

# Risk mitigation in a volatile US equity market: A comparative analysis of hedging with index futures and investing in gold as a safe haven

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**Abstract.** The aim of this paper is to compare two alternative strategies for protecting a US stock market portfolio against market risk during four major stock market crashes: the Global Financial Crisis, the European Sovereign Debt Crisis, the COVID-19 pandemic and the Russia-Ukraine war. Hedging with the S&P 500 index futures is compared with investing in gold as a safe haven based on the risk minimisation criterion. The effectiveness of the protection strategies is verified for portfolios that differ in terms of the number of constituents which range from single-asset to well-diversified portfolios. The results vary depending on the specific crisis, portfolio size and time horizon. However, hedging with index futures tended to provide more effective long-term protection, particularly for larger portfolios.

**Keywords:** hedging, safe haven, futures, gold, portfolio

**JEL:** G11, G13, C58

## 1. Introduction

In the last 20 years, four major events have shaken global financial markets. The most significant was the Global Financial Crisis (GFC) of 2007–2009, triggered by subprime lending, inflated housing prices and poorly regulated mortgage-backed securities. The bankruptcy of the Lehman Brothers on 15th September 2008 was the most dramatic moment, pulling financial markets down. The European Sovereign Debt Crisis (ESDC) was a multi-year financial crisis that began after the GFC, affecting several Eurozone member states. It was triggered by a combination of high government debt, vulnerable banking systems and weak economic growth, particularly in countries such as Greece, Ireland, Italy, Portugal, and Spain. In early August 2011, global financial markets sharply declined due to the growing fears of a Greek default and contagion throughout the Eurozone. A more recent crisis was caused by the COVID-19 pandemic, which was officially declared as such by the World Health Organization on 11th March 2020. It led to an unprecedented global economic shock. Widespread lockdowns, supply chain disruptions

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and a collapse in consumer demand triggered a global recession. The fourth crisis began with Russia's invasion of Ukraine on 24th February 2022, causing further economic disruption across Europe. The conflict resulted in a major energy crisis, involving spikes in gas, electricity and crop prices, which accelerated inflation.

Portfolio diversification aims to reduce risk by investing in uncorrelated assets. However, it cannot eliminate systematic risk, which affects the entire market. During market crashes, both volatility and correlations between assets tend to increase (Sandoval & Franca, 2012). Thus, diversification weakens when most needed, prompting investors to shift from risky assets like stocks and corporate bonds to safe havens. According to Baur and Lucey (2010), safe-haven assets are uncorrelated or negatively correlated with the reference asset in times of a market crash.

Based on the Modern Portfolio Theory, such investments allow investors to reduce the overall risk of their portfolios. A wide range of instruments have been analysed in the recent literature, including precious metals (Baur & Lucey, 2010; Baur & McDermott, 2010; Beckmann et al., 2015; Boubaker et al., 2020; Echaust & Just, 2022), bonds (Drobetz et al., 2020; Pisedtasalasai, 2021), oil (Batten et al., 2021; Disli et al., 2021), agricultural commodities (Ali et al., 2020), foreign exchange rates (Cho & Han, 2021; Dong et al., 2021; Siemaszkiewicz, 2023), cryptocurrencies (Będowska-Sójka & Kliber, 2021; Conlon & McGee, 2020; Goodell & Goutte, 2021; Just & Echaust, 2024; Kliber et al., 2019; Mizerka et al., 2020), and other non-traditional assets (Siemaszkiewicz & Let, 2020). Among them, gold is perceived as the traditional and most efficient safe haven.

A large body of literature emphasises the role of gold as a safe-haven asset. Baur and McDermott (2010) examine the safe-haven properties of gold for equities across 53 emerging and developed countries over a 30-year period (1979–2009). They show evidence that gold is a strong-form safe haven for most major developed stock markets. Boubaker et al. (2020) provide long-run evidence, showing that gold functions as a hedge against risk over a period of more than seven centuries, depending on the model specification. Similarly, Klein (2017) finds that gold and silver act as safe havens in developed markets, although their effectiveness weakens after 2013. Beckmann et al. (2015) show that gold's safe-haven property is market-specific across 18 stock markets and five regional indices, likely reflecting differences in market structure and capital flows. Focusing on crisis episodes, Dong et al. (2021) and Ji et al. (2020) demonstrate that gold can protect portfolios against extreme equity losses, particularly during the GFC and the COVID-19 pandemic. Banerjee and Pradhan (2024) confirm the similar safe-haven behaviour of gold for U.S. equities using high-frequency data during COVID-19. In emerging markets, Wen and Cheng (2018) also support the safe-haven role of this commodity

using copula methods, while Ryan et al. (2024) show that gold should be used as a safe haven against S&P 500 risk during periods of macroeconomic uncertainty.

An alternative investment strategy used to protect portfolios against losses during market crashes is hedging with derivatives. It allows investors to temporarily offset portfolio losses with profits generated from these derivatives. The use of linear derivatives such as forwards or futures requires, unlike safe-haven instruments, a high correlation between these contracts and the underlying risk exposure. Correlation is therefore a key factor in determining whether a particular asset is suitable for a given hedging strategy, as periods of financial distress not only disrupt the functioning of financial markets through the transmission of shocks but also significantly increase market interdependence (Forbes & Rigobon, 2002). This heightened interdependence may amplify crises, disrupting both financial markets and real economic activity. This proves that accounting for market interconnections is crucial to understanding financial market behaviour (Faldziński & Pietrzak, 2015).

Numerous studies have examined the same protective assets – predominantly gold, oil and cryptocurrencies – against stock market risk, considering their roles in long positions as safe havens as well as in short positions as hedging instruments. However, these two strategies require assets with fundamentally different characteristics. While instruments like gold, oil or cryptocurrencies may serve as safe havens, they are inappropriate for a hedging strategy against stock market risk. Echaust et al. (2024) show that these instruments cannot compete with index futures in a hedging role. We aim to compare the effectiveness of both strategies within the minimum variance framework in the context of four major market crashes: the Global Financial Crisis, the European Sovereign Debt Crisis, the COVID-19 pandemic, and the Russia–Ukraine war. Unlike previous studies, we compare gold as a safe-haven asset with S&P 500 futures as a hedging instrument against the risk exposure of equity portfolios of varying sizes. Although the safe-haven strategy may be perceived as offering longer-term protection than hedging, Baur and Lucey (2010) showed that in practice gold is a safe haven only in the short run. Safe havens have been one of the most extensively explored subjects in the financial literature in recent years (Anas et al., 2024); therefore, it appears both reasonable and indeed necessary to compare these two key risk-mitigation strategies.

We contribute to the existing literature in two ways. First, we compare a safe-haven strategy in the spirit of Baur and Lucey (2010) with a hedging strategy. We evaluate their effectiveness in the U.S. stock market during four global stock market crashes. Our study considers gold as a safe haven and S&P 500 futures as a hedging instrument against stock portfolio risk. These assets are likely to be the first choice for many investors, which makes our study highly relevant from a practical perspective. Second, unlike most existing studies, we examine the protection of

stock portfolios that vary in the number of constituents, rather than using broad market indices. Limiting the analysis to market indices would place the hedging strategy in a favourable position, as it would result in near-perfect hedging. However, small equity portfolios typically show lower correlation with index futures, which limits the effectiveness of hedging.

## 2. Methods

### 2.1. The choice of a stock portfolio

Let us assume that an investor on the brink of a financial crash (the last day of the portfolio construction period shown in Figure 1) holds a stock portfolio ranging from a single stock to a fully diversified one including all S&P 500 constituents. The stocks in the portfolio are selected based on a variance minimisation criterion calculated from a one-year sample, with short selling restricted. This assumption ensures consistency in our subsequent analysis of portfolio protection, as both the hedging ratio and the optimal weights of the safe-haven asset are determined using the same criterion. Furthermore, we consider two approaches to portfolio construction: the first is the minimum variance portfolio according to Markowitz (1952), and the second is based on equal-weighting.

### 2.2. Investing in a safe haven

Let us assume that the investor holds a long stock portfolio position chosen according to Subsection 2.1. and a long position in a safe haven. The overall portfolio (after adding a safe-haven asset to the stock portfolio) return  $r_{P,t}$ , at time  $t$  is

$$r_{P,t} = w_{S,t} \cdot r_{S,t} + w_{SH,t} \cdot r_{SH,t}, \quad (1)$$

where  $r_{S,t}$  and  $r_{SH,t}$  denote return on the stock portfolio and the safe haven, respectively and  $w_{S,t}$  and  $w_{SH,t}$  denote their weights in a portfolio, such that  $w_{S,t} + w_{SH,t} = 1$ .

The formula for an optimal (minimum variance) weight of safe-haven asset is given by

$$w_{SH,t} = \begin{cases} 0 & \text{for } w_{SH,t}^* \leq 0 \\ w_{SH,t}^* & \text{for } 0 < w_{SH,t}^* \leq 1, \\ 1 & \text{for } w_{SH,t}^* > 1 \end{cases} \quad (2)$$

where

$$W_{SH,t}^* = \frac{\sigma_{S,t}^2 - Cov(r_{S,t}, r_{SH,t})}{\sigma_{SH,t}^2 + \sigma_{S,t}^2 - 2 \cdot Cov(r_{S,t}, r_{SH,t})}, \quad (3)$$

where  $\sigma_{S,t}^2$ ,  $\sigma_{SH,t}^2$  denote variances of the stock portfolio and the safe haven, respectively and  $Cov(r_{S,t}, r_{SH,t})$  denotes the covariance between the stock portfolio and safe haven returns.

### 2.3. Short hedging with futures

Let us assume that the investor holds a long stock portfolio position chosen according to Subsection 2.1. and adopts a short futures position to hedge the portfolio. The overall portfolio (after hedging the stock portfolio with a futures contract) return  $r_{P,t}$ , at time  $t$  is

$$r_{P,t} = r_{S,t} - h_t \cdot r_{F,t}, \quad (4)$$

where  $r_{S,t}$  and  $r_{F,t}$  denote returns on the stock portfolio and the futures, respectively, and  $h_t$  denotes the hedge ratio. The minimum variance optimal hedge ratio at time  $t$  is as follows:

$$h_t = \frac{Cov(r_{S,t}, r_{F,t})}{\sigma_{F,t}^2}. \quad (5)$$

### 2.4. Effectiveness of the protection strategy

In order to compare the performance of the chosen protection strategy, we employ a well-known performance measure for the variance minimisation problem, designed to evaluate the effectiveness of a hedging strategy (Ederington, 1979), namely:

$$Effectiveness = 1 - \frac{\text{Variance of a protected portfolio}}{\text{Variance of a stock portfolio}}. \quad (6)$$

This ratio reflects how well a protective strategy minimises the variance (in percentage terms) of a stock portfolio. The ratio of 0 means the protection strategy provides no risk reduction, while the ratio of 1 indicates perfect effectiveness, fully reducing the variance of the underlying exposure.

### 3. Data

The data were obtained from the Refinitiv Eikon database and include stock prices, E-mini S&P 500 Index Futures (ESc1) and gold spot prices quoted in U.S. dollars per ounce (XAU=). The sample includes S&P 500 constituents with complete price histories, defined separately at the onset of each crisis. For each crisis, the portfolio construction period is defined as the one-year interval immediately preceding the crisis onset, while the portfolio evaluation begins on the crisis onset date (15th September 2008, 1st August 2011, 11th March 2020 and 24th February 2022). The evaluation will be conducted over four different horizons: weekly, monthly, semi-annual and annual. The details are provided in Table 1 and Figure 1.

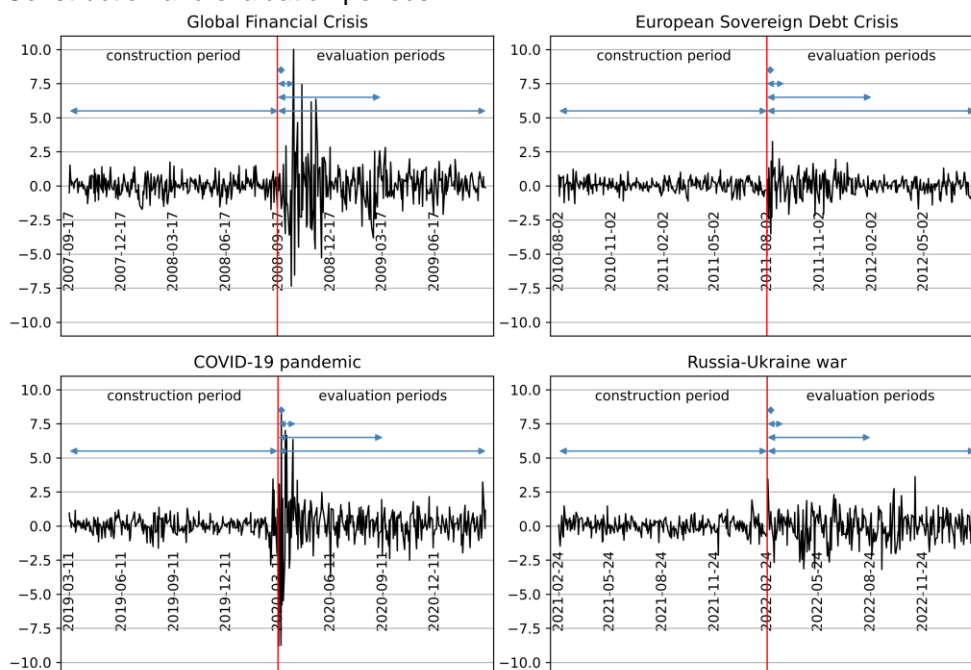
**Table 1.** Portfolio construction and evaluation periods

Crisis	Construction period	Full evaluation period
Global Financial Crisis	15-09-2007 – 14-09-2008	15-09-2008 – 14-09-2009
European Sovereign Debt Crisis	01-08-2010 – 31-07-2011	01-08-2011 – 31-07-2012
COVID-19 pandemic	11-03-2019 – 10-03-2020	11-03-2020 – 10-03-2021
Russia-Ukraine war	24-02-2021 – 23-02-2022	24-02-2022 – 23-02-2023

Source: authors' work.

Figure 1 clearly shows that the assumed crash start dates separate the period of low volatility (the portfolio construction period) prior to the outbreak of the crash from the subsequent increase in volatility during the evaluation periods. The most spectacular increase in volatility is observed during the periods of the GFC and COVID-19 pandemic.

**Figure 1.** Construction and evaluation periods



Note. The figure illustrates returns of the Markowitz portfolio comprising 10 constituents.  
Source: authors' work.

## 4. Empirical study

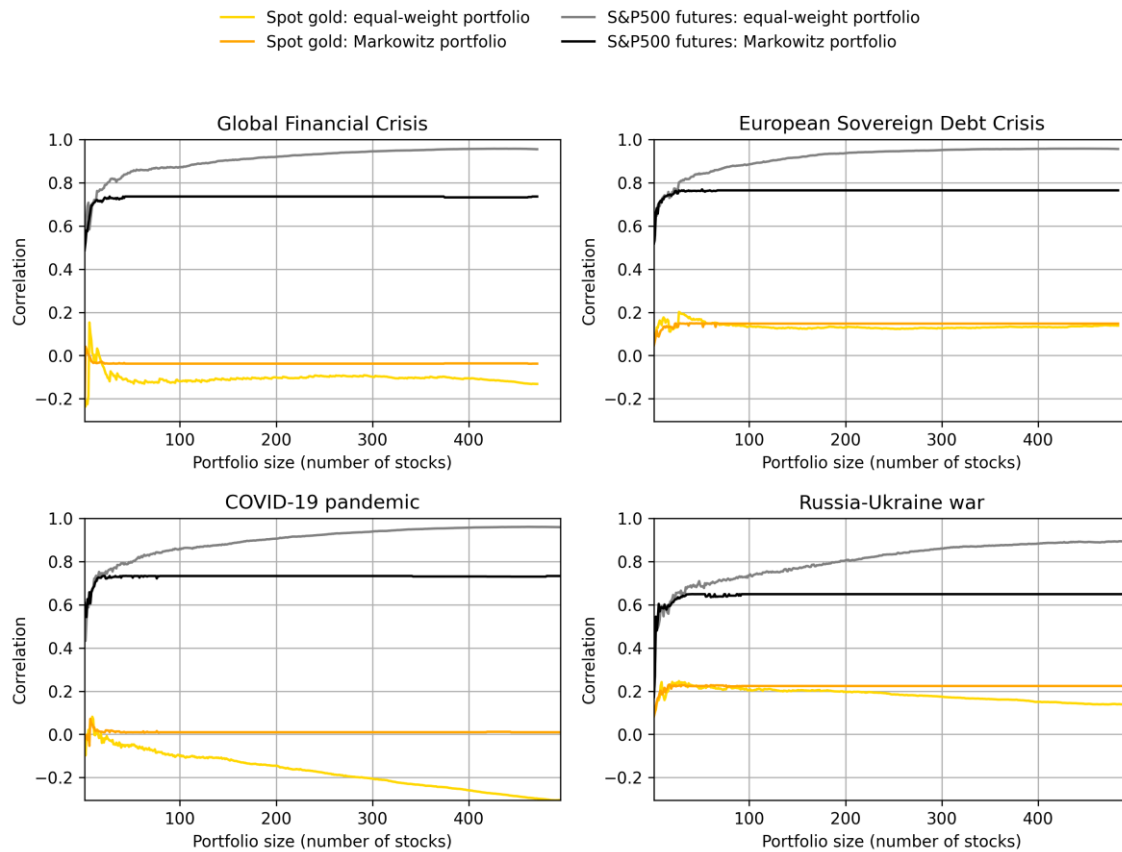
This section focuses on the construction phase, analysing such key factors as correlation and variance used in the static approach. Weights and hedge ratios are calculated on the last day of the construction period and remain fixed throughout the evaluation phase.

### 4.1. Correlations, variances, weights and hedge ratios during portfolio construction phases

Figure 2 shows the correlations between stock portfolios of different sizes and index futures or gold during construction phases and Figure 3 shows variances of stock portfolio, futures, and gold. The correlation between stock portfolios and gold is expected to be close to zero or negative, whereas the correlation between stock portfolios and index futures should be close to one for effective hedging. Correlations with Markowitz portfolios tend to stabilise when the number of assets exceeds a few dozen constituents, as beyond a certain portfolio size, the optimisation algorithm adds new stocks but assigns them zero weight. For equally-weighted portfolios, correlations with gold decrease, while correlations with index futures increase along with the number of constituents.

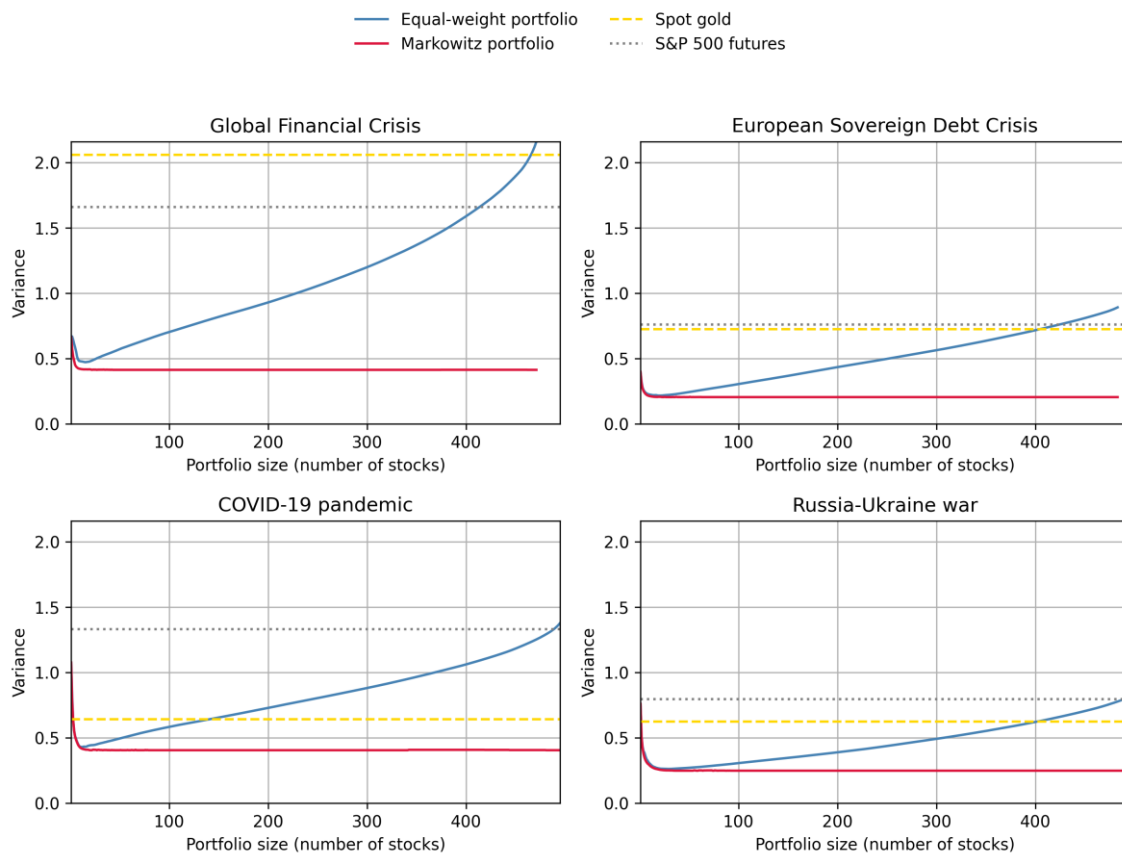
Markowitz portfolios consistently exhibit lower variance than equally-weighted portfolios (see Figure 3). The variance of a Markowitz portfolio consisting of several stocks becomes nearly constant. This supports the findings of Elton and Gruber (1977) and Eom et al. (2021) that a portfolio should consist of maximum 20–50 stocks to significantly reduce the unsystematic risk through diversification. To further reduce variance, the portfolios will be combined with a safe-haven asset in a long position or an index futures contract in a short position.

**Figure 2.** Correlations between stock portfolios and gold, and between stock portfolios and index futures during the construction period



Source: authors' work

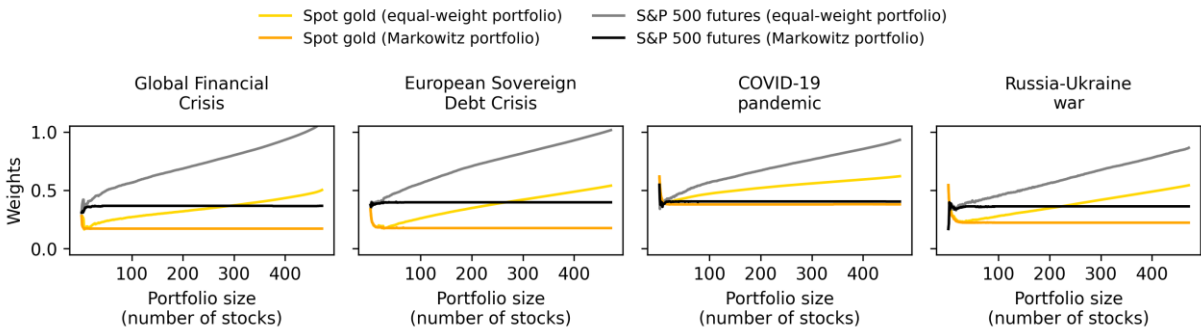
**Figure 3.** Variances of stock portfolios, gold, and S&P 500 futures across the construction phases



Source: authors' work.

Figure 4 presents the asset weights in portfolios composed of a stock portfolio and a protection asset. For the Markowitz portfolio, the compositions are fairly stable, except for small portfolios. Equally-weighted portfolios, on the other hand, change with each added stock. The weight of gold in an equally-weighted portfolio reaches minimum for a portfolio consisting of several stocks and then increases as the portfolio size expands. The hedging ratio for an equally-weighted portfolio systematically converges to one as the portfolio size increases, indicating full hedging. The highest gold weights are observed in the year preceding the onset of the COVID-19 pandemic. These results, on the one hand, from the lowest correlation between the constructed portfolios and gold (Figure 2) and, on the other hand, from the exceptionally low volatility of gold (Figure 3). Assuming these relationships persist throughout the evaluation period, gold may be expected to be the most effective safe haven during this crash period.

**Figure 4.** Portfolio weights (hedging ratios in the case of the hedging strategy) for the considered strategies



Source: authors' work.

**4.2. Effectiveness of protection strategies**

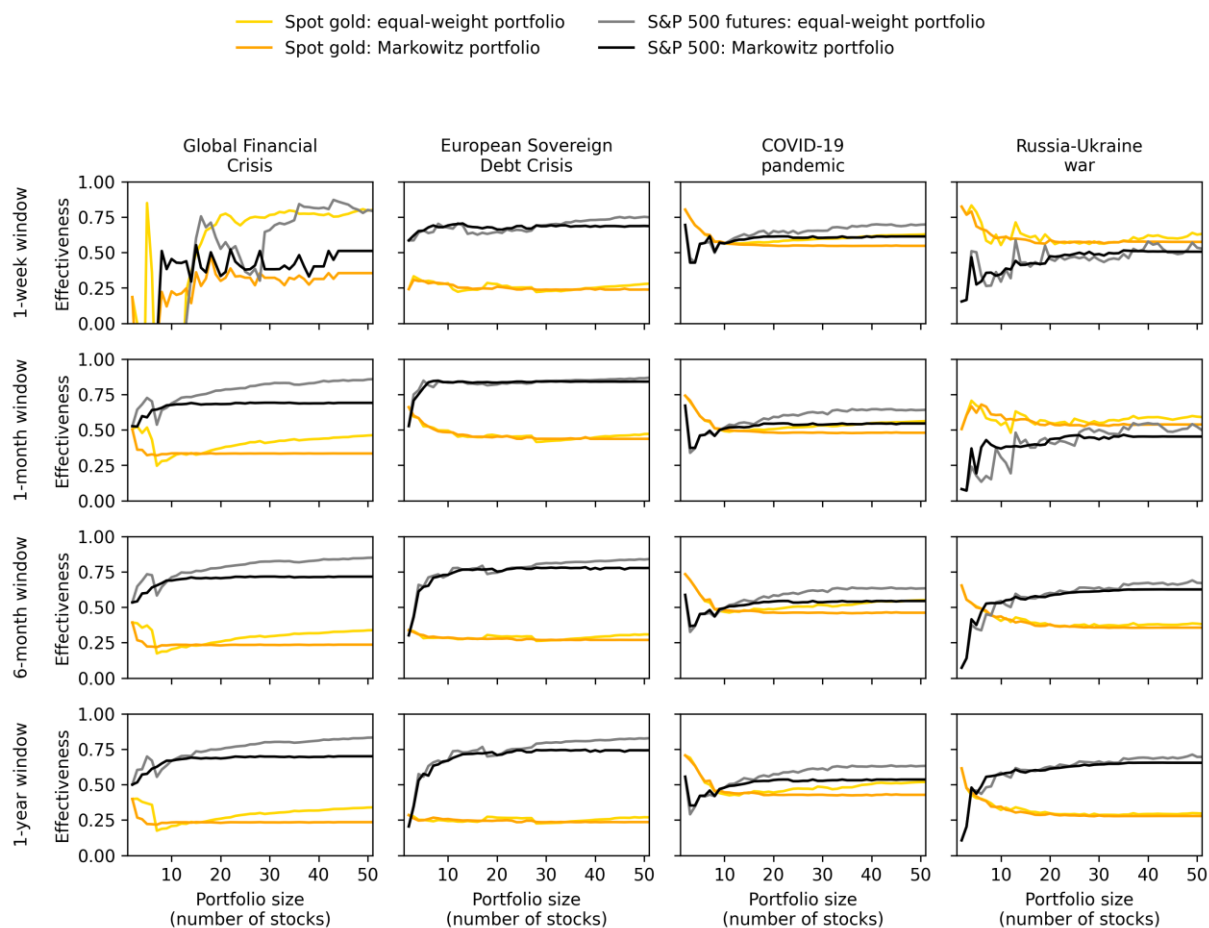
Given the predetermined portfolio allocations, we examine the out-of-sample performance of the protected portfolio across four time horizons: one week, one month, six months and one year, beginning at the onset of the market crash. The evaluation focuses on portfolios containing no more than 50 stocks, as clear trends form in this range and small portfolios exhibit the highest volatility.

The effectiveness shown in Figure 5 depends on the chosen portfolio construction method, the portfolio size, the tested crash and the time horizon of the analysis. For small portfolios, the effectiveness measure fluctuated considerably, especially at the beginning of the GFC. During its first week, gold proved to be an effective safe haven, in particular for an equal-weighted stock portfolio with at least a dozen constituents. Its effectiveness was on average 72% (for portfolios of 15-30 stocks), compared to around 55% for futures contract. In the other horizons

considered, the S&P 500 futures contract reduced portfolio volatility much more effectively, regardless of the portfolio's size. During the ESDC, hedging with futures definitely outperformed the safe-haven strategy with the effectiveness higher by 35%-57% (for portfolios of 11-50 stocks), regardless of the hedging horizon. During the COVID-19 pandemic, gold offered superior protection for small portfolios (up to nine constituents). For larger portfolios, hedging turned out as the more effective risk-mitigation strategy, although the differences between both approaches were considerably less pronounced than during the two preceding crises. The analysis of the last crash, caused by the Russia-Ukraine war, yields more ambiguous results. Gold provided better protection during the initial phase of the war (week and month). The effectiveness advantage of gold over futures was significantly higher (by 19%-67%) for portfolios of 1-10 stocks, but this advantage diminished as the size of the portfolio increased. In the long run, gold also outperformed futures for portfolios with a few constituents. However, for larger portfolios it was the opposite: the larger the portfolio, the greater the efficiency advantage in favour of the hedging strategy.

In all the cases, variance reduction was greater for equally-weighted portfolios than for Markowitz portfolios. This could be expected, as equally-weighted portfolios are typically characterised by higher variance. Furthermore, when hedging with index futures, the reduction in the variance increases along with the growing portfolio size. However, for safe-haven assets, the variance reduction mostly decreases as the portfolio grows. The most effective strategy is hedging an equally-weighted stock portfolio with index futures. This result is intuitive: as the number of stocks increases, the equally-weighted portfolio converges towards the S&P 500 index, which can then be effectively hedged with index futures. In contrast, the safe-haven strategy is found to be the least effective for the Markowitz portfolio among all strategies considered. A safe-haven strategy is essentially the diversification through a specific asset. For a well-diversified portfolio, gold is not able to further significantly enhance the diversification effect.

**Figure 5.** Effectiveness (static approach – 1-year construction window)



Note. The figure illustrates the effectiveness of the protection strategies calculated according to Equation (6).

Source: authors' work.

The results presented in Table 2 show the average effectiveness of the considered protection strategies for underlying portfolios consisting of 1–50 constituents. Comparisons across crises should be interpreted with some caution, as each of these periods was characterised by a different level of underlying risk. It is therefore more informative to examine effectiveness across evaluation horizons. Except for the weekly horizon, which is very short and produces highly varied results, a general pattern can be observed: as the protection horizon increases, in most cases the effectiveness of the protection strategies tends to decrease. This likely reflects market conditions changing over time, while the optimal strategy does not fully adjust to these dynamics, leading to a gradual loss of effectiveness.

**Table 2.** Average effectiveness of protection strategies.

Evaluation period	Crisis	Static approach				Dynamic approach			
		Gold EW	Sp500 EW	Gold M	SP500 M	Gold EW	SP500 EW	Gold M	SP500 M
1W	GFC	0.59	0.51	0.28	0.38	0.55	0.67	0.21	0.76
	ESDC	0.26	0.69	0.25	0.68	0.40	0.70	0.40	0.70
	COVID-19	0.61	0.64	0.57	0.60	0.93	0.71	0.92	0.63

	RUS_IN V	0.61	0.45	0.60	0.45	0.71	0.42	0.61	0.39
1M	GFC	0.40	0.78	0.34	0.67	0.60	0.85	0.59	0.81
	ESDC	0.47	0.83	0.46	0.83	0.66	0.88	0.65	0.87
	COVID- 19	0.54	0.59	0.50	0.53	0.77	0.56	0.76	0.48
	RUS_IN V	0.58	0.42	0.56	0.41	0.57	0.34	0.54	0.33
6M	GFC	0.29	0.78	0.24	0.70	0.43	0.83	0.41	0.80
	ESDC	0.29	0.77	0.28	0.74	0.35	0.78	0.33	0.75
	COVID- 19	0.52	0.58	0.48	0.52	0.73	0.56	0.72	0.48
	RUS_IN V	0.40	0.58	0.39	0.57	0.53	0.61	0.53	0.62
1Y	GFC	0.29	0.75	0.24	0.68	0.42	0.79	0.39	0.77
	ESDC	0.25	0.74	0.24	0.70	0.31	0.75	0.29	0.70
	COVID- 19	0.49	0.57	0.45	0.51	0.66	0.55	0.64	0.47
	RUS_IN V	0.32	0.61	0.31	0.60	0.42	0.69	0.42	0.69

Note. This table presents the average effectiveness of the analysed strategies. The left panel (static approach) shows the results for the strategies described in this section, while the right panel the results for the strategies described in Section 5.2. EW denotes equal-weighted portfolios, while M refers to Markowitz portfolios.

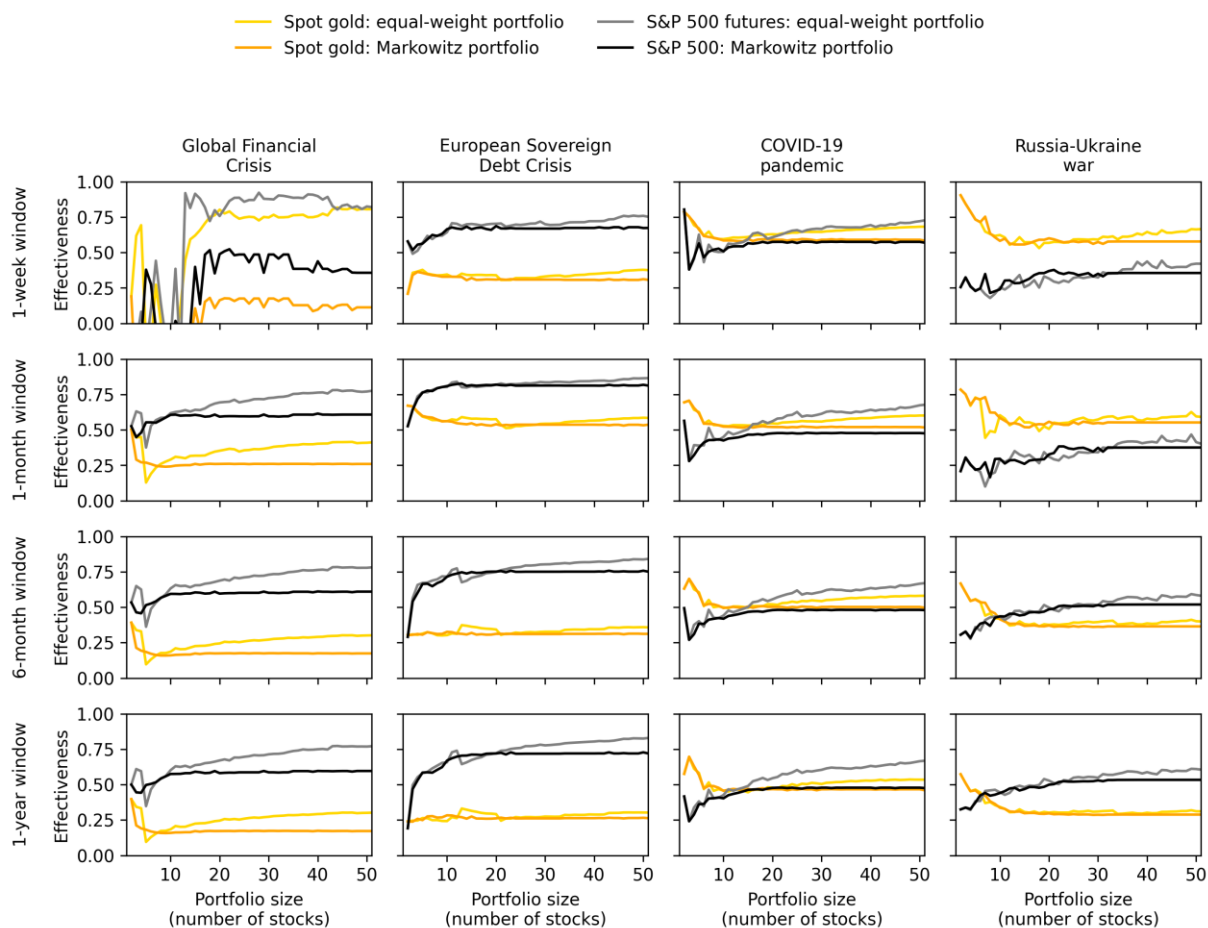
Source: authors' work.

## 5. Robustness check

### 5.1. Effectiveness of protection strategies: 6-month construction period

In this subsection, we present the results as in the baseline approach, replacing the annual construction period with a half-year period. The results of the protection effectiveness are presented in Figure 6. Overall, the findings remain largely unchanged, indicating that the results are robust to the length of the portfolio formation window. The relative effectiveness of hedging and safe-haven strategies across different crashes, portfolio sizes, and investment horizons remains virtually unchanged compared to the baseline specification, with only negligible differences observed.

**Figure 6.** Effectiveness (static approach – 6-month construction window)



Note. The figure illustrates the effectiveness of the protection strategies calculated according to Equation (6).

Source: authors' work.

## 5.2. Effectiveness of protection strategies – dynamic approach

In this subsection, we extend the baseline analysis by allowing the protection strategy to be rebalanced on a daily basis. On the one hand, this approach does not restrict the evaluation of the strategy's effectiveness to only two dates, thereby enabling investors to adjust their decisions in a more flexible and potentially more efficient manner. On the other hand, more frequent rebalancing generates higher expenses and may reduce investment profitability (Latoszek & Ślepaczuk, 2020). While in the baseline approach transaction costs have only a limited impact on the effectiveness of the strategy, as they occur only once at the initial stage, daily rebalancing would substantially increase their importance. Nevertheless, in this part of the study we do not consider transaction costs either. Transaction costs in the futures market are typically charged on a per-contract basis rather than as a percentage of transaction value, as is commonly the case in equity markets. Accounting for such costs would therefore require additional assumptions regarding the portfolio size.

We compare the effectiveness of both strategies using a dynamic approach based on the Engle (2002) DCC model. Model-driven futures-hedging strategies are not explicitly addressed in this

study. However, related evidence described in Michańkó et al. (2023) shows that the effectiveness of hedging depends to a large extent on the choice of signal-generation methods within algorithmic investment strategies. Their results indicate that model-based forecasts can materially affect diversification and hedging performance, particularly in turbulent markets. However, incorporating such model-specific elements introduces a range of additional issues that move the analysis away from the generality of the presented results.

Let us denote a two-dimensional vector by  $\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \epsilon_{2,t})'$ . The DCC model assumes that:

$$\boldsymbol{\epsilon}_t | \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H}_t), \mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \quad (7)$$

where  $\mathbf{D}_t = \text{diag}(\sqrt{h_{11,t}}, \sqrt{h_{22,t}})$  and conditional variance  $h_{ii,t}$  is modelled using the GARCH-type model. The conditional correlation matrix  $\mathbf{R}_t$  is expressed by

$$\mathbf{R}_t = (\text{diag}(\mathbf{Q}_t))^{-\frac{1}{2}} \mathbf{Q}_t (\text{diag}(\mathbf{Q}_t))^{-\frac{1}{2}}, \quad (8)$$

with

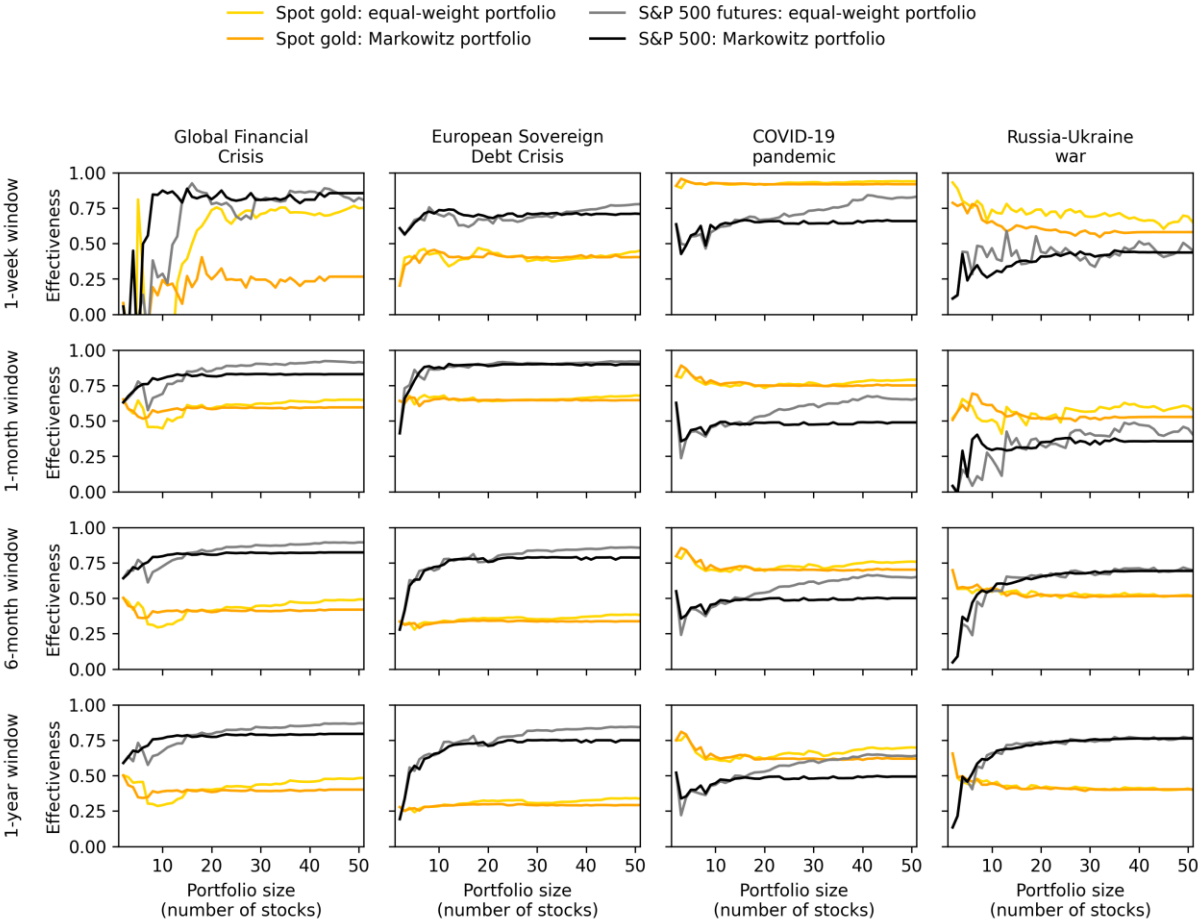
$$\mathbf{Q}_t = (1 - a - b) \mathbf{Q}^* + a \mathbf{z}_{t-1} \mathbf{z}'_{t-1} + b \mathbf{Q}_{t-1}, \quad (9)$$

where  $\mathbf{Q}^*$  is the unconditional covariance matrix of  $\mathbf{z}_t$  ( $z_{i,t} = \epsilon_{i,t} / \sqrt{h_{ii,t}}$ ), and  $a, b$  are parameters such that  $a, b \geq 0$  and  $a + b < 1$ .

We use the standard GARCH(1,1) approach proposed by Bollerslev (1986) with Gaussian innovations to model the conditional volatility. We extend the estimation window to two years to provide a sufficient amount of data for this model (Hafner & Reznikova, 2012). For the starting sample (two years prior to the onset of each crash), we estimate the parameters of the model and compute one-day-ahead forecasts of the conditional covariance matrix, which are necessary for calculating the hedge ratios and the weights of gold. Subsequently, the estimation sample is updated by including a new observation and removing the oldest one. Next, we re-estimate the model and generate forecasts using the updated estimation sample. This algorithm is repeated iteratively until forecasts are obtained for the last day of the evaluation window. The entire procedure enables us to compute the forecast of returns of the protected portfolio. Similarly to the baseline approach, portfolio variances and, consequently, effectiveness are computed using the same weekly, monthly, semi-annual and annual horizons starting with the crash. The final results are presented in Figure 7. The effectiveness shown in these plots behaves

in the same way as in the static approach during the first two crises and the last one. The hedging strategy outperforms the safe-haven strategy, with minor exceptions already discussed in Subsection 4.2. During the COVID-19 pandemic, investing in gold outperformed the hedging strategy in all cases. However, the differences in the efficiency between the strategies became less pronounced as the number of portfolio constituents was growing.

**Figure 7.** Effectiveness (dynamic approach)



Note. This figure illustrates the effectiveness of the protection strategies calculated according to Equation (6).

Source: authors' work.

The results presented in the right panel of Table 2 for the dynamic strategy are similar to those obtained for the static strategy. Excluding the weekly horizon, a longer protection horizon is associated with a gradual decline in effectiveness. However, when comparing the static and dynamic approaches, the latter exhibits higher effectiveness in the majority of cases. Rebalancing the optimal protection strategy is associated with a greater ability to follow market trends and to adjust the investment allocation to changing market conditions. However, in practice such a strategy would also generate transaction costs, which in turn would reduce its net effectiveness.

### 5.3. Maximum drawdown

Although variance is the most widely used risk measure in the context of this study, it represents only the volatility dimension of the overall risk. Therefore, in this section we analyse the level of risk reduction, understood as the portfolio loss mitigated through the implementation of a protection strategy. Maximum drawdown (MDD) was selected as the risk measure, as it represents the largest peak-to-trough decline in portfolio value over a given period. In practice, this measure represents the worst-case scenario for an investment over a specific time horizon:

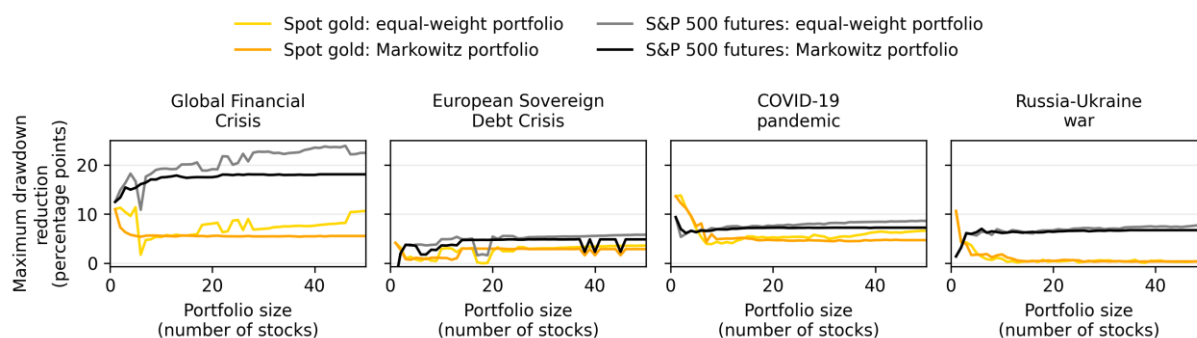
$$\text{MDD} = \frac{V_{\text{peak}} - V_{\text{trough}}}{V_{\text{peak}}}. \quad (10)$$

In our study, peak value,  $V_{\text{peak}}$ , corresponds to the last day of the construction period, while the specific time horizon refers to the full evaluation period, namely one year. We introduce the effectiveness of the protection strategy in the following way:

$$\text{Effectiveness} = \text{MDD of a stock portfolio} - \text{MDD of a protected portfolio}. \quad (11)$$

The final results showing the effectiveness of the protection strategies are shown in Figure 8. The maximum drawdown duration (the number of days from peak to trough) differed only slightly across the portfolios, regardless of the portfolio size. For the unprotected portfolios, the most frequent (modal) maximum drawdown duration was 171 days during the GFC, 9 days during the ESDC, 12 days during COVID-19, and 163 days during the war. The plots presented in this figure strongly resemble those in Figure 5 for variance. The similarity is particularly visible for the semi-annual/annual periods during the GFC and the war, as well as for the weekly/monthly periods during COVID-19, which corresponds to the mode of the MDD duration. Regardless of the type of crisis, hedging effectiveness exceeded that of investing in gold for portfolios containing at least several assets. During the ESDC, hedging with index futures generally outperformed the safe-haven strategy, although the differences between the two were less pronounced than under the variance-minimisation criterion.

**Figure 8.** Effectiveness measured using the maximum drawdown metric



Note. The figure illustrates the effectiveness of the protection strategies calculated according to Equation (11).

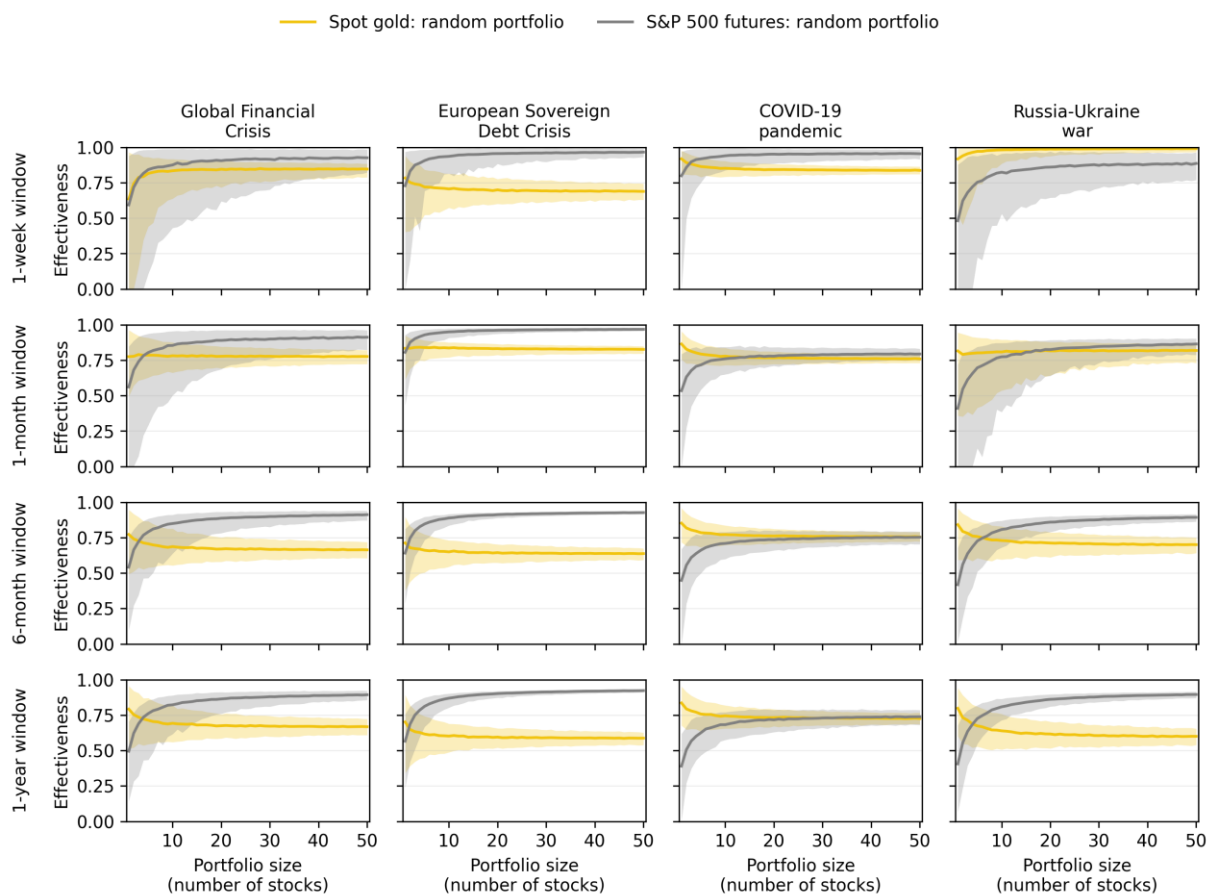
Source: authors' work.

## 5.4. Random portfolios

The main objective of this study is to compare the effectiveness of hedging strategies with investment in a safe-haven asset. The vast majority of previous studies address this problem from the perspective of variance minimisation; therefore, equity portfolios are constructed following the same criterion. In this subsection, we compare the effectiveness of these strategies relative to equally-weighted portfolios generated using Monte Carlo simulations. Portfolios of varying sizes were randomly selected from all constituents of the S&P 500 index and their risk is mitigated using either index futures or gold. This process is repeated 1,000 times to ensure robust statistical results. The results are indicated in Figure 9 as shaded bands representing the 95%-confidence interval, with the central line denoting the median.

Overall, the shape of the plots closely resembles those obtained previously for minimum variance portfolios, suggesting that the observed relationships are robust to the portfolio construction method. As shown in Figure 9, the confidence intervals for effectiveness do not overlap for the semi-annual and annual evaluation horizons and for larger portfolios, indicating that the difference between the two protection strategies is statistically significant. Hedging effectiveness increases as the number of assets in the portfolio grows, while the effectiveness of gold as a safe haven gradually declines. The exception is the pandemic period, where the differences between the effectiveness of the two strategies are not statistically significant (the confidence intervals overlap), except for the weekly horizon. In contrast, during ESDC, hedging outperforms investing in a safe haven in all evaluation periods for portfolios containing more than a few stocks. An important finding is that there is no case, at the 5% significance level, in which investing in gold is more effective for risk mitigation than hedging with futures contracts.

**Figure 9.** Effectiveness of random portfolios



Note. This figure illustrates the effectiveness of the protection strategies calculated according to Equation (6).

Source: authors' work.

## 6. Conclusions

This study compares the effectiveness of a hedging strategy relying on S&P 500 index futures with a safe-haven strategy based on gold, according to the definition of a safe haven proposed by Baur and Lucey (2010). Therefore, risk minimisation is the only criterion common to all of the considered strategies.

The empirical results indicate that hedging with index futures generally provided more effective protection, particularly for larger and equally-weighted portfolios. Gold as a safe-haven outperformed hedging with futures only for small portfolios and over short time horizons, particularly in the early stages of the GFC and during the Russia–Ukraine war; however, this result is not statistically significant. In general, if risk minimisation is the only criterion considered by investors, hedging with index futures is recommended as a more effective strategy. This result is particularly robust over longer horizons (six months to a year) and becomes stronger as the level of diversification increases. The evidence remains largely consistent across the range of the conducted robustness tests and carries important implications

for investors and portfolio managers. These findings align with the traditional hedging theory, which states that derivative instruments such as index futures provide direct and efficient risk reduction due to their strong linkage with the underlying asset and their ability to precisely replicate market exposure. The results also suggest that theoretical definitions based solely on correlation may be insufficient, as fail to fully capture the economic effectiveness of hedging strategies under realistic portfolio constraints and they do not reflect investors' true expectations toward safe-haven assets. Analysing Bitcoin's ability to serve as a safe haven, Baur et al. (2022) highlight the limitations of a correlation-based definition by showing that extreme volatility can destroy its hedging properties even when the correlation is negative. Conlon and McGee (2020) confirm this finding, showing that including highly volatile instruments in stock portfolios increases downside risk exposure despite their negative correlation with equities. Incorporating potential profits and evaluating the utility of a protective strategy may better reflect investors' expectations, in which case the assessment presented in this study would likely be different. It is worth noting that in the recent literature, in addition to the traditional approach based on risk minimisation, an alternative definition of a safe-haven asset has emerged, which is based on the maximisation of the expected utility in the context of the prospect theory (Echaust et al., 2026). This perspective will constitute an important direction for our further research.

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