer's ESG parameters needed by the bank in the current period?; 2. Does the bank currently have access to customer ESG parameters?; 3. What is the source of a given ESG parameter used by the bank? Optional: a) Client's ESG/non-financial report publicly available b) Information obtained directly from the client – in the form of a survey/interview c) Market sources / ESG information bases, if so, which ones?... d) Other sources, what kind?...; 4. Will the indicated data be needed by the bank in 2026 (according to the bank's current knowledge and predictions)?.
Governance	
40. Code of Ethics; 41. Anti-corruption policy; 42. Number and type of identified cases of corruption; 43. Description of the supply chain; 44. Supplier evaluation mechanism in the field of ESG/CSR; 45. Certificates used in the supply chain; 46. Diversity of the members of the management and supervisory staff in terms of gender; 47. Diversity of the members of the management and supervisory staff in terms of age; 48. Diversity of the members of the management and supervisory staff in terms of nationality; 49. Production quality management certificates (if relevant to the client's business); 50. Product safety certifications; 51. Certified production/services (type of certification and share of certified production); 52. Number of customer information breaches; 53. Number of cases of complaints and violations in the field of marketing communication; 54. Other?;	Options to answer: 1. Are the customer's ESG parameters needed by the bank in the current period?; 2. Does the bank currently have access to customer ESG parameters?; 3. What is the source of a given ESG parameter used by the bank? Optional: a) Client's ESG/non-financial report publicly available b) Information obtained directly from the client in the form of a survey/interview c) Market sources/ESG information bases, if so, which ones?... d) Other sources, what kind?...; 4. Will the indicated data be needed by the bank in 2026 (according to the bank's current knowledge and predictions)?.

Table 1. Survey structure and questions (cont.)

PART I Data / information / parameters / ESG information of bank customers (cont.)	
Managing ESG	
55. ESG/sustainability/non-financial report; 56. ESG ratings that the client submits to and their current performance;	
PART II Information on support obtained by the bank in accessing ESG data	
57. What tools/solutions and market changes in the field of customer ESG data analytics does the bank currently need? 57.1. For climate risk analysis 57.2. For the purpose of analyzing and defining the level of ESG risk, the bank needs:	Type of support/needed solutions currently identified by the bank (yes/no) Tech tools/analytical support Access to databases Customer actions Regulator support
57.3. For the purpose of identifying assets compliant with the EU Taxonomy (GAR – green asset ratio calculations), the bank needs:	
57.4. For the purpose of identifying sustainable assets, in accordance with the internal processes currently implemented by the bank, the bank needs:	Type of support/needed solutions currently identified by the bank (yes/no) Tech tools/analytical support Access to databases Customer actions Regulator support
57.5 What other challenges and needs in terms of access to data and ESG analytics, not mentioned in the study, are currently identified by the bank?.	

Source: authors' work

3.2. Research sample

The questionnaire was distributed to all commercial banks operating in Poland targeting the ESG/sustainability managers and directors. We obtained questionnaires from the nine largest commercial banks with the largest number of clients and the largest assets representing 50% of the population on commercial banks operating in Poland. We acknowledge the limitation of the study given its exploratory approach. Yet, our intention was to map the concerns that the banks raised in the process of accessing, interpreting and analyzing the ESG data obtained from their corporate clients. While the magnitude of the sustainability disclosure debate refers to the reporting practice at the company level, there is an insufficient understanding of the applicability of ESG data by banks, investors and other financial institutions. The characteristics of banks covered by the study is briefly presented in Table 2.

Table 2. Research sample

Bank	Name (in alphabetical order)	Ownership structure	Value
Bank 1	Alior Bank, as a universal bank, addresses its services to both individual and business customers. Founded in 2008 by an Italian group, Carlo Tassara. It has been a subsidiary of the PZU SA insurance company since 2015. Alior Bank is one of the fastest growing banks in Poland. It is the first start-up bank which from the very beginning of its activity has been committed to innovation and trend-setting in online banking in the Polish market.	Alior Bank SA is listed on the Warsaw Stock Exchange (WSE) since 14th December 2012. Shareholder composition at the end of 2022 (Alior Bank, n.d.b): • PZU S.A. Group 31.91% • Nationale-Nederlanden OFE 9.47% • Allianz OFE 8.83% • Generali OFE 5.56% • Other shareholders 44.23%	*'In our day-to-day activities, we combine principles of traditional banking with innovative solutions, through which we systematically strengthen our market position and set new directions for development of the Polish banking sector.'* (Alior Bank, n.d.a).
Bank 2	Bank Gospodarstwa Krajowego (BGK) is a state development bank, established in 1924, supporting social and economic development of Poland and its public sector. BGK provides funding for infrastructure investments and thus supports the growth of this sector of the economy. It thereby creates an important link in the provision of funding and support for areas such as housing infrastructure, sustainable energy and public utilities. BGK supports Polish exporters by taking on part of the risk related to trading activities of Polish companies.	Stated-owned development bank.	*'Our mission is to support sustainable social and economic growth of our country. In 2025, we want to be the leader in creating and implementing the programs that enhance a stable and competitive economy, supported by strong social capital.* *We respond to major development challenges in key areas of the economy, we cooperate with the market and stabilize it, and also adjust our activities to the emerging needs, trends and market challenges through BGK strategy pillars and program.'* (BGK, n.d.).
Bank 3	BNP Paribas Bank Polska (formerly Bank Gospodarki Żywnościowej SA-BGŻ Bank) is a universal commercial bank providing innovative financial solutions in a responsible manner to help clients and to support the local economy. BNP Paribas Bank Polska offers savings and investment products as well as a wide range of loans to individual and business clients (micro enterprises, SMEs and corporates). BNP Paribas Bank Polska also pays special attention to companies from the food and agricultural sectors.	Listed on the WSE, the Bank's shareholder structure as of 5th April 2023 (BNP Paribas, n.d.b): • BNP PARIBAS in total: 87.35% • Other shareholders 12.65%	*Mission* *'We implement positive banking into the lives of our Clients, responding to their financial needs and making it easier for them to achieve their goals. We operate in a simple, thoughtful and safe manner, caring for the society and the environment.'* (BNP Paribas, n.d.a).

Table 2. Research sample (cont.)

Bank	Name (in alphabetical order)	Ownership structure	Value
Bank 4	Credit Agricole is a universal bank that has been present in the Polish market since 2021. It offers banking services to retail customers and corporations, to farmers and SMEs, as well as consumer finance services.	Part of the Credit Agricole Group – the 10th largest financial group in the world.	The bank motto, is: *'Working every day in the interest of our customers and society'. This means that the customer is at the center of our attention: we listen carefully to our customers and deliver the solutions they need.'* (Credit Agricole Bank, n.d.).
Bank 5	ING – in June 2001, ING Bank became a major shareholder of Bank Śląski and on 6th September 2001, Bank Śląski, a member of the ING Group, started operating under a new name – ING Bank Śląski SA. ING Bank Śląski is a universal bank, offering a broad range of retail, corporate and private banking products and services to individual and corporate customers.	Listed on the WSE since 1994, current shareholder structure: (ING Bank Śląski, n.d.b): • ING Bank N. V. 75.00% • Allianz Polska Otwarty Fundusz Emerytalny 9.30% • Other shareholders 15.70%	*'Our mission: at ING Bank Śląski, we support customers to be one step ahead in life and in business, including in ESG. That is why ESG strategy is part of our business strategy. It is important to us because the future is our shared responsibility.'* (ING Bank Śląski, n.d.a).
Bank 6	mBank was established in 2000 as BRE Bank, the bank was rebranded to mBank in 2013. mBank is one of the largest Polish banks, operating as a universal bank and serving all client groups, leading in terms of mobility and innovations. mBank provides a comprehensive range of products and services in retail, business, corporate and private banking.	Listed on the WSE since 1992. At the end of 2022, CommerzBank AG owned 69.17%, meantime 30.83% of shares was in free float. (mBank, n.d.b).	*Values of mBank Group:* (mBank, n.d.a). • *Authenticity* • *Empathy* • *Responsibility* • *Courage* • *Cooperation*
Bank 7	Bank Millennium is a nationwide universal bank, offering its services to all market segments through a network of branches, individual advisors and electronic banking. Bank Millennium was established in 1989 as Bank Inicjatyw Gospodarczych. In 1997 it changed its name to BIG Bank GDAŃSKI and in 2003 to the present name	Bank Millennium has been listed on the WSE since 1992; it was the first Bank ever to float its shares on the WSE. Banco Comercial Portugues (Millennium bcp) – Portugal's largest commercial bank and Bank Millennium's strategic shareholder (owner of 50.10% of the Bank's shareholding).	*'The mission of Bank Millennium is to support our customers to succeed in their financial present and future. The Bank's goal, stemming from its mission, is to deliver premium quality universal financial services to all customer groups and live up to the development challenges of Poland's financial services market, which should bring constant increase of the Bank's value to Shareholders.'* (Bank Millennium, n.d.).

Table 2. Research sample (cont.)

Bank	Name (in alphabetical order)	Ownership structure	Value
Bank 8	Bank Pekao, an international universal bank, is the largest corporate bank and the leader of the private banking market in Poland. The bank was founded in 1929 by the Ministry of Treasury as a national bank, mainly to provide financial services to Poles living abroad. In 1939, the bank had branches in virtually every capital city of the countries where Poles lived. Historically, the Italian bank UniCredit used to own 59% of the company, then it was sold. In 2017, the process of acquiring Bank Pekao S.A. shares by Powszechny Zakład Ubezpieczeń and the Polish Development Fund was completed.	Since 1998, the bank has been listed on the WSE. Shareholding structure as of the 2022 Report: • Powszechny Zakład Ubezpieczeń S.A. 20.00% • Polski Fundusz Rozwoju S.A. 12.80% • Funds managed by Nationale-Nederlanden Powszechne Towarzystwo Emerytalne S.A. 6.40% • Funds managed by Powszechne Towarzystwo Emerytalne Allianz Polska S.A. 5.91% • Subsidiares managed by BlackRock, Inc 5.04% • Other shareholders 49.85%	Mission: *'Simple and safe banking world.* *For almost a century we are setting the standards on the market. We are a reliable partner in the everyday life of millions of Poles. We help to make dreams come true and to pursue passions. We have positive impact on the economy, we build strong relationships with entrepreneurs and provide safety to our customers. We put innovative services into practice in an easy and friendly way'* (Bank Pekao, n.d.).
Bank 9	Santander Bank Polska (formerly Bank Zachodni WBK) was established in 2001 as a result of a merger between Bank Zachodni S.A. and Wielkopolski Bank Kredytowy S.A. (WBK). In 2011, Bank Zachodni WBK became a member of Santander Group. Santander Bank Polska SA offers financial solutions for individuals, micro enterprises and SMEs, as well as Polish and international corporations.	Shareholders structure (Santander, n.d.a): • Banco Santander S.A. 67.41% • Nationale-Nederlanden OFE 5.01% • Other shareholders 27.58%	• *Purpose* *To help people and businesses prosper. To be the best open financial services platform by acting responsibly and earning the lasting loyalty of employees, customers, shareholders and communities.* • *Values* *Simple, Personal, Fair* (Santander, n.d.b).

Source: authors' work based on the websites of the sample banks.

Polish banks attempt to align with the UN Sustainable Development Goals (SDGs), although this sector lacks a universally accepted model, with approaches varying between internal resource management and external portfolio steering. The overview of the sample banks, however, indicates a shift from treating ESG as a side project to making it an important element of their business models. For instance, Santander Bank Polska has become the first bank in Poland to issue sustainable debt securities totaling PLN 750 million. In a similar vein, mBank has rewritten its rulebook for green transformation and allocated PLN 22 billion to sustainable investments, doubling its original strategic targets. As the largest market player, PKO BP is working through its 2025–2027 strategy to bring over half of its large loan portfolio into the transformation plan, cutting its own Scope 1 and 2 emissions and increasing the use of green bonds. Finally, BNP Paribas Bank Polska, moved beyond the goals with its 'Accelerate 2030' strategy and targeted the decarbonization of the agrifood sector through a dedicated Sustainable Development and Agribusiness Area, while Bank Pekao S.A. ensured a stable, reliable source of funding for complex ESG projects.

3.3. Results

Addressing the first research question (RQ1), we asked the respondents about the types of the ESG data which the bank requires now and the ones it will need to obtain, as demanded by EU regulations. As shown in Table 1, we divided the topics of ESG data according to environmental, social and governance dimensions. When dealing with the environmental dimensions, respondents indicated that the company ESG parameters needed by the bank currently include data on greenhouse gas emissions/CO2 equivalent in scope 1 and scope 3, followed by information about the identified physical risks and environmental management policies/procedures (each selected by 4 banks). Next, we learned that the banks predict a growing demand for additional ESG data according to the currently known legislation procedures. The highest number of answers was depicted for greenhouse gas emissions on FTE and greenhouse gas emissions per unit of product produced (all sample banks), biodiversity policy (8 banks), the energy mix of the subject and energy consumption per unit of area/production, water consumption, waste management and circular economy (7 banks).

Next, RQ1 was used to analyze the responses on social information within ESG data. The number of items that banks found important now is similar to the case of the environmental information. Four banks pointed at the information about fines in employee-related matters, while three banks emphasized the importance of the information on the number of employees/FTE, occupational health and safety policy/procedure, the number and severity of accidents, as well as the number of cases of discrimination. The greatest interest in the social information for 2026 notified by the respondents refers to the information on the employment structure by nationality (9 banks) and by age, information on the whistleblowing system and the number and type of irregularities reported, the number of training hours per employee/FTE and total gender pay gap (8 banks), followed by information on the employment structure by gender, occupational health and safety policy/procedure, the number of accidents, the number of cases of discrimination, gender pay gap in base salary and complaints and controversies in employee-related matters (7 banks).

With respect to governance information data, which banks perceived as needed currently covers code of ethics (5 banks), followed by the information on anti-corruption policy (4 banks) and the number and type of identified cases of corruption, certified production/services, diversity of members of the management and supervisory staff in terms of gender (3 banks). Banks submitted the demand for the following ESG metrics – diversity of members of the management and supervisory staff in terms of age and nationality (8 banks), followed by the number and type of identified cases of corruption, the description of the supply chain, the supplier evaluation mechanism in the field of ESG/CSR, certificates used in the supply chain, production quality management certificates, product safety certifications, certified production/services and number of customer information breaches (7 banks).

Overall, while banks demand ESG data now, the survey results indicate that they anticipate a significant increase in ESG data availability. The increase refers to all dimensions with social metrics identified as the most dynamically growing aspects of ESG data. Our findings show that, as indicated by the sustainable banking theory, the sample banks anticipate the need for more detailed ESG information from corporate clients (Weber, 2017), proceeding from a defensive approach (focused on addressing reputational risks) to a preventive and proactive approach (Bouma et al., 2017; Jeucken, 2001). It should be noted that the increased demand for data corresponds with the banks' shift toward integrating environmental risks into credit risk models and portfolio management according to material principles on environmental performance. Additionally, the evidence points to increased social data needs, which aligns with the global trend in banking toward assessing human capital quality as a component of long-term creditworthiness. The existing literature demonstrates that weaknesses in workforce management, discrimination or health and safety translate into operational and legal risks that may affect a borrower's financial stability (Goyal & Joshi, 2011). Banks' increasing demand for social metrics on diversity, pay ratios and whistleblowing systems provide a means to strengthening the monitoring of social controversies that may undermine their credibility or reputation (Wang, 2023). A similar logic drives the demand for governance-related metrics, particularly covering anti-corruption systems, board diversity and supply-chain oversight.

The second research question (RQ2) referred to the sources of ESG data used by banks to obtain necessary information. According to the survey results, respondents pointed at non-financial reports published by companies as the main sources of these four areas of ESG company data. In addition, one bank uses the klimada.pl publicly available website to learn about the physical risks, two banks conduct a separate questionnaire among their clients to obtain any necessary data. Moreover, two banks claimed that they obtained data directly from companies and one bank emphasized the role of personal relations to get access to ESG data from the client company. These findings suggest the ESG data infrastructure is in its transitional stage, during which banks still rely heavily on self-reported information, despite the known problems of inconsistency, selective disclosure and greenwashing (Haji et al., 2023). The literature on the non-financial disclosure by banks themselves points to similar risks, stressing that a high level of ESG disclosure does not necessarily imply alignment with sustainability performance or transparency. The reliance on client questionnaires and relational channels confirms that bank-level ESG integration cannot progress to the next, proactive stage until standardized, machine-readable and assured data are obtained. Such a practice has been widely noted also in studies on institutional investors who directly approach their portfolio companies to collect reliable and customized ESG data (Aluchna, 2024; Semenova, 2023).

The final research question, RQ3 focuses on the support that the sampled banks require in accessing ESG data. Respondents pointed to all the formulated support for all three dimensions, emphasizing access to databases, customer actions and regulator support (8 banks), followed by technical tools and analytical support (7 banks). We interpret these observations as a recognition of ESG data management as a critical capability gap requiring structural investment. As the sustainable finance literature suggests, banks need enhanced data platforms, advanced analytics and supervisory guidance to translate ESG information into coherent risk assessments (Steiner & Makarenko, 2025, pp. 65–102). The demand for regulatory support, reflected by the sector's expectations, involve developing efficient and coherent reporting standards, assurance requirements and supervisory frameworks that would reduce data asymmetries and increase reliability across corporate borrowers. The results of the survey are summarized in Table 3.

Table 3. Summary of survey findings

Research aspect	Main findings	Implication for banks
Type of ESG data required	All three types, i.e. environmental, social, governance. Concerns about controversies or incompliance.	Increasing demand for data on environmental and social impacts, as well as value chains. Inclusion of ESG into banks' policies.
Source of ESG data	Company, non-financial reports, bank's direct access to company via questionnaires.	Growing importance of metrics to enhance comparability. Bank's demand for stronger infrastructure.
Support in accessing the ESG data	Databases, customer actions and regulatory support.	Growing importance for technical and analytical tools supporting banks.

Source: authors' work.

The research results show that banks are well aware of the growing need for ESG data, driven by the increasing impact of ESG information (Xu et al., 2025) on financial performance and reputation (Samaniego-Medina Reyes & Giráldez-Puig, 2022). Furthermore, ESG transparency is also determined by EU regulations, which will most likely lead to the further expansion of the scope of the reported information. The enhancement of disclosure is noted within all of the ESG dimensions: environmental data providing information on emissions, waste management, water consumption across the value chain, including the performance of suppliers and subcontractors. The demand for social information with regard to the characteristics of employees (gender, age), anti-discrimination measures, gender pay gap and occupational health and safety will also increase. Again, the scope of the data will also include information from suppliers and subcontractors. Finally, additional attention will be given to governance data referring to anti-corruption measures and policies, cases of corruption, diversity of boards, as well as certification along supply chains. The regulatory change results in a significant demand for data on ESG performance and policies leading to the development of a reporting framework, reporting standards and for metrics and indices. To ensure regulatory compliance, both companies and banks have to engage IT specialists, experts in quantitative methods and auditors to avoid any structural constraints (Menicucci & Paolucci, 2023).

In summary, the increasing demand for ESG data, efforts to enhance data accessibility and infrastructure development appear to follow the trajectory conceptualized by Jeucken (2001), moving from defensive behavior toward a proactive strategy of ESG data integration into banks' policies. The increasing demand for detailed, value-chain-wide ESG indicators reflects the attempts of the banking sector to integrate sustainability into credit assessments, pricing policies and portfolio management (Bouma et al., 2017; Goyal & Joshi, 2011). Supporting banks with metrics, methodology, assurance and infrastructure determines their position since the associated costs and structural shortcomings may undermine their potential to contribute to sustainability (Nykvist & Maltais, 2022).

4. Conclusions

Addressing stakeholder expectations for greater transparency and substantive actions towards decreasing the negative impact of business on society and environment creates a strong demand for ESG data. ESG data provide a bigger picture of company performance and become an essential tool used to manage a company's transition towards sustainable economy.

These trends have been strengthened in recent years in the EU with a package of regulations that pressure companies to redevelop their business models to comply with the principles of sustainability. In addition, to accelerate the transition process, more emphasis was put on the financial sector in order to redirect the capital towards sustainable projects. As a result, banks as well as other financial institutions are required to report the composition of their credit and investment portfolios. The goal of the regulation is to limit the financing for companies with a negative impact and channel the capital toward green solutions.

The goal of our study was to confront the regulatory framework for ESG data with the practice and plans of the banks operating in the Polish market. The survey carried out on a sample of nine commercial banks representing about 50% of the market share revealed that the banks are aware of the emerging expectations. The results document the evolving requirements for ESG data by banks. While at present the findings indicate the need for selected types of ESG information, they also signal a significant increase of demand for ESG data. The answers collected from sustainability experts in the sample banks indicate that the form, accuracy and credibility of the reported ESG information is expected to increase in the following years.

Our study provides significant implications for both practitioners and policymakers. Firstly, the results suggest that banks move along the sustainable-banking maturity path outlined in Jeucken's (2001) framework, evolving from defensive, reputation-oriented behavior toward proactive ESG integration. Given the central role of banks in the financial system, our results imply that banks can function as transmitters of sustainability norms by requiring standardized and assured ESG data from their corporate clients.

Secondly, given the magnitude of the change, we suggest policymakers consider the institutional and expertise support for companies and banks which will operate under sustainability regulations. Adjusting the reporting and IT systems will require a significant effort from both companies and banks. The assistance provided by policymakers would support the enforcement of sustainability regulations and assure a more effective implementation of the new practices. There is a growing need and market hope for building a central ESG database, whose goal would be to gather the data from entities (reporting companies) in one place. Access would be provided to investors, regulators and financial institutions, including banks, to allow them the use of reliable data, necessary for their ESG analysis. The provision of effective and systematic solutions in ESG data management will require a strong collaboration between the players in the financial market, coupled with support and supervision from regulatory bodies.

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Report from the 34th Scientific Conference of the Classification and Data Analysis Section of the Polish Statistical Association

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1. Introduction

The 34th Scientific Conference of the Classification and Data Analysis Section of the Polish Statistical Association took place on 23rd–24th September 2025, in Wrocław, Poland. It was organized by the Classification and Data Analysis Section (Sekcja Klasyfikacji i Analizy Danych – SKAD) of the Polish Statistical Association (Polskie Towarzystwo Statystyczne – PTS) and the Wrocław University of Economics and Business (WUEB). Basic information about the conference is available at https://www.skad.edu.pl/skad2025/.

Krzysztof Jajuga, PhD, DSc, ProfTit (WUEB), chaired the organising committee, which included Andrzej Dudek, PhD, DSc, Assoc. Prof. at WUEB, Katarzyna Kuziak, PhD, DSc, Assoc. Prof. at WUEB, Aleksander Mercik, PhD, DSc, Assoc. Prof. at WUEB, the Scientific Secretary of the Conference, Marcin Pełka PhD, Assoc. Prof. at WUEB and Marek Walesiak, PhD, DSc, ProfTit.

The aim of the SKAD conference was to provide a platform for the exchange of ideas related to the theoretical and applied aspects of classification and data analysis, to present the latest research in this area and to indicate its possible development directions. The following topics were addressed during the conference:

- theoretical aspects of classification and data analysis: taxonomy, graphical methods, discriminant analysis, linear ordering methods, multivariate statistical analysis, methods of analyzing continuous and discrete variables, symbolic data analysis, machine learning methods, and artificial intelligence;
- application: financial data analysis, marketing data analysis, spatial data analysis, computer application of statistical methods, and other areas of the application of data analysis such as medicine, psychology and archaeology.

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The conference featured 51 participants, including faculty members and doctoral students from the following universities and institutions: Wroclaw University of Economics and Business, Leiden University, Statistics Poland, University of Rzeszów, University of Lodz, Institute of Environmental Protection – National Research Institute, University of Szczecin, Krakow University of Economics, Jan Kochanowski University of Kielce, Nicolaus Copernicus University in Toruń, University of Warsaw, Mazovian Academy in Płock, Poznań University of Economics and Business, Statistical Office in Poznań, University of Economics in Katowice, Warsaw University of Life Sciences, University of Gdańsk, Federal University of Bahia, Adam Mickiewicz University, Poznań, Maria Curie-Skłodowska University in Lublin, StatSoft Poland, and EMPIRICA Ltd.

A total of 32 presentations introducing research results related to the theory and application of classification and data analysis were made during five plenary sessions, six parallel sessions, and a poster session. The sessions were chaired by Józef Pociecha, Eugeniusz Gatnar, Paweł Lula, Krzysztof Najman, Marek Walesiak, Andrzej Dudek, Marcin Salamaga, Grażyna Dehnel, Kamila Migdał-Najman, Katarzyna Kuziak and Joanna Landmesser-Rusek.

Below is a list of all papers presented during the conference:

- Mark de Rooij, *Reduced Rank Regression for Mixed Predictor and Response Variables*;
- Marek Cierpiał-Wolan, *Statistics in a Hyper-Turbulent Era*;
- Józef Pociecha, Marek Walesiak, Krzysztof Jajuga, *The Section of Classification and Data Analysis of the Polish Statistical Association – a bit of history*;
- Andrzej Dudek, *Data Preprocessing Step in Forecasting with Deep Learning Models*;
- Paweł Rokita, Radosław Pietrzyk, Piotr Grzybowski, *Toxic Order Flow, VPIN Spikes, and Price Changes: Implications for Market Makers in Unregulated Crypto-Token Market*;
- Beata Bieszk-Stolorz, *Survival Time Entropy in Assessing the Chances of Taking Up Employment*;
- Krzysztof Dmytrów, *Multidimensional Evaluation of Linear Ordering Methods in Location Selection During Order Picking*;
- Joanna Michalak, Małgorzata Szczepaniak, *Profiling Attitudes Toward Income Inequality in Poland Using Cluster Analysis*;
- Volodymyr Melnykov, *Conditional Mixture Modelling with Applications;*
- Paweł Lula, *Methods for Classifying Text Documents*;
- Krzysztof Najman, Kamila Migdał-Najman, *Explainable Artificial Intelligence – A Step Toward Increasing Trust in AI Models*;

- Joanna Trzęsiok, *On Discrimination Without Bias*;
- Katarzyna Raca, Alisson Soares, *Discussions Around the "Great Reset" Theory on Reddit: Content and Dynamics Analysis*;
- Janusz Wątroba, *Accounting for Additional Sources of Variability When Assessing Statistical Significance of Effects*;
- Marta Dziechciarz, Marcin Pełka, *Linear Ordering Based on Symbolic Histogram Data*;
- Monika Święciochowska, Łukasz Smaga, *Random Projections Method in Multifactor Analysis of Variance for Functional Data*;
- Joanna Landmesser-Rusek, *GAT Neural Networks in Exchange-Rate Modelling*;
- Marcin Salamaga, *Examining the Impact of Firms' Internationalization on Their Digital Innovativeness Using Structural Equation Modelling*;
- Dorota Rozmus, *Impact of Current and Expected Inflation on Assets of Public-Sector Manufacturing Enterprises*;
- Tomasz Klimanek, Sylwia Filas-Przybył, *Assessing the Impact of Changes in Temporal Accessibility on Permanent-Residence Migration*;
- Kamil Zientarski, *On the Correctness of Selecting and Weighting Diagnostic Variables in Taxonomic Procedures*;
- Marcin Pełka, Aneta Rybicka, *Studying Beer Consumer Preferences Using Latent Profile Analysis*;
- Aleksander Mercik, Krzysztof Piontek, *Volatility Spillovers Between Stock Exchanges in Digital Asset Markets*;
- Marek Walesiak, Grażyna Dehnel, *Achieving SDG 4 in EU countries in relation to the target year 2030: A multivariate indicator analysis using a hybrid approach*;
- Katarzyna Kopczewska, *Spatial Association Rules for Uncovering the Urbanization-Greening Relationship: Land-Use Changes Under Population Change in Europe (2006–2018)*;
- Jacek Batóg, Barbara Batóg, *Multidimensional Analysis of Public Debt of the World's Largest Economies*.

During the poster session, the following papers were presented:

- Andrzej Geise, *Digitalization and Innovation Impact on Export of High-Tech Products in European Countries – A Panel Data Approach*;
- Dominik Krężołek, *Nonparametric Risk Classification Methods for Selected Companies Listed on the Warsaw Stock Exchange*;
- Norbert Jaworski, *A Novel Density-Based Metric for Measuring Spatial Competition in Urban Agglomerations*;
- Michał Boda, Marta Karaś, Michał Stachura, *Beyond Traditional Risk Models A Robust Interaction-Based Framework for Systemic Risk Assessment*;

- Elżbieta Antczak, Agnieszka Sobol, *Energy Poverty in Poland – Assessment of the Situation and Spatial Trends Using a Composite Development Measure*;
- Paweł Kaczmarczyk, *VAR Models for Approximating and Forecasting the Telecommunication Market: Empirical Results on Effectiveness*.

On the first day of the conference, the members of SKAD held their annual meeting. Prof. Andrzej Dudek, the president of SKAD, chaired the meeting, which included the following agenda items:

1. Report on the activities of SKAD PTS;
2. Information on the planned national and international conferences;
3. Organization of the SKAD PTS conference in 2026 and 2027;
4. Election of the SKAD PTS Representative to the IFCS Council for the 2026–2029 term;
5. Miscellaneous matters.

Opening the session, the Chair welcomed the participants and confirmed the agenda, emphasizing the importance of continued collaboration within the scientific community and the active role of SKAD members in national and international initiatives.

The first item on the agenda was a comprehensive report on the activities of SKAD. The report summarized the developments in June 2024–September 2025. It included membership statistics, administrative matters related to GDPR declarations, maintenance of communication channels, and the availability of information resources for the members. The Chair emphasized the ongoing engagement and thanked the members for their support and involvement in the SKAD initiatives.

According to the report, SKAD currently has 231 members. Any bylaws and membership applications are available on the SKAD website. Then, a moment of silence was observed in memory of SKAD members, Alina Karska, PhD, and Cyprian Kozyra, PhD, who have recently passed away.

The report concerning the SKAD conference (which was held in Kraków on 5th–6th June 2024) could be found in issue 2/2025 of the *Przegląd Statystyczny. Statistical Review* journal (Dudek, A., Lula, P., & Pawełek, B. (2025). Report from the 33rd Scientific Conference of the Classification and Data Analysis Section of the Polish Statistical Association. *Przegląd Statystyczny. Statistical Review*, *71*(2), 39–42. https://doi.org/10.59139/ps.2024.02.3.).

The growing SKAD members' international presence was also emphasized, evidenced by their active participation in major global conferences, strengthening the visibility of the Polish research in the area of classification and data analysis, and the related fields.

The second item on the agenda concerned the planned national and international conferences. Information on important upcoming scientific meetings in Poland was presented, which included the Multivariate Statistical Analysis 2025 (4th–6th November 2025) and the Aleksander Zeliaś International Conference (11th–14th May 2026), as well as other international events: the IFCS 2026 (15th–19th July 2026), ECDA 2026 (9th–11th September 2026), the German-Polish Seminar on Data Analysis (9th–11th September 2026), and CLADAG 2027. The SKAD members were encouraged to actively participate in these events.

The third item on the agenda related to the organization of future SKAD conferences. The members were informed that the negotiations concerning the SKAD 2026 conference were underway, with mid-September 2026 as the most likely date for the event to take place. Preliminary considerations for the organization of the SKAD 2027 meeting were also briefly mentioned, with details to be determined in due course.

The election of the SKAD representative to the IFCS Council for the 2026–2029 term was the fourth item on the agenda. A scrutinizing committee composed of Prof. Eugeniusz Gatnar and Prof. Marcin Pełka was appointed. Voting was carried out among the eligible participants, ensuring a transparent and orderly procedure, consistent with the SKAD rules.

The results of the election were subsequently announced. A total of 18 votes were cast, all valid. The candidate, Prof. Andrzej Dudek, received 17 votes in favour, 1 abstention, and no votes against, thus becoming elected as the SKAD representative to the IFCS Council for the 2026–2029 term. The Chair thanked the members for their trust and expressed readiness to continue supporting international collaboration and the strategic presence of SKAD within the IFCS structures.

In the final part of the meeting, participants were invited to submit additional remarks and proposals. Appreciation was expressed for the organizational efforts related to the conference and the functioning of the section. The Chair concluded the meeting by thanking all the attendees for their active participation and constructive contributions, reaffirming the shared commitment to advancing research in classification and data analysis. Prof. Andrzej Dudek and Prof. Jacek Białek (University of Lodz) invited the members to the next SKAD conference, which will be held in Łódź or Bełchatów in September 2026.