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Peaks over Threshold Approach with a time-varying scale parameter and range-based volatility estimator for Value-at-Risk and Expected Shortfall estimation

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Abstract. Exploiting daily high-low range has become increasingly popular among volatility models due to valuable information about volatility dynamics. It has been shown in the literature that range-based volatility estimators can improve volatility and covariance forecasts, and thus models that use high and low prices can outperform standard volatility models solely based on closing prices. This paper incorporates a range-based volatility estimator in an extreme value theory framework to provide better estimates of the tails of daily asset returns. We introduce the Peaks over Threshold model with a range-based volatility estimator depicting the volatility of extreme returns that can contribute to more accurate tail risk estimation. We evaluate the proposed model based on the Monte Carlo simulation and long-period sample of the empirical financial time series by forecasting the Value-at-Risk and Expected Shortfall. We provide evidence that the proposed model can lead to better risk measure forecasts, especially for high tail probabilities.
Keywords: GARCH, Value-at-Risk, Expected Shortfall, Peaks over Threshold, Extreme Value Theory
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1. Introduction

Volatility plays an important role in many areas of economics and finance, where there are countless models and methods of estimating volatility. This topic still attracts many researchers, who want to find new ways of describing volatility, to better understand its behaviour, and to be able to leverage that in practice. The GARCH model is the most popular time-varying volatility model introduced by Engle (1982) and Bollerslev (1986). The GARCH models are

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formulated solely on closing prices, whereas more accurate estimates of variance can be constructed from daily low and high prices (Parkinson, 1980). The use of high and low prices and volatility estimators constructed on the basis of the range of a maximum and minimum prices provided more accurate volatility models (see, e.g., Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fiszeder & Perczak, 2016; Fiszeder et al., 2023a, 2023b; Molnár, 2016; Xie, 2019). Daily low and high prices are almost always commonly available with closing prices for financial time series, therefore their application in volatility models is important from a practical point of view, and in most cases is relatively easy to implement. The application of such prices has also economic consequences (see Chou & Liu, 2010; Wu & Liang, 2011). All in all, the literature showing that range-based volatility models outperform models based on closing prices has recently been gaining popularity and expanding (see the reviews in Chou et al.; 2015; Fałdziński et al., 2023; Petropoulos et al., 2022).

Extreme quantile estimation has been one of the main focuses of risk management for researchers and financial institutions, especially in the aftermath of the 2008 financial crisis. Effective risk forecasting plays a role of immense importance, not only in meeting regulatory requirements, but also to providing optimal capital allocation and investment decisions. For this purpose, several risk measures have been introduced that require extreme quantiles estimation, specifically in the left tail of the return distribution. It turns out that Value-at-risk (VaR) and Expected Shortfall (ES) are two of the most widely used risk measures in quantitative risk management. Many different VaR and ES forecasting models and methods have been proposed in the literature. They can be divided into four main groups: parametric, non-parametric, semi-parametric and hybrid (see overviews for VaR in Abad et al., 2014; Nieto & Ruiz, 2016). Standard parametric methods that use an entire dataset for the estimation of the returns distribution are not the best choice for high-quantile estimation. In such cases, a model is fitted to the data better where most of the data points reside, and not surprisingly, it is the mid-regions of the distribution. On the other hand, for risk measures, we focus specifically on the extreme quantiles where there are a few observations, so we need more specialised approaches.

The extreme value theory (EVT) is a probabilistic theory with the principal role of describing extreme observations and providing models and methods built specifically for such extraordinary observations and their dynamics. This theory focuses on the tails of the distribution by taking advantage of the limiting laws of extremes. The EVT has been applied to many areas in finance (see an overview in Candia & Herrera, 2024; Echaust & Just, 2020a, 2020b; Herrera & Clements, 2020; Herrera & Schipp, 2013; Rocco, 2014), but its prevailing purpose is extreme quantiles estimation, as it is well suited to estimating and predicting the tails of the distribution, thus being a natural candidate for VaR and ES estimation.

Fisher and Tippett (1928) and Gnedenko (1943) proved that the distribution of the extreme values that are i.i.d.[1] for an unknown cumulative distribution function F converges to a Generalized Extreme Value (GEV) distribution that comprises three distributions. Interestingly, the type of asymptotic distribution of extreme values does not entirely depend on the exact cumulative distribution function F. This major advantage of the EVT enables us, in a way, to 'neglect' the exact form of F.

Another reason why EVT-based models and methods can be more accurate in estimating tail-risk measures is that each tail of the distribution is estimated independently, hence being more flexible and taking into account possible skewness of the data[2]. The main criticism of the EVT, however, stems from the fact that the underlying probabilistic theory holds for i.i.d. samples, whereas financial time series are time-dependent. A naive application of the EVT to the raw time series of returns tends to produce poor estimates of the VaR and ES (see, for instance, Chavez-Demoulin et al., 2014). Consequently, there are two main approaches to modelling the tails of the time-varying conditional return distribution in the literature. First, we focus on an EVT-based model for standardised residuals, where the conditional mean and the conditional volatility are described by some other model (mainly a volatility model) () – presented for instance in McNeil & Frey (2000). This approach assumes that a volatility model removes the time dependence of a time series rendering standardised residuals i.i.d. The second approach involves modelling the behavior of extreme values directly and taking into consideration the dependence structure of the data (see, for instance, Chavez-Demoulin et al., Bee et al., 2019; Bień-Barkowska, 2020; Bień-Barkowska, 2024; Chavez-Demoulin et al., 2005; Chavez-Demoulin et al., 2014; Tomlinson et al., 2024). This approach is commonly defined as the duration between consecutive extreme events, and it considers the magnitude of large losses occurring over a high threshold. Bień-Barkowska (2024) proposed a discrete-duration version of the autoregressive conditional duration peaks-over-threshold model, where duration between the extremes is treated as discrete. On the other hand, these approaches in most cases do not consider the possibility of time-varying parameters to capture short-term shocks during changing market conditions (see Fuentes et al., 2023). Attempts were made to overcome this

[1] independent and identically distributed.

[2] Skewness in financial time series is one of the properties that are exhibited in such data (see Hansen, 1994; Harvey & Siddique, 1999).

limitation by using a class of score-driven models introduced by Creal et al. (2013), which have become increasingly popular in recent years.

Researchers also tried to apply a score-driven model to extreme-events modelling. Massacci (2016) proposed a score-driven Generalized Pareto framework to model the magnitude of extremes using a one-factor model. Zhang and Schwaab (2016) criticized one-factor model as not justified empirically, and they introduced a score-driven framework based on two stages. Similarly, Bee et al. (2019) proposed a Peaks over Threshold approach based on realized measures obtained from intraday returns, including autoregressive terms using a score-driven framework. D'Innocenzo et al. (2024) also introduced a score-driven model with time-varying tail parameters, but with no pre-filtering for volatility. Lately, Fuentes et al. (2023) proposed a Marked Point Process model for extreme events with time-varying parameters, whose dynamics are functions of the observations through the score function of the predictive density and possibility to incorporate realized volatility measures. The use of realized volatility measures in the modelling framework has been gaining popularity in the literature recently (see, for instance; Bauwens & Xu, 2023; Bee et al., 2019; Yao et al., 2019). Empirical application of such approaches is limited, as it requires availability of intraday data, which is not common, and these type of data have other drawbacks (see for instance Fantazzini, 2011).

This paper introduces an extension of the first approach by incorporating information from volatility of extreme returns into the EVT-based model. The motivation behind such an approach is that time-varying volatility of returns is an intrinsic property of financial time series, hence also extreme observations exhibit time-varying volatility. Therefore, extreme observations are not heterogeneous from a time point of view, and taking into account extreme time-varying volatility in an EVT-based model should be beneficial for tail-risk measures. We propose a model that uses a standard GARCH model to describe the conditional mean and variance and the Generalized Pareto Distribution (GPD) with the Parkinson estimates of the magnitudes of threshold exceedances to describe the dynamics of extreme values (referred to as the GARCH-GPD-P further in the text).

We carry out the Monte Carlo simulation based on the stochastic volatility (SV) model and analyse how efficient the proposed model is for VaR and ES estimation compared to three benchmarks, i.e., the GARCH models with the normal (Gaussian) and t-distributed errors and the model proposed by McNeil and Frey (2000), i.e. the combination of the GARCH model and EVT-based Peaks over Threshold method with the GPD. Additionally, we perform an empirical analysis for a relatively large sample of stock indices, currencies and cryptocurrencies to study their usefulness in empirical cases.

The paper further consists of: Section 2, describing the applied models (i.e. GARCH-GPD and the newly-proposed GARCH-GPD-P), Section 3, which provides information on Value-at-Risk and the Expected Shortfall and their backtesting procedure, Section 4 that compares the GARCH-GPD-P model against three benchmarks by carrying out a Monte Carlo simulation to analyse the effects of their specifications on the Value-at-Risk and Expected Shortfall forecasting, and Section 5, comparing performance of the models to empirical financial time series, i.e. stock indices. The article's conclusions and summary are provided in Section 6.

2. Theoretical background

2.1. GARCH models

The GARCH model of Bollerslev (1986) is the most popular univariate volatility model, and it is based solely on closing prices. We apply this model in the paper as a benchmark for comparison reasons. The GARCH model describes the dynamics of the conditional variance of returns.

Let us assume that the ε_t is the univariate innovation process for the conditional mean (or, in a particular case, the return process) and can be written as:

$$\varepsilon_t|\psi_{t-1}\sim N(0,h_t), \tag{1}$$

where ψ_{t-1} is the set of all information available at time $t-1$, N is the conditional normal distribution, and h_t is the conditional variance. The GARCH(1,1) model is the one most frequently used in empirical studies. It may be presented as:

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{2}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$.

The parameters of the GARCH model can be estimated by the quasi-maximum likelihood (QML) method. The log-likelihood function can be written as:

$$L(\boldsymbol{\theta}) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(\ln h_t + \frac{\varepsilon_t^2}{h_t}\right), \tag{3}$$

where $\boldsymbol{\theta}$ is a vector containing unknown parameters of the model, and n is the number of daily observations used in the estimation. The estimates obtained by the QML method are consistent and asymptotically normal (see Bollersle & Wooldridge, 1992; Straumann, 2005; Weiss, 1986).

Instead of the conditional normal distribution, the Student's *t*-distribution can be applied to better describe fatter tails and leptokurtosis of unconditional distributions of many empirical financial time series (Bollerslev, 1987). The log-likelihood function (Bollerslev, 1987) can be written as:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{n}\left(\ln\left[\Gamma\left(\frac{\upsilon+1}{2}\right)\right] - \ln\left[\Gamma\left(\frac{\upsilon}{2}\right)\right] - \frac{1}{2}\ln[\pi(\upsilon-2)] - \frac{1}{2}\ln(h_t) - \frac{\upsilon+1}{2}\ln\left[1+\frac{\varepsilon_t^2}{(\upsilon-2)h_t}\right]\right), \quad (4)$$

where $\Gamma(\cdot)$ is the Gamma function and υ are the degrees of freedom parameter. To ensure that the second-order moment exists, the constraint $\upsilon > 2$ is imposed.

2.2. Peaks over Threshold (POT) Approach

A natural choice for modelling extreme values is to focus on values that are in the tail of the distribution, i.e. the observations above some high threshold. In the Peaks over Threshold (POT) approach, we are interested in the exceedances over threshold u, conditional on the fact that u is exceeded. Let X_1, X_2 ... be a sequence of i.i.d. random variables, having a marginal distribution function F_u. As shown by Balkema and de Haan (1974) and Pickands (1975), the excess distribution over threshold u corresponding to a random variable X is

$$F_u(x) = P(X-u|X>u) = \frac{F(x+u)-F(u)}{1-F(u)}, \quad 0 \le x < x_{sup}-u, \quad (5)$$

where $x_{sup} = \sup\{x \in \mathbb{R}: F(x) < 1\}$. The asymptotic distribution of F_u is the GPD with shape parameter γ and scale parameter σ:

$$GPD_{\gamma,\sigma} = \begin{cases} 1-\left(1+\gamma\frac{x}{\sigma}\right)^{-\frac{1}{\gamma}}, \gamma \neq 0 \\ 1-\exp\left(-\frac{x}{\sigma}\right), \gamma = 0 \end{cases} \quad (6)$$

where $x \ge 0$ if $\gamma \ge 0$ and $0 \le x \le -\sigma/\gamma$ if $\gamma < 0$ and $\sigma > 0$. When $\gamma > 0$, F_u has a Pareto-type upper tail with a tail index $1/\gamma$. The assumption of i.i.d. is rather restrictive, but fortunately, Leadbetter et al. (1983) proved it for stationary random variables. An estimate of the tail probability can be obtained in the following way (McNeil & Frey, 2000):

$$H_{\hat{\gamma},\hat{\sigma}} = \left(1 + \hat{\gamma}\frac{x}{\hat{\sigma}}\right)^{-\frac{1}{\hat{\gamma}}} \tag{7}$$

where $\hat{\gamma}$ and $\hat{\sigma}$ are the estimates of the GPD parameters.

This parametric approach consists of two steps:

1. given a sample of $X_1, \dots, X_n$, choose a threshold u and set $Y_i = X_i - u$, where $i = 1, \dots, N_u$ and N_u denotes the number of extreme values above the threshold u,
2. fit the GPD to the sequence $Y_1, \dots, Y_{N_u}$ of exceedances to obtain estimates $\hat{\gamma}, \hat{\sigma}$ of the parameters γ, σ.

The parameters of GPD can be estimated by a maximum likelihood (Hosking & Wallis, 1987; Smith, 1985) with the log-likelihood function:

$$L(\gamma, \sigma) = -N_u ln\sigma - (1 + 1/\gamma) \sum_{i=1}^{N_u} ln(1 + \gamma y_i/\sigma) \tag{8}$$

provided $(1 + \sigma^{-1}\gamma y_i) > 0$ for $i = 1, \dots N_u$. Other estimation methods may be used, like probability-weighted moments (PWM) (Hosking et al., 1985). One drawback of the POT method is that the estimates of GPD are sensitive to the choice of threshold u. The choice of threshold u involves a trade-off between bias and variance for the estimates. There are different methods of choosing the threshold – for instance, on the basis of the mean excess plot, by minimising the mean squared error of the estimator (see Beirlant et al., 1996; Jansen & de Vries, 1991; Koedijk et al., 1990), or a widely-used approach that boils down to 10%–15% of the data points that fall in the tail of the distribution (see Smith, 1987). Chavez-Demoulin and Embrechts (2004) show that small variations in the value of the threshold typically have little impact on the estimation.

2.3. GARCH-POT Approach

The POT approach is sometimes called the unconditional Peaks over Threshold method, as we fit GPD directly to observations that are above threshold u, disregarding the potentially time-varying mean and variance nature of the observations. The time-dependent structure of observations is assumed to be i.i.d., which in many cases is not true for financial time series. To circumvent this problem, McNeil and Frey (2000) proposed to filter the data by using the ARMA-GARCH model, and then to apply the POT approach to the standardised residuals that should be i.i.d. The main idea behind this method is the assumption that we are dealing with strictly stationary time series of the form $r_t = \mu_t + h_t^{1/2}\varepsilon_t$, with μ_t and h_t being the conditional mean, and variance and ε_t a strict white noise process of unknown distribution. This method will be further referred to in the text as GARCH-GPD, and involves two steps:

1. estimate the ARMA-GARCH(1,1) model with normally distributed errors to model the conditional mean and variance and obtain the standardised residuals $\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2}$;
2. from the standardised residuals $\tilde{\varepsilon}_t$, where $t = 1, \dots, n$, obtain extremes residuals that are above a high threshold u, for which the exceedances are $\{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\}$, and define threshold excesses as $\check{\varepsilon}_i = \tilde{\varepsilon}_i - u$, where $i = 1, \dots N_u$;
3. fit GPD distribution to the extreme standardised residuals, i.e. $\check{\varepsilon}_i \sim GPD(\gamma, \sigma)$ to obtain estimates $\hat{\sigma}_0$, $\hat{\sigma}_1$ and $\hat{\gamma}$.

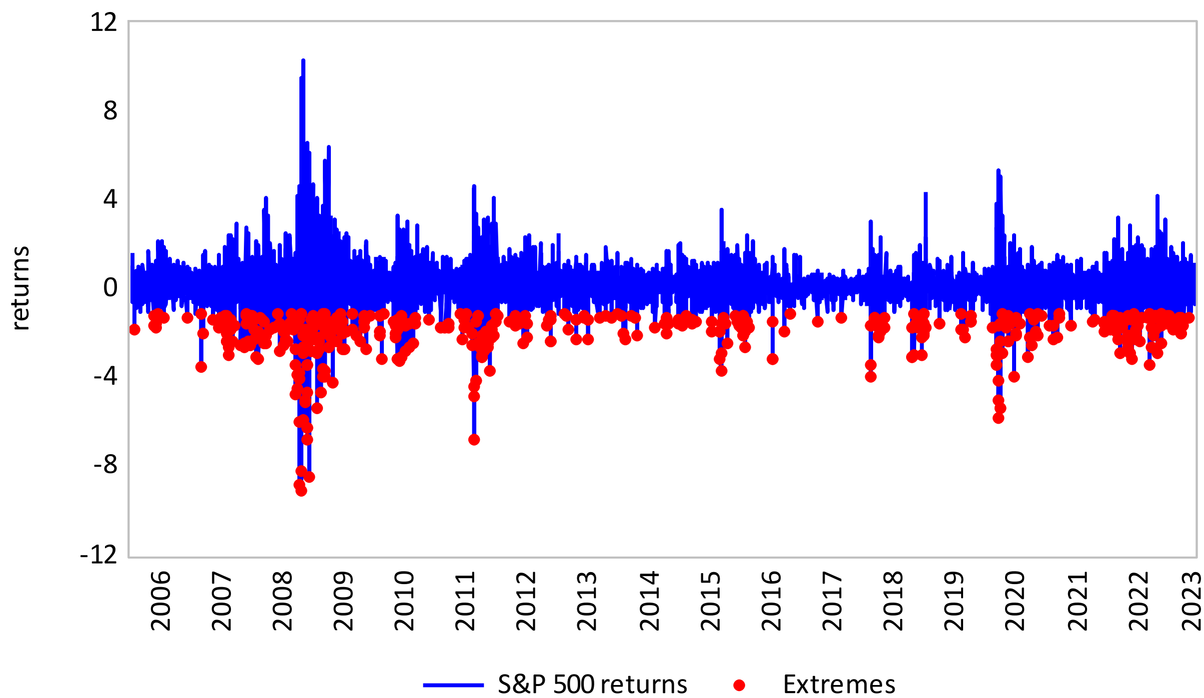
Importantly, Jalal and Rockinger (2008) show that even when the ARMA-GARCH model is misspecified, the GARCH-GPD approach provides good results, which indicates this method is relatively robust. The GARCH-GPD method has been present in the literature, and in most cases, has generated more accurate estimates of tails than other methods (see, Bali, 2007; Chan & Gray, 2006; Kuester et al., 2006).

The use of volatility model is not limited to the standard GARCH(1,1) model, as other specifications may be used, for instance the asymmetric GARCH models, i.e. GJR (Glosten et al., 1993; Pagan & Schwert, 1990), EGARCH (Nelson, 1991) or RGARCH (Molnár, 2016), where lagged squared residuals are replaced with the range-based volatility estimator, or even a CARR model (Chou, 2005), a popular univariate volatility model based on a price range.

2.4. GARCH-POT approach with GDP has a time-varying scale parameter

The unconditional POT approach assumes that the extremes are stationary, so the parameters γ, σ are constant over time. This is likely not the case for financial time series, as the extreme values used for the POT method come from different groups that are above a given threshold u. From an empirical point of view, volatility clustering is a major phenomenon behind financial time series, observing the grouping of high and low volatility across time. It means that clusters with high volatility will have more observations falling in the tail of the distribution, thus being more likely above threshold u than other clusters. We could expect that extreme observations above threshold u should be a part of high-volatility groups formed across the time frame and most likely distant in time from other groups. In EVT, this behavior is well known as the ability of extremes to create clusters. There are methods, like the extremal index (see, for instance, Embrechts et al., pp. 124–135, 2003; Ferro & Segers, 2003), to estimate how extreme observations form series. Figure 1 presents S&P returns with identified extreme values based on the 10th quantile of return distribution as a threshold. Not surprisingly, there are more extreme observations identified for the subperiods like 2008-2009 (financial crisis), 2011 (sovereign crisis), or 2020 (COVID-19 outbreak), and less extreme observations for subperiods 2006, 2014 or 2016–2017. In the literature, there are works employing a time-varying Generalized Pareto distribution with different covariates to model extremes (Bee et al., 2019; Chavez-Demoulin et al., 2005; Chavez-Demoulin et al., 2014; Massacci, 2016; Zhang & Schwaab, 2016), but these models describe extreme values and the dependence in the original data in a single framework. Modelling volatility itself has often proven to be a challenge; hence, it seems that modelling the conditional mean and the conditional variance together but separately from modelling extremes is a more appropriate approach. In this paper, we propose an extension of the GARCH-GPD model of McNeil and Frey, by extending GPD to include time-varying parameters to account for the dynamics of extreme observations.

Figure 1. S&P daily returns with extreme values from 3rd January 2006 to 31st May 2023. Red dots indicate days for which a threshold set at the 10th quantile of distribution is not exceeded



Source: author's work based on the data from www.finance.yahoo.com site.

Following Coles (2001), the GPD with time-varying parameters σ_i and γ_i for a series of extremes x, where $i = 1, \dots, N_u$ (the number of extremes) can be written as[3]:

$$GPD_{\gamma_i,\sigma_i} = \begin{cases} 1 - \left(1 + \gamma_i \frac{x_i}{\sigma_i}\right)^{-\frac{1}{\gamma_i}}, \gamma_i \neq 0 \\ 1 - \exp\left(-\frac{x_i}{\sigma_i}\right), \gamma_i = 0 \end{cases}, \tag{9}$$

where $x_i \geq 0$ if $\gamma_i \geq 0$ and $0 \leq x_i \leq -\sigma_i/\gamma_i$ if $\gamma_i < 0$ and $\sigma_i > 0$. The time-varying shape parameter γ_i is some function f_{γ_i} with a constant and covariates:

$$\gamma_i = f_{\gamma_i}(\boldsymbol{X}'_{\gamma_i}\boldsymbol{\gamma}), \tag{10}$$

where $\boldsymbol{X}'_{\gamma_i} = \left[1, \boldsymbol{X}'_{\gamma_i,1}, \dots, \boldsymbol{X}'_{\gamma_i,l}\right]$ is a vector of covariates and $\boldsymbol{\gamma} = [\gamma_0, \gamma_1, \dots, \gamma_l]$ is a vector of l parameters to be estimated.

Time-varying scale parameter σ_i is some function f_{σ_i} with a constant and covariates:

$$\sigma_i = f(\boldsymbol{X}'_{\sigma_i}\boldsymbol{\sigma}), \tag{11}$$

where $\boldsymbol{X}'_{\sigma_i} = \left[1, \boldsymbol{X}'_{\sigma_i,1}, \dots, \boldsymbol{X}'_{\sigma_i,k}\right]$ is a vector of covariates, and $\boldsymbol{\sigma} = [\sigma_0, \sigma_1, \dots, \sigma_k]$ is a vector of k parameters to be estimated. The parameters of time-varying GPD can be estimated by a maximum-likelihood method with the following log-likelihood function (see, Coles, 2001):

$$L(\gamma_i, \sigma_i) = -N_u ln\sigma_i - (1 + 1/\gamma_i) \sum_{i=1}^{N_u} ln(1 + \gamma_i y_i/\sigma_i), \tag{12}$$

provided $(1 + \sigma_i{}^{-1}\gamma_i y_i) > 0$ for $i = 1, \dots N_u$.

The simplest case of GPD_{γ_i,σ_i} is when there is only a constant for both shape and scale parameters, thus it reduces to the classical GPD given in (6). The question arises as to what covariates and functions f_{γ_i}, f_{σ_i} should be specified to model time-varying parameters. It is usually difficult to estimate time-varying shape parameter γ, so, advisably, it should be kept constant to stabilise the results (see Chavez-Demoulin et al., 2005). It means that we are going to consider the idea of the time-varying scale parameter σ_i only. A natural choice for f_{σ_i} can be a linear additive or logarithmic function, where the latter ensures that σ_i is always positive.

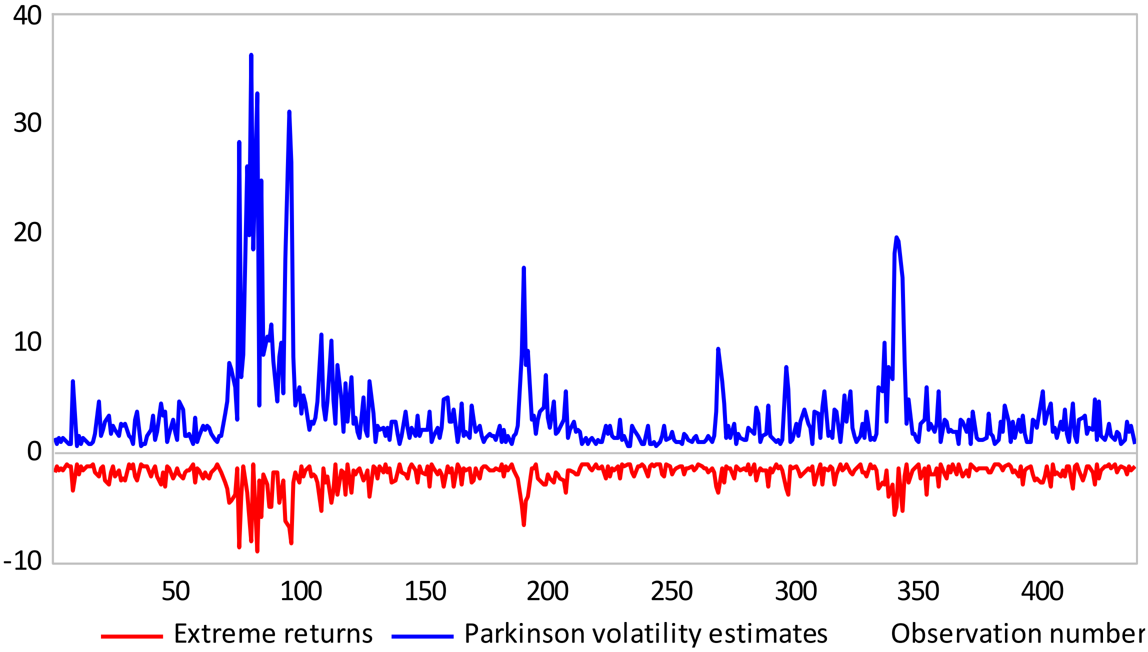
[3] It is worth emphasizing that i here denotes time for the extremes and not the time for all observations of the underlying process.

A more important decision to be made is with covariates, as these should, in theory, describe the dynamic behaviour of extreme observations. We propose to use a range-based estimator that can describe return volatility relatively accurately due to the use of high and low prices. A range-based estimator can show the correct volatility, especially on turbulent days with drops and recoveries in the markets, while the traditional close-to-close volatility indicates a low level. It should be even more pronounced for extreme observations, as these occur when market volatility is particularly high. We propose to use the Parkinson volatility estimator (Parkinson, 1980) in the form of

$$\sigma_{P,i}^2 = [ln(H_i/L_i)]^2/(4\ln 2), \tag{13}$$

where H_i and L_i are the high and low prices at a given day i. In the literature, there is growing evidence that the use of range-based volatility estimators can lead to more accurate conditional volatility and covariance estimates and forecasts, in both univariate (Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fałdziński et al., 2024; Fiszeder & Perczak, 2016; Molnár, 2012, 2016) and multivariate frameworks (Asai, 2013; Chou & Cai, 2009; Chou et al., 2009; Fiszeder et al., 2019; Fiszeder et al., 2023a, 2023b; Su & Wu, 2014). Moreover, there are range-based volatility models (based on range instead of returns) that outperform classical models based on closing prices (see the reviews in Chou et al., 2015; Petropoulos et al., 2022). Different estimators based on daily low, high, or additionally open and closing prices can be employed (Garman & Klass, 1980; Rogers & Satchell, 1991; Yang & Zhang, 2000). The Garman-Klass estimator is sensitive to microstructure effects associated with low liquidity during the start of quotations, and Molnár (2016) showed that the Garman-Klass estimator does not improve results compared to the Parkinson estimator. On the other hand, the Rogers-Satchell estimator can take a zero value despite the high volatility during the day. It happens when the opening price is equal to the low price and the closing price is equal to the high price or vice versa, i.e., the opening price is equal to the high price and the closing price is equal to the low price. The Yang-Zhang estimator requires estimating an additional parameter and assumes constant variance over time, which is untrue. Moreover, the Yang-Zhang estimator cannot be estimated for a single day. For these reasons, we focus here on the Parkinson estimator.

Figure 2. S&P 500 extreme observations and Parkinson volatility estimates that are ordered consecutively



Source: author's work based on the data from www.finance.yahoo.com site.

To justify the use of the range-based estimator, Figure 2 presents the Parkinson daily volatility estimates associated with extreme observations found for the S&P 500 index from the time range presented in Figure 1, where extremes are ordered as they occurred in time (in total there are 438 extreme observations). The red solid line illustrates extreme returns, and the blue solid line Parkinson's volatility estimates. High and low Parkinson volatility estimates are concurrent with high and low extreme daily returns identified and it seems to provide a good approximation of daily extreme-returns volatility. Therefore, we propose the following time-varying scale equation σ_i for GPD:

$$\sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2 \qquad \text{where } i = 1, \dots, N_u \tag{14}$$

where $\sigma_0 > 0$ and $\sigma_1 \geq 0$ to ensure that σ_i is positive. It is worth noting that the Parkinson's volatility estimates $\sigma_{P,i}^2$ are contemporaneous with extreme residuals. It is possible to consider the past Parkison volatility estimates, but concurrent values to extreme returns should be preferred as the contemporaneous values are available at a given time i and should provide a better fit than the past ones. In this regard, it is worth noting that extremes are a sub-sample of available observations.

The proposed method will be referred to further in the text as GARCH-GPD-P, and consists of the following steps:

1. estimate the ARMA-GARCH(1,1) model to obtain both the conditional mean μ_t and conditional variance h_t;

2. obtain the standardised residuals $\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2}$;

3. from the standardised residuals $\tilde{\varepsilon}_t$, where $t = 1, \dots, n$ obtain extremes residuals that are above a high threshold u, for which the exceedances are $\{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\}$, and define threshold excesses as $\check{\varepsilon}_i = \tilde{\varepsilon}_i - u$, where $i = 1, \dots N_u$;

4. fit GPD distribution to the extreme standardized residuals, i.e. $\check{\varepsilon}_i \sim GPD(\gamma, \sigma_i)$ where $\sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2$ and $\sigma_{P,i}^2$ is the Parkinson estimator at observation i (noting that $i \leq n$) to obtain estimates $\hat{\sigma}_0$, $\hat{\sigma}_1$ and $\hat{\gamma}$.

The GPD and GPD-P method rely on extremes as a sub-sample of all observations above a threshold u. Given two samples that imperfectly overlap with each other, the sub-samples of extremes above threshold may have a perfect overlap, some overlap or, in edge case, no overlap in extremes. Consequently, the GPD-P estimates are based on the sub-sample of observations that is deemed as extreme at a particular time.

The proposed framework GARCH-GPD-P can be concisely formulated as:

$$r_t = \mu_t + \varepsilon_t, \varepsilon_t | \psi_{t-1} \sim N(0, h_t), t = 1, \dots, n \tag{15}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{16}$$

$$\tilde{\varepsilon}_t = (r_t - \mu_t)/h_t^{1/2} \tag{17}$$

$$\check{\varepsilon}_i = \tilde{\varepsilon}_i - u, \text{ for } i = 1, \dots N_u \text{ where } \{\tilde{\varepsilon}_t : \tilde{\varepsilon}_t > u\} \tag{18}$$

$$\check{\varepsilon}_i \sim GDP(\gamma, \sigma_i), \text{ where } \sigma_i = \sigma_0 + \sigma_1 \sigma_{P,i}^2 \,. \tag{19}$$

3. Value-at-Risk and Expected Shortfall

3.1. Value-at-Risk and Expected Shortfall estimation

Tail-based risk measures such as the Value-at-Risk (VaR) and the Expected Shortfall (ES) are mostly used in quantitative risk management, from the perspective of the regulatory and financial institution. The Basel Accords explicitly use VaR and ES as risk measures and oblige financial institutions to implement and report them to monitor risk and determine the amount of capital that is subject to regulatory supervision.

Let $\alpha \in [0,1]$ denote the coverage level (or probability level). The α level VaR is defined as $VaR_t(\alpha) = P(r_t \leq -VaR_t) = \alpha$, so the $VaR_t(\alpha)$ is the α quantile of the r_t returns distribution that is negative. The VaR has been criticised for not being able to show the average potential loss but only whether losses are larger than the VaR. This was one of the reasons why the ES has been proposed to measure the size and the likelihood of losses, unlike VaR. ES is defined as the expected loss given that the loss is greater than VaR, and it may be written as $ES_t(\alpha) = -E[-r_t > VaR_t(\alpha)]$. A more useful representation of $ES_t(\alpha)$ is as follows:

$$ES_t(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} VaR_t(u)\, du. \tag{20}$$

$ES_t(\alpha)$ comprises information from the left tail of the returns distribution, by integrating VaR from 0 to α. In practice, risk managers specify parametric conditional versions of VaR and ES. For the GARCH model, VaR and ES are given by:

$$VaR_{t,cond}(\alpha) = -\mu_t - \sqrt{h_t}F^{-1}(\alpha), \tag{21}$$

$$ES_{t,cond}(\alpha) = -\mu_t - \sqrt{h_t}m(\alpha), m(\alpha) = E[\varepsilon_t | \varepsilon_t \leq F^{-1}(\alpha)], \tag{22}$$

where $F^{-1}(\alpha)$ is the α-quantile of the inverse cumulative distribution function. In this paper, we are using the normal distribution and Student's t-distribution function with υ degrees of freedom. The driving force behind the VaR and ES estimates variability is the conditional variance (see So & Yu, 2006), as the conditional mean is, in most cases, close to zero (or omitted) and α-quantile of the inverse cumulative distribution function used as a constant value (for instance for the normal distribution it is -1.64 at a 5% probability level). Thus, to obtain better estimates of VaR and ES, we can only achieve it by improving variance estimates, as a quantile from the normal or Student's t-distribution is constant at a given probability.

To obtain the VaR and ES with the GPD approach, we need an inverse of the cumulative GPD function given by equation (7) and the $\hat{\gamma}$ and $\hat{\sigma}$ estimates. Then, the unconditional VaR and ES with GPD (following McNeil & Frey, 2000) are given as:

$$VaR_{unc}(\alpha) = \hat{u} + \frac{\hat{\sigma}}{\hat{\gamma}}\left[\left(\frac{n}{N_u}\alpha\right)^{-\hat{\gamma}} - 1\right], \tag{23}$$

$$ES_{unc}(\alpha) = \frac{VaR_{unc}(\alpha)}{1-\hat{\gamma}} + \frac{\hat{\sigma}-\hat{\gamma}\hat{u}}{1-\hat{\gamma}}, \tag{24}$$

where $\hat{u}$ is the threshold estimate, n is the number of observations and N_u is the number of extremes.

Consequently, the unconditional VaR and ES with time-varying GPD_{γ_i,σ_i} can be written as:

$$VaR_{unc}(\alpha) = \hat{u} + \frac{\hat{\sigma}_i}{\hat{\gamma}_i}\left[\left(\frac{n}{N_u}\alpha\right)^{-\hat{\gamma}_i} - 1\right], \tag{25}$$

$$ES_{unc}(\alpha) = \frac{VaR_{unc}(\alpha)}{1-\hat{\gamma}_i} + \frac{\hat{\sigma}_i-\hat{\gamma}_i\hat{u}}{1-\hat{\gamma}_i}, \tag{26}$$

where $\hat{\sigma_i}$ and $\hat{\gamma}_i$ are estimates of σ_i and γ_i for $i = 1, \dots N_u$. In the proposed framework, we are using the latest available extreme for the unconditional VaR and ES calculation, i.e. for $i = N_u$. For VaR and ES calculation when a new extreme observation is available, VaR and ES estimates are impacted not only by the change in the conditional mean and the conditional variance, but also by the change in scale parameter $\hat{\sigma}_i$ through the change in the GPD-P quantile.

The conditional one-day-ahead VaR and ES with the GARCH-GPD and GARCH-GPD-P approaches are given by:

$$VaR_{t+1,cond}(\alpha) = -\mu_{t+1} - \sqrt{h_{t+1}}VaR_{unc}(\alpha), \tag{27}$$

$$ES_{t+1,cond}(\alpha) = -\mu_{t+1} - \sqrt{h_{t+1}}ES_{unc}(\alpha), \tag{28}$$

where μ_{t+1} and h_{t+1} are the one-day-ahead forecasts of the conditional mean and the conditional variance of returns, respectively.

The advantage of GPD and time-varying GPD-P stems from the fact that the unconditional VaR and ES are tail-based estimates depending on parameter estimates for GPD and GDP-P, respectively. The difference between GPD and time-varying GPD-P is that the latter takes into account the magnitudes of threshold exceedances measured by the Parkinson estimator, thus we can expect more accurate estimates of the unconditional VaR and ES, as the variability of extremes should be described more accurately by the time-varying scale σ_i parameter. In other words, to obtain better VaR and ES estimates for the GARCH-GPD or GARCH-GPD-P, we can improve either or both the conditional variance and tail-based estimates from the GPD or GPD-P.

3.2. Value-at-Risk and Expected Shortfall backtesting

There is already a wide spectrum of methods and models to estimate tail-based risk measures, like VaR and ES. The evaluation of forecasting accuracy is of great importance when it comes to risk measures, for practitioners and regulatory institutions, to ensure that financial institutions have adequate capital to deal with large unexpected losses. The literature provides information on many various ways to assess the accuracy of VaR estimates by developing statistical tests, methods and measures known as backtesting. We can divide backtesting methods into three categories: a) statistical tests verifying the validity of VaR assumptions, b) measures to assess VaR accuracy, and c) statistical tests to determine which of the competing models are superior to others.

The hit variable (or violation variable) associated with the ex-post observation of a $VaR_t(\alpha)$ at time t, denoted $I_t(\alpha)$ is defined as:

$$I_t(\alpha) = \mathbf{1}\big(r_t \leq -VaR_t(\alpha)\big) \tag{29}$$

where $\mathbf{1}(\cdot)$ is the indicator function. Kupiec (1995) shows that to assess VaR validity it is possible to test whether the hit sequence $I_t(\alpha)$ follows two conditions: a) unconditional coverage (UC) $P[I_t(\alpha) = 1] = E[I_t(\alpha)] = \alpha$, and b) independence property (IND) that the variable $I_t(\alpha)$ has to be independent of the variable $I_{t-k}(\alpha), \forall k \neq 0$. These two conditions are necessary but not sufficient of the VaR definition. The most popular backtesting test
`s are: the unconditional coverage LR_{UC} proposed by Kupiec (1995) and the independence LR_{ind} and conditional coverage LR_{CC} tests by Christoffersen (1998). It has been documented that these tests exhibit law power (see de la Pena et al., 2007; Pérignon & Smith, 2008; Pritsker, 2006). Alternatively, Candelon et al. (2011) proposed the unconditional, independence and conditional coverage tests (denoted here as J_{UC}, J_{IND} and J_{CC}, respectively) based on the duration of the hit sequence and showed that their GMM-based tests are of greater statistical power than classically used ones. Additionally, they encourage obtaining simulated p-values instead of asymptotic ones, by applying Dufour's approach (Dufour, 2006) to ensure the correct test size.

Besides testing the hit process, loss functions can be used to select a model that produces accurate Value-at-Risk estimates. Lopez (1998) suggested measuring the accuracy of VaR forecasts by the distance between observed returns and forecasted VaR. A model is penalized if a violation takes place and is preferred to another one because it gives a lower loss value. In the general form, Lopez proposes the following formula:

$$LF_t = \begin{cases} f(r_t, VaR_t(\alpha)) \text{ if } r_t < -VaR_t(\alpha) \\ g(r_t, VaR_t(\alpha)) \text{ if } r_t \geq -VaR_t(\alpha) \end{cases} \tag{30}$$

where $f(x, y)$ and $g(x, y)$ are such that $f(x, y) \geq g(x, y)$. The best model is the one that minimizes $LF = \sum_{t=1}^{T} LF_t$. Lopez in 1998 proposed the following loss measure:

$$RLF(L) = \begin{cases} 1 + (VaR_t - r_t)^2 \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{31}$$

Sarma et al. (2003) and Caporin (2008) proposed loss functions from two perspectives: the regulator's loss function (RLF) and the Firm's Loss Function (FLF).

$$RLF(STS) = \begin{cases} (r_t - VaR_t)^2 \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{31}$$

$$RLF(C1) = \begin{cases} \left|1 - \left|\frac{r_t}{VaR_t}\right|\right| \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{32}$$

$$RLF(C2) = \begin{cases} \frac{(|r_t| - |VaR_t|)^2}{VaR_t} \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{33}$$

$$RLF(C3) = \begin{cases} |r_t - VaR_t| \text{ if } r_t < -VaR_t \\ 0 \text{ if } r_t \geq -VaR_t \end{cases} \tag{34}$$

$$FLF(STS) = \begin{cases} (r_t - VaR_t)^2 \text{ if } r_t < -VaR_t \\ -ococVaR_t \text{ if } r_t \geq -VaR_t \end{cases}, ococ \text{ is the opportunity cost of capital} \tag{35}$$

$$FLF(C1) = \left|1 - \left|\frac{r_t}{VaR_t}\right|\right| \tag{36}$$

$$FLF(C2) = \frac{(|r_t| - |VaR_t|)^2}{VaR_{t|}} \quad (37)$$

$$FLF(C3) = |r_t - VaR_t| \quad (38)$$

Şener et al. (2012) propose a loss function that penalizes the magnitude of the errors, the autocorrelation between the errors, and excessive capital allocations. The penalization measure is of the form:

$$PM(\varphi, VaR) = \frac{1}{T^*}[(1-\varphi)PM_{VS} + \varphi PM_{SS}] \quad (39)$$

where PM_{VS} and PM_{SS} is the penalization measure for the violation space and the safe space, respectively, φ is the weighting parameter and T^* is the number of all negative returns. The weighting parameter φ is assumed to be set to the coverage level α, thus violations have more importance than non-violations which is expected from the regulator's and financial institution perspectives. The penalization measure for the violation space PM_{VS} can be written as:

$$PM_{VS} = \sum_{i=1}^{n_c-1} \sum_{j=1}^{n_c} \frac{1}{d_{i,i+j}} \left(\prod_{k=1}^{l_i} (1 + lf_{k,i}) \prod_{k=1}^{l_{i+j}} (1 + lf_{k,i+j}) - 1 \right) \quad (40)$$

where $lf_t = (VaR_t(\alpha) - r_t)$ given $r_t < -VaR_t(\alpha)$, n_c is the number of violation clusters, $d_{i,i+j}$ is the time between i-th and j-th violations clusters and l_i is the length of violation cluster i.

The penalization measure for the violation space PM_{VS} focuses on the magnitude of unexpected losses and clusters of unexpected losses (autocorrelation) and is calculated only for violations. On the other hand, the penalization measure for the safe space PM_{SS} may be given as:

$$PM_{SS} = \sum_{t=1}^{T} (r_t - VaR_t(\alpha))[\mathbf{1}(r_t > VaR_t(\alpha) | r_t < 0)] \quad (41)$$

where $\mathbf{1}$ is the indicator function and T is the number of all observations for which VaR forecasts have been obtained. This measure takes into account excessive capital allocation for returns that are not a violation and are negative. The idea behind the penalization measure is to have the flexibility to capture both the regulator's and risk manager's perspectives while being able to give different weights to each.

Furthermore, to determine which of the competing models produces superior VaR estimates: Sarma et al. (2003) proposed to use the Diebold and Mariano test (Diebold & Mariano, 1995), and Şener et al. (2012) introduced a predictive ability test for the penalization measure $PM(\varphi, VaR)$ that does not require a benchmark model, thus allowing for the simultaneous comparison of several models. The test is based on White's framework (White, 2000) as an extension of Diebold and Mariano test. The null hypothesis states that the loss series generated by any chosen forecasting method is statistically no worse than the others.

When it comes to backtesting of Expected Shortfall, the situation is quite different from Value-at-Risk, where the literature is scarce in this regard. More recently, Du and Escanciano (2016) introduced the unconditional DE_{UC} and the conditional DE_{IND} tests based on cumulative violations sequence. The cumulative violation process is defined as

$$H_t(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} I_t(u)du \tag{42}$$

where $H_t(\alpha)$ has mean equal to $\alpha/2$. Then, the unconditional backtest UC_{ES} is a t-test for the hypothesis $E[H_t(\alpha)] = \alpha/2$. The test statistic is given by:

$$DE_{UC} = \frac{\sqrt{n_f}(\bar{H}(\alpha) - \alpha/2)}{\sqrt{Var(H_t(\alpha))}} \sim N(0,1) \tag{43}$$

where $\bar{H}(\alpha)$ denotes the sample mean of $H_t(\alpha)$, n_f is the number of ES estimates and $Var(H_t(\alpha))$ is the variance of $H_t(\alpha)$ with the standard normal asymptotic distribution $N(0,1)$. The conditional backtest of independence DE_{IND} is based on the lag-j autocovariance and autocorrelation of $H_t(\alpha)$ for $j \geq 0$ that are defined as follows:

$$cov_{n_f,j} = \frac{1}{n_f - j}\sum_{t=1+j}^{n_f}(H_t(\alpha) - \alpha/2)\big(H_{t-j}(\alpha) - \alpha/2\big) \text{ and } \rho_{n_f,j} = \frac{cov_{n_f,j}}{cov_{n_f,0}} \tag{44}$$

The test statistic is given as:

$$DE_{IND}(m) = n_f \sum_{j=1}^{m} \hat{\rho}_{n_f,j} \tag{45}$$

where $\hat{\rho}_{n_f,j}$ is the sample estimate of $\rho_{n_f,j}$ with the limiting chi-square distribution χ_m^2 with m degrees of freedom.

4. Monte Carlo simulation

We conduct a Monte Carlo simulation to analyse the finite sample properties of the proposed model, i.e. the GARCH-GPD-P versus the competing models (the GARCH model with normally and Student's *t*-distributed errors denoted as GARCH-n and GARCH-t, respectively, and McNeil and Frey's GARCH-GPD). We choose the stochastic volatility (SV) model as the data-generating process due to its flexibility, and quite often this model is used for simulation purposes in the literature (see for instance Alizadeh et al., 2002; Buescu et al., 2013; Molnár, 2016; Shu & Zhang, 2006). The main advantage of the SV model over the GARCH model is that it assumes two innovation processes (for the conditional mean and the conditional volatility). In the SV model, the volatility is a random variable, hence this model can be more flexible than the GARCH model. It is believed that the SV model is more effective in describing empirical properties of financial time series (see Danielsson, 1994; Kim et al., 1998). Assuming the SV model as the data generating process does not favour any of the competing models.

Daily volatility is simulated by the stochastic volatility model that can be given as (see Melino & Turnball, 1990; Taylor, 1990):

$$ln(P_t/P_{t-1}) = \mu_{sv,t} + \sigma_{sv,t}\varepsilon_t\,, \tag{46}$$

$$ln(\sigma^2_{sv,t}) = \alpha_{sv} + \phi_{sv} ln\sigma^2_{sv,t-1} + \sigma_\eta\eta_t, \tag{47}$$

where ε_t and η_t are mutually independent and i.i.d. following the normal distribution with zero mean and unit variance $N(0,1)$. We assume the following set of values for the parameters: $\mu_{sv,t} = 0.001$, $\alpha_{sv} = 0.02$, $\phi_{sv} = 0.95$ and $\sigma^2_\eta = 0.065$ that are consistent with the empirically observed values for the stochastic volatility model. As we need to obtain not only daily close prices but also low and high prices, we simulate intraday price paths following the geometric Brownian motion based on the simulated daily volatility and mean from the stochastic volatility model.

We simulate 1,600 daily price paths with their volatilities following the SV model (Equations 46 and 47) where for each day we generate 100,000 intraday prices based on the geometric Brownian motion. The first 100 observations are dropped to remove the impact of the starting values. Then, we use the next 500 observations (so from 101 to 600) to estimate the parameters of all four competing models (the GARCH-n, GARCH-t, GARCH-GPD and GARCH-GDP-

P). This step includes obtaining the Parkinson volatility estimates based on simulated high and low prices and estimating the conditional VaR and ES for the next day by Equations (21), (22), (27) and (28), where one-day-ahead forecasts of the conditional mean and the conditional volatility are used. For the GARCH-GPD and the GARCH-GPD-P models, we set the threshold as the 12% cut-off point of the most negative standardised residuals. The threshold was set based on the mean excess plot for the empirical times series used in section 3. We repeat this process for each next day by applying the rolling window approach, where one observation from the beginning of the sample is removed and one observation is added to the end of the sample, thus having a fixed size of 500 observations in the sample. This way, we obtain 1,000 VaR and ES daily estimates for one iteration of the simulation. These 1,000 VaR and ES estimates are backtested using methods and measures described in subsection 3.2 for 5% and 10% coverage levels. Lastly, we repeat the process above 1,000 times, which is the number of iterations in the Monte Carlo simulation. The final results presented in the paper are the averages for all 1,000 iterations. All in all, we obtain and evaluate 1,000,000 VaR and ES estimates as a basis for the backtesting procedures.

4.1. Evaluation of models based on the Monte Carlo simulation

For in-sample comparisons, we are going to focus on the results of two models, i.e. the GARCH-GPD and GARCH-GPD-P, as the GARCH-n and GARCH-t models are benchmarks for risk measure purposes. As described in Section 4, the parameters of all models are estimated 1,000 times for each of the 1,000 repetitions of the Monte Carlo simulation based on the rolling window approach. For all repetitions, we compute the average and standard deviation of the estimated parameters and the robust standard errors which are presented in Table 1. Scale parameter σ for the GPD and σ_1 for the GPD-P are highly significant. Moreover, the constant scale parameter for the GPD-P model is considerably lower than the σ scale parameter for the GPD. We perform the likelihood ratio test for each estimated model for all repetitions and the average values are presented in Table 1. The null hypothesis is rejected even at a high significance level indicating that the GPD-P model is better fitted to the extreme observations than the GPD model. It means that the information comprised of high and low prices associated with extreme observations provides considerable insight into the dynamic behaviour of the extremes.

The out-of-sample analysis involves the evaluation of the VaR and ES forecasts at 5% and 10% probability levels. For each repetition in the simulation, we evaluate the 1,000 obtained

VaR and ES forecasts and we backtest them by testing their statistical properties, calculating the loss measures and testing the superiority of the VaR forecasts against the others. We repeat this process for all 1,000 iterations and compute the average of the obtained results.

Table 1. The results of the parameter estimates for the GPD and GPD-P for Monte Carlo simulation

Statistics	GPD			GPD-P				LM p-value
	σ	γ	ln L	σ_0	σ_1	γ	ln L	
Mean	0.6475* (0.1169)	-0.0922 (0.1139)	-27.3616	0.1171 (0.0448)	0.0590* (0.0175)	-0.2181* (0.1091)	-4.6664	0.0000*
St. dev.	0.1169 (0.0310)	0.1139 (0.0274)	7.0713	0.2708 (0.0658)	0.0533 (0.0591)	0.1074 (0.0872)	6.9171	0.0032

Note. * indicates that the null hypothesis is rejected at a 5% significance level, the robust Huber-White standard errors are reported in parentheses, St. dev. - the standard deviation, ln L - logarithm of the likelihood function, LM p-value is the p-value from the likelihood ratio test based on the logarithm of the likelihood function for GPD vs GPD-P.
Source: author's work.

Table 2. The results of backtesting tests for VaR(10%) and VaR(5%) based on the Monte Carlo simulation

VaR coverage level	Statistic	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
		p-value	p-value	p-value	p-value
10%	LR_{UC}	0,3153	0,5606	0,6408	0,6623
	LR_{IND}	0,5434	0,5716	0,5648	0,5576
	LR_{CC}	0,3872	0,6062	0,6538	0,6591
	J_{UC}	0,3334	0,5546	0,6258	0,6644
	J_{IND}	0,3519	0,5557	0,5809	0,5931
	J_{CC}	0,3413	0,5476	0,5744	0,5898
5%	LR_{UC}	0,5845	0,5351	0,6268	0,6985
	LR_{IND}	0,5015	0,5121	0,4897	0,5012
	LR_{CC}	0,5796	0,5361	0,5952	0,6425
	J_{UC}	0,5838	0,5091	0,6133	0,6913
	J_{IND}	0,5733	0,5757	0,5740	0,5746
	J_{CC}	0,5638	0,5758	0,5737	0,5814

Note. * indicates that the null hypothesis is rejected at a 5% significance level. LR_{UC} is the unconditional coverage test proposed by Kupiec (1995), LR_{IND}, LR_{CC} are the independence and conditional coverage tests, respectively, proposed by Christoffersen (1998). J_{UC}, J_{IND}, J_{CC} are the unconditional coverage, independence and conditional coverage tests, respectively, proposed by Candelon et al. (2011). For J_{IND} and J_{CC}, the number of moments is fixed to 5, p-values for J_{UC}, J_{IND}, J_{CC} are obtained through Dufour's (2006) Monte Carlo procedure involving 10,000 repetitions.
Source: author's work.

Table 2 shows the results of testing statistical properties of VaR at 10% and 5% coverage levels. At both levels, all the competing models seem to perform quite well as the null hypothesis is not rejected for all tests, although the p-values for the GARCH-GPD-P and GARCH-GPD are generally higher than for the GARCH-n and the GARCH-t models. Table 3 presents the results of the loss functions used for VaR forecast evaluation. To that end, we utilise the following measures split into two groups i.e. regulator's loss functions (RLF): $RLF(L)$ by

Lopez (1998), $RLF(STS)$ by Sarma et al. (2003), $RLF(C1)$, $RLF(C2)$ and $RLF(C3)$, all three proposed by Caporin (2008) and FLFs: $FLF(STS)$ by Sarma et al. (2003), $FLF(C1)$, $FLF(C2)$ and $FLF(C3)$ all three proposed by Caporin (2008). At a 10% coverage level, the GARCH-n model leads to the smallest values of the regulator's loss functions, but at the same time, the FLFs are the highest across the models. The GARCH-GPD-P and GARCH-GPD perform quite similarly for all loss functions, although the values of loss functions are lower for the GARCH-GDP-P model. There are two cases ($FLF(C2)$ and $FLF(C3)$) where the GARCH-GPD-P model have the lowest values of all models. The poorer performance at lower coverage levels is not surprising as the EVT-based methods are designed to accurately model high tails, i.e. 5%, 1% or even 0.5%. At a 5% coverage level, we can observe that the GARCH-GPD-P model produces the best estimates of VaR according to all regulators' loss functions. On the other hand, we can see that the proposed model may lead to some overestimation based on the firm's loss functions. This is in line with an empirical observation from other studies where the POT approach is applied.

Table 3. The average results of the loss measures for VaR(10%) and VaR(5%) based on the Monte Carlo simulation

VaR coverage level	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
10%	RLF(L)	**111,0527**	126,4374	127,7731	127,5317
	RLF(STS)	**22,2427**	25,3874	25,6931	25,3156
	RLF(C1)	**43,0243**	52,2956	53,2825	53,0290
	RLF(C2)	**30,1174**	36,6573	37,3801	37,2902
	RLF(C3)	**31,3574**	35,7206	36,1238	36,0265
	FLF(STS)	**56,4412**	57,0688	57,1631	57,0823
	FLF(C1)	572,6937	**569,5876**	569,7450	569,6610
	FLF(C2)	322,0481	306,4964	305,5258	**303,8595**
	FLF(C3)	809,4938	772,2083	769,1470	**766,4545**
	PM	**0,0297**	0,0313	0,0310	0,0303
	PM(VS)	**5,9594**	7,8347	7,0591	7,0421
	PM(SS)	184,0021	165,2148	163,6847	**161,3238**
5%	RLF(L)	62,3923	66,0560	63,7472	**58,9353**
	RLF(STS)	12,1923	12,9660	12,5772	**11,0953**
	RLF(C1)	18,5921	20,2220	19,4369	**18,4008**
	RLF(C2)	12,9681	14,1229	13,5645	**12,5804**
	RLF(C3)	17,3157	18,3933	17,8284	**16,1446**
	FLF(STS)	57,9073	**57,5883**	57,8911	61,5843
	FLF(C1)	607,8039	**603,7517**	606,3970	620,8263
	FLF(C2)	432,7660	**420,4871**	428,5252	509,8466
	FLF(C3)	993,6257	**975,5646**	986,9144	1 082,4124
	PM	0,0309	0,0304	0,0308	**0,0294**
	PM(VS)	1,7541	1,9833	1,8841	**1,0103**
	PM(SS)	276,3196	**267,1751**	272,9081	273,3008

Note. The lowest values of loss functions are marked in bold. RLF(L) – the loss function proposed by Lopez (1998), RLF(STS), FLF(STS) — the loss functions proposed by Sarma et al. (2003), RLF(C1), RLF(C2), RLF(C3), FLF(C1), FLF(C2) and FLF(C3) – loss functions proposed by Caporin (2008), PM, PM_{VS} PM_{SS} – penalisation measure, the penalisation measure for the violation space and the penalisation measure for the safe space proposed, respectively, by Şener et al. (2012).

Source: author's work.

The best values of the firm's loss functions are obtained for the GARCH-t model. It is worth noting that the GARCH-GPD-P model has the best value of the penalisation measure, mainly because in case of violations, the GARCH-GPD-P model is the least underestimated.

Table 4 shows the results of the predictive ability test of Şener et al. (2012) for VaR(5%) and VaR(10%). At both levels, we do not reject the null hypothesis, but we may see that the GARCH-GPD-P and the GARCH-t have the highest p-values at a 5% and 10% probability, respectively. This means that it is difficult to find significant statistical differences in VaR forecasting among the tested models.

Table 4. The average *p*-values of the predictive ability test (Şener et al., 2012) for VaR(5%) and VaR(10%) based on the penalisation measure–: the Monte Carlo simulation

VaR coverage level	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
10%	0.6551	0.8687	0.7258	0.7938
5%	0.3895	0.7878	0.6010	0.8529

Source: author's work.

Table 5 presents the results of backtesting for the Expected Shortfall at 10% and 5% levels. At both levels, we do not reject the null hypothesis for the unconditional and independent tests, although we may observe that the p-values for the GARCH-GPD-P are the highest, thus indicating that this model may produce better properties of ES. The mean of cumulative violation process H_t for the GARCH-GPD-P is closer to the desired level (i.e. $\alpha/2$) than any other competing model. It suggests that the forecasts of the Expected Shortfall are most accurate from the GARCH-GPD-P model.

Table 5. The results of backtesting for ES(10%) and ES(5%) based on Du and Escanciano (2016): the Monte Carlo simulation

ES coverage level	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean H_t	p-value	Mean H_t	p-value	Mean H_t	p-value	Mean H_t
10%	DE_{UC}	0.5758	0.0497	0.5583	0.0520	0.6524	0.0515	0.6989	0.0510
	DE_{IND}	0.4870	-	0.4680	-	0.4702	-	0.5240	-
5%	DE_{UC}	0.3746	0.0290	0.5043	0.0273	0.6033	0.0263	0.6551	0.0245
	DE_{IND}	0.5587	-	0.5505	-	0.6755	-	0.7904	-

Note. For independence test DE_{IND}, we calculate the statistics up to 5 lags. DE_{UC}, DE_{IND} – unconditional coverage and independence test, respectively, proposed by Du and Escanciano (2016), H_t – cumulative violation process.

Source: author's work.

All in all, it is impossible to select the best model for VaR and Expected Shortfall forecasting. This is also a prevailing conclusion from other studies that compare risk measures from different perspectives (see for instance Abad et al., 2014; Nieto & Ruiz, 2016). The performance of the GARCH-GPD-P model in the Monte Carlo simulation boils down to the advantage at a higher probability level (5%) where the results indicate more accurate VaR and ES forecasts than the ones obtained from other competing models.

5. Analysis of stock indices, currencies and cryptocurrencies

5.1. Data

We apply the analysed models to real financial data, i.e. five stock indices, three currencies and four cryptocurrencies. The set of data consists of three classes of assets: five selected U.S. stocks: Amazon, Apple, Google, Microsoft and NVIDIA, three currencies: EUR-USD, GBP-USD, USD-JPY and four cryptocurrencies: BTC-USD, ETH-USD, LTC-USD and XRP-USD. The dataset comprises daily data spanning over sixteen and a half years, i.e. from 3rd January 2006 to 31st May 2023 (4,382 observations) for stocks, from 3rd January 2006 to 31st May 2023 (4,512 observations) for currencies, from 3rd January 2015 to 31st May 2023 (3,073 observations) for BTC-USD, 3rd January 2016 to 31st May 312023 (2,708 observations) for LTC-USD and 3rd January 32018 to 31st May 2023 (1,977 observations) for ETH-USD and XRP-USD. These long periods consist of high-volatility events (like the financial crisis, the European sovereign debt crisis and COVID-19), but also low-volatility periods, where the latter is more prominent over time. Table 6 presents the descriptive statistics for the logarithmic returns calculated as $r_t = 100\ln(c_t/c_{t-1})$, where c_t is a closing price at time t. All return series appear to have heavy tails and they do not follow normal distribution. The time series exhibit non-zero skewness and kurtosis greater than three. In the majority of the cases, stocks and cryptocurrencies time series are autocorrelated, whereas currencies do not seem to be autocorrelated. The three groups of time series share similarities, but also differences, such as higher volatility for cryptocurrencies and lower volatility for currencies compared with the stocks volatility. These three asset classes give the opportunity to show the performance of the proposed model across somewhat different groups of time series.

Table 6. Summary statistics of the daily returns

Time series	Mean	Standard deviation	Minimum	Maximum	Skewness	Excess kurtosis	Ljung-Box
Amazon	0.0896	2.4206	23.8621	-24.6182	0.4308*	15.5456*	8.2049
Apple	0.0958	2.0509	13.0194	-19.7470	-0.2751*	9.0581*	21.5800*

Google	0.0553	1.8849	18.2251	-12.3685	0.2457*	11.2674*	18.4530*
Microsoft	0.0572	1.7748	17.0626	-15.9453	-0.0420	12.1607*	68.4090*
NVIDIA	0.1090	3.1091	26.0876	-36.7109	-0.3207*	12.3159*	7.4354
EURSUD	-0.0022	0.5741	3.41572	-2.94799	0.0635	5.4851*	2.1372
GBP/USD	-0.0072	0.6133	3.130041	-9.50501	-1.0561*	18.3437*	11.6850
USD/JPY	0.0037	0.6255	5.23658	-4.13554	-0.2202*	8.3409*	8.6468
BTC/USD	0.1452	3.8282	22.5119	-46.473	-0.7935*	14.1622*	9.1718
ETH/USD	0.0448	4.9436	23.06952	-55.0732	-1.0068*	13.4112*	20.6500*
LTC/USD	0.1201	5.4522	51.14174	-44.9062	0.2625*	14.3996*	22.3370*
XRP/USD	-0.0775	5.7847	44.47556	-55.0503	-0.0698	16.3753*	6.0308

Note. The sample period is 3rd January 2006 to 31st May 2023, * indicates that the null hypothesis is rejected at a 5% significance level, Ljung-Box – the Ljung-Box statistic for 5 lags.
Source: author’s work based on the data from www.finance.yahoo.com site.

5.2. In sample evaluation based on empirical data

Firstly, we evaluate the proposed model, i.e. the GARCH-GPD-P against GARCH-GPD for the whole range of data. The estimation results of the GPD-P and the GPD are presented in Table 7. Parameter σ_1, responsible for the dynamics of extremes based on the Parkinson volatility estimates is highly significant and positive for all time series. This means that the dynamic behaviour of extreme values is observed and takes part in explaining the tail of the distribution. The σ_0 estimates in the GPD-P are considerably lower (in many cases 2-3 times lower) than those obtained for the GPD. We compare the likelihood functions of the competing models and for all the considered time series, the likelihood ratio test indicates that GPD-P is significantly better fitted to the data (extreme observations) than the GPD.

Table 7. The results of the parameter estimates for the GPD and GPD-P for stock indices

Time series	GPD			GPD-P				LM *p*-value
	σ	γ	ln L	σ_0	σ_1	γ	ln L	
Amazon	0.5994* (0.0359)	0.0166 (0.0493)	-221.1060	0.2320* (0.0484)	0.0704* (0.0080)	-0.2350* (0.0303)	-151.3465	0.0000
Apple	0.6480* (0.0369)	-0.0583 (0.0391)	-222.4223	0.2484* (0.0373)	0.0838* (0.0111)	-0.2255* (0.0289)	-172.1086	0.0000
Google	0.5659* (0.0432)	0.1332* (0.0547)	-246.9619	0.1102* (0.0349)	0.1373* (0.0137)	-0.1838* (0.0368)	-166.4483	0.0000
Microsoft	0.6010* (0.0408)	0.0339 (0.0434)	-229.8312	0.1581* (0.0350)	0.1480* (0.0132)	-0.2454* (0.0312)	-145.2000	0.0000
NVIDIA	0.5789* (0.0402)	0.0186 (0.0444)	-206.6774	0.2151* (0.0399)	0.0361* (0.0045)	-0.1859* (0.0343)	-146.0543	0.0000
EURSUD	0.5222* (0.0308)	0.0313 (0.0463)	-172.1332	0.1897* (0.0337)	0.6258* (0.0736)	-0.1426* (0.0308)	-114.2083	0.0000
GBP/USD	0.5895* (0.0338)	0.0519 (0.0585)	-236.0500	0.2599* (0.0507)	0.7039* (0.0778)	-0.3022* (0.0292)	-157.9306	0.0000
USD/JPY	0.5611* (0.0353)	0.1114* (0.0435)	-240.6455	0.1887* (0.0447)	0.5820* (0.0740)	-0.1669* (0.0339)	-158.1730	0.0000
BTC/USD	0.6908* (0.0639)	0.1837* (0.0748)	-249.8602	0.2351* (0.0582)	0.0220* (0.0027)	-0.2401* (0.0435)	-181.7658	0.0000

ETH/USD	0.7550* (0.0774)	0.1130 (0.0814)	-164.7198	0.1974* (0.0679)	0.0171* (0.0015)	-0.4946* (0.0566)	-99.9482	0.0000
LTC/USD	0.6707* (0.0593)	0.1149 (0.0660)	-193.9026	0.0556 (0.0380)	0.0156* (0.0018)	-0.3292* (0.0346)	-121.0097	0.0000
XRP/USD	0.6322* (0.0730)	0.2089* (0.0808)	-148.5824	0.2201* (0.0661)	0.0125* (0.0023)	-0.2655* (0.0586)	-110.2027	0.0000

Note. Robust Huber-White standard errors are reported in parentheses, * indicates that the null hypothesis is rejected at a 5% significance level, ln L - the logarithm of the likelihood function, LM *p*-value is the *p*-value from the likelihood ratio test based on the logarithm of the likelihood function for GPD vs GPD-P.
Source: author's work.

5.3. Forecasting Value-at-Risk

In this subsection, we compare the proposed model (the GARCH-GPD-P) against the GARCH-GPD and two benchmarks, namely the GARCH-n and the GARCH-t, for VaR forecasting. We formulate out-of-sample one-day-ahead forecasts of the conditional VaR (5% and 10% coverage level) based on the GARCH-n, GARCH-t, GARCH-GPD, and GARCH-GDP-P models, where parameters are estimated separately each day based on a rolling sample of a fixed size of 500 (approximately two years) and 1,000 separately. Then, the first observation from the sample is dropped and one is added to the end of the sample (the rolling window approach) to obtain the VaR forecasts. This process is repeated iteratively until all observations are exhausted, i.e. till 31st May 2023. Table A1 in the Appendix summarises the forecasting start and end dates as the number of forecasts used in the empirical study. We present the results only for the first group (500 observations used for the parameters estimation), as the results for the second group are similar and do not change the conclusions.

For backtesting purposes, we evaluate the VaR forecasts by testing their statistical properties, calculating loss measures and testing the superiority of VaR forecasts over the others. The statistical adequacy of VaR forecasts is verified by: unconditional coverage LR_{UC} proposed by Kupiec (1995), independence LR_{ind} and conditional coverage LR_{CC} tests designed by Christoffersen (1998), unconditional coverage J_{uc}, independence J_{ind} and conditional coverage J_{cc} tests devised by Candelon et al. (2011). Under Basel Accords (Basel Committee on Banking Supervision, 2011, 2019), financial institutions that report too many violations during the last year, need to apply additional capital charges directly linked to the number of these violations. It means that the unconditional coverage property is of paramount importance from the regulators' and financial institutions' point of view. In other words, rejecting the null hypothesis of the unconditional coverage test would result in too many violations and additional capital charges. A model leading to such outcome is by far undesirable for the market participants, regulators and financial institutions.

Firstly, Table A2, Table A3 and Table A4 (Appendix) present the results of the statistical properties of VaR for 10% and Table A5, Table A6 and Table A7 (Appendix) for 5%. Generally speaking, VaR forecasts from the GARCH-GPD-P, GARCH-GPD and GARCH-t models have better statistical properties than the ones obtained from the GARCH-n. Only VaR forecasts from the GARCH-GDP-P model meet both criteria i.e. unconditional coverage and independence properties at a 5% significance level for both coverage levels. In many cases VaR forecasts from the GARCH-n model have a significantly different number of violations and are not independent across time.

Secondly, we evaluate methods for VaR forecasting based on the same set of loss functions that are used in the simulation. Moreover, we calculate penalisation measure PM and its components, i.e. the penalisation measure for violation space $PM(VS)$ and safe space $PM(SS)$ proposed by Şener et al. (2012). The results for VaR(10%) are given in Tables 8–10 and for VaR(5%) in Tables 11–13. At a 10% coverage level, in many cases (mainly stocks and currencies), the GARCH-GPD-P model generates VaR forecasts that lead to the smallest loss functions from the regulator's perspective (RLF measures). The second most accurate model in terms of the regulator's loss functions is the GARCH-n model, especially for cryptocurrencies. For the firm's loss functions (FLFs) it is difficult to indicate a single best model, but the GARCH-t model seems to be the most prominent. The lowest values of penalisation measure PM are obtained for the proposed GARCH-GPD-P model (in the case of stocks and currencies) and for the GARCH-n model (in the case of cryptocurrencies). It is not surprising that for such a low coverage level as 10%, the standard GARCH model can produce more accurate VaR forecasts, as EVT-based methods are believed to be better at describing extreme quantiles such as 5%, 1%, 0.5% or even higher.

For a 5% coverage level, the situation is quite different, as the GARCH-GPD-P generates the most accurate VaR forecasts based on many of the regulator's loss functions for all three asset classes. When it comes to the FLFs, VaR forecasts from the GARCH-t model have the lowest values in most cases. For all the selected time series, penalisation measure PM is also the smallest for the GARCH-GPD-P model. The second most accurate model for the PM is either the GARCH-GPD or the GARCH-n model. It seems like the proposed model tends to overestimate the VaR because for most FLFs other models produce more accurate results. At a 5% coverage level, the results show that the GARCH-GPD-P is generally better than the competing models. The probable reason is that the use of high and low prices in the form of the Parkinson estimator for extreme observations generates a quick reaction to what is happening in the markets. If there is a jump in volatility, it will have an immediate reaction on the time-varying scale parameter in the GPD, thus producing higher VaR estimates. In turbulent times this mechanism is going to provide more accurate VaR estimates and result in a smaller number of violations (as reported in the unconditional coverage tests). On the other hand, in periods of low volatility, it could lead to VaR overestimation.

Table 8. The results of the loss measures for VaR(10%): stocks

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-
Amazon	RLF(L)$\cdot 10^{-3}$	1.1673	1.2924	1.3647	**1.1618**
	RLF(STS) ·	0.8163	0.9174	0.9657	**0.7798**
	RLF(C1) ·	**0.1795**	0.2090	0.2187	0.1973
	RLF(C2) ·	**0.3658**	0.4299	0.4515	0.3891
	RLF(C3) ·	**0.3730**	0.4092	0.4296	0.3757
	FLF(STS) ·	1.2030	1.2905	1.3210	**1.1609**
	FLF(C1) ·	2.1919	2.1958	2.1856	**2.1835**
	FLF(C2) ·	3.5947	3.5611	**3.4529**	3.6330
	FLF(C3) ·	9.3595	9.1635	**8.8889**	9.2737
	PM	0.2811	0.3544	0.3809	**0.2226**
	$PM_{VS} \cdot 10^{-3}$	0.4681	0.6243	0.6851	**0.3496**
	$PM_{SS} \cdot 10^{-3}$	2.0649	1.9534	**1.8303**	2.0350
Apple	RLF(L) $\cdot 10^{-3}$	1.0551	1.1739	1.0762	**0.9370**
	RLF(STS) ·	0.6661	0.7519	0.6822	**0.5700**
	RLF(C1) ·	0.2029	0.2406	0.2101	**0.1886**
	RLF(C2) ·	0.3341	0.3974	0.3468	**0.2999**
	RLF(C3) ·	0.3535	0.3956	0.3602	**0.3186**
	FLF(STS) ·	0.9882	1.0542	1.0026	**0.9133**
	FLF(C1) ·	2.1956	**2.2009**	2.2012	2.2084
	FLF(C2) ·	2.9798	**2.8910**	2.9837	3.1468
	FLF(C3) ·	7.9454	**7.6484**	7.9225	8.2488
	PM	0.2115	0.2631	0.2235	**0.1598**
	$PM_{VS} \cdot 10^{-3}$	0.3289	0.4370	0.3529	**0.2200**
	$PM_{SS} \cdot 10^{-3}$	1.5980	**1.4580**	1.5882	1.7500
Google	RLF(L) $\cdot 10^{-3}$	0.9718	1.0987	1.0865	**0.9658**
	RLF(STS) ·	0.6158	0.6957	0.6865	**0.5968**
	RLF(C1) ·	**0.1955**	0.2421	0.2322	0.2104
	RLF(C2) ·	**0.3624**	0.4444	0.4325	0.3802
	RLF(C3) ·	**0.3176**	0.3621	0.3516	**0.3176**
	FLF(STS) ·	0.9213	0.9759	0.9727	**0.9118**
	FLF(C1) ·	2.2319	**2.2209**	2.2231	2.2441
	FLF(C2) ·	2.8904	**2.7375**	2.7851	3.0595
	FLF(C3) ·	7.3364	**6.9504**	7.0520	7.5121
	PM	0.1650	0.2005	0.1931	**0.1581**
	$PM_{VS} \cdot 10^{-3}$	0.2464	0.3286	0.3110	**0.2272**
	$PM_{SS} \cdot 10^{-3}$	1.6669	**1.4705**	1.5216	1.7676
Microsoft	RLF(L) $\cdot 10^{-3}$	0.8631	1.0015	0.9621	**0.8597**
	RLF(STS) ·	0.5061	0.5885	0.5591	**0.4797**
	RLF(C1) ·	**0.1958**	0.2477	0.2333	0.2128
	RLF(C2) ·	**0.3208**	0.4028	0.3785	0.3345
	RLF(C3) ·	0.2977	0.3435	0.3299	**0.2951**
	FLF(STS) ·	0.7930	0.8514	0.8283	**0.7691**
	FLF(C1) ·	2.2076	2.2072	**2.2033**	2.2148
	FLF(C2) ·	2.7333	**2.6242**	2.6446	2.8044
	FLF(C3) ·	7.0240	**6.6725**	6.7629	7.0730
	PM	0.1489	0.1790	0.1682	**0.1344**
	$PM_{VS} \cdot 10^{-3}$	0.2148	0.2824	0.2592	**0.1845**
	$PM_{SS} \cdot 10^{-3}$	1.4652	**1.2994**	1.3404	1.4999
NVIDIA	RLF(L) $\cdot 10^{-3}$	1.9801	2.2310	2.1285	**1.8861**
	RLF(STS) ·	1.6191	1.8300	1.7335	**1.5191**
	RLF(C1) ·	**0.1858**	0.2202	0.2136	0.1941
	RLF(C2) ·	0.5282	0.6300	0.5992	**0.5263**
	RLF(C3) ·	0.5082	0.5726	0.5511	**0.5006**
	FLF(STS) ·	2.1426	2.3193	2.2342	**2.0558**
	FLF(C1) ·	2.2109	**2.2057**	2.2092	2.2115
	FLF(C2) ·	4.8917	**4.7013**	4.7701	5.0555
	FLF(C3) ·	12.6114	**12.1108**	12.2747	12.8274
	PM	0.4731	0.6044	0.5293	**0.3832**
	$PM_{VS} \cdot 10^{-3}$	0.8069	1.0835	0.9285	**0.6206**
	$PM_{SS} \cdot 10^{-3}$	2.7036	**2.4547**	2.5349	2.8170

Note. The lowest values of loss functions are marked in bold. RLF(L) is the loss function proposed by Lopez (1998), RLF(STS), FLF(STS) are the loss functions proposed by Sarma et al. (2003), RLF(C1), RLF(C2), RLF(C3), FLF(C1), FLF(C2) and FLF(C3) are the loss functions proposed by Caporin (2008),

PM, PMVS PMSS are the penalisation measure, the penalisation measure for the violation space and the penalisation measure for the safe space, respectively, proposed by Şener et al. (2012).
Source: author's work.

Table 9. The results of the loss measures for VaR(10%): currencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
EUR/USD	RLF(L)$\cdot 10^{-3}$	0.4436	0.4858	0.4693	**0.4365**
	RLF(STS) ·	0.0686	0.0728	0.0703	**0.0625**
	RLF(C1) ·	0.1699	0.1874	0.1814	**0.1640**
	RLF(C2) ·	0.1005	0.1089	0.1055	**0.0939**
	RLF(C3) ·	0.1105	0.1188	0.1150	**0.1038**
	FLF(STS) ·	0.2030	0.2020	**0.2014**	0.2017
	FLF(C1) ·	2.2485	**2.2368**	2.2382	2.2568
	FLF(C2) ·	1.2112	**1.1740**	1.1836	1.2616
	FLF(C3) ·	3.1510	**3.0790**	3.0995	3.2246
	PM	0.0304	0.0313	0.0304	**0.0283**
	PMVS $\cdot 10^{-3}$	0.0271	0.0308	0.0284	**0.0207**
	PMSS$\cdot 10^{-3}$	0.6984	**0.6619**	0.6714	0.7321
GBP/USD	RLF(L) $\cdot 10^{-3}$	**0.5191**	0.5710	0.5731	0.5307
	RLF(STS) ·	**0.1651**	0.1800	0.1721	0.1567
	RLF(C1) ·	**0.1867**	0.2268	0.2142	0.1948
	RLF(C2) ·	**0.2045**	0.2516	0.2266	0.2073
	RLF(C3) ·	0.1334	0.1480	0.1446	**0.1310**
	FLF(STS) ·	0.3045	0.3104	**0.3031**	0.3080
	FLF(C1) ·	2.2694	2.2597	**2.2530**	2.2617
	FLF(C2) ·	1.3585	1.3087	**1.3058**	1.5933
	FLF(C3) ·	3.2785	3.1399	**3.1500**	3.5113
	PM	0.0390	0.0456	0.0407	**0.0367**
	PMVS $\cdot 10^{-3}$	0.0433	0.0614	0.0504	**0.0318**
	PMSS$\cdot 10^{-3}$	0.7517	**0.6763**	0.6862	0.8779
USD/JPY	RLF(L) $\cdot 10^{-3}$	**0.4808**	0.5446	0.5400	0.4915
	RLF(STS) ·	0.1268	0.1496	0.1410	**0.1225**
	RLF(C1) ·	**0.1774**	0.2454	0.2216	0.1991
	RLF(C2) ·	**0.1550**	0.2339	0.1880	0.1637
	RLF(C3) ·	**0.1305**	0.1541	0.1496	0.1330
	FLF(STS) ·	**0.2651**	0.2762	0.2682	0.2692
	FLF(C1) ·	2.3518	2.3696	**2.3459**	2.3673
	FLF(C2) ·	1.4067	1.3781	**1.3441**	1.5687
	FLF(C3) ·	3.3052	**3.1276**	3.1338	3.4714
	PM	0.0369	0.0448	0.0401	**0.0368**
	PMVS $\cdot 10^{-3}$	0.0376	0.0587	0.0487	**0.0344**
	PMSS$\cdot 10^{-3}$	0.7497	**0.6608**	0.6657	0.8057

Note. As in Table 8.
Source: author's work.

Table 10. The results of the loss measures for VaR(10%)–: cryptocurrencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
BTC/USD	RLF(L)$\cdot 10^{-3}$	**4.9515**	6.3602	6.3294	5.9225
	RLF(STS) ·	**4.7695**	6.0912	6.0794	5.6985
	RLF(C1) ·	**0.1526**	0.2963	0.2445	0.2315
	RLF(C2) ·	**1.2292**	2.0489	1.8486	1.6898
	RLF(C3) ·	**0.5781**	0.7856	0.7752	0.7212
	FLF(STS) ·	**5.3468**	6.5286	6.5325	6.2507
	FLF(C1) ·	**1.7127**	1.8737	1.7573	1.7743
	FLF(C2) ·	7.0829	6.9557	**6.7321**	6.5502
	FLF(C3) ·	13.9326	**11.6914**	11.9423	11.7111
	PM	**1.4267**	3.5851	3.8839	2.6811
	$PM_{VS} \cdot 10^{-3}$	**1.6313**	4.4137	4.7848	3.2228
	$PM_{SS} \cdot 10^{-3}$	3.2737	**2.2540**	2.3797	3.1668
ETH/USD	RLF(L) $\cdot 10^{-3}$	5.2748	6.3902	6.3386	**5.2233**
	RLF(STS) ·	5.1718	6.2482	6.1946	**5.1053**
	RLF(C1) ·	**0.0701**	0.1124	0.1105	0.0851
	RLF(C2) ·	**0.8087**	1.1761	1.2140	0.8351
	RLF(C3) ·	**0.4167**	0.5482	0.5357	0.4409
	FLF(STS) ·	**5.5904**	6.5766	6.5308	5.5920
	FLF(C1) ·	0.9177	**0.9107**	0.9187	0.9516
	FLF(C2) ·	4.6726	**4.1980**	4.3716	4.3941
	FLF(C3) ·	10.0032	**8.5455**	8.6827	9.4151
	PM	**1.8349**	4.9249	4.5914	1.9632
	$PM_{VS} \cdot 10^{-3}$	**1.2472**	3.5957	3.3420	1.3051
	$PM_{SS} \cdot 10^{-3}$	2.3948	**1.7145**	1.7918	2.4200
LTC/USD	RLF(L) $\cdot 10^{-3}$	**7.4716**	9.3778	10.3435	7.6661
	RLF(STS) ·	**7.3226**	9.1648	10.1135	7.5031
	RLF(C1) ·	**0.0947**	0.1589	0.1760	0.1107
	RLF(C2) ·	**1.0165**	1.5178	1.8399	1.1282
	RLF(C3) ·	**0.6411**	0.8814	0.9372	0.6621
	FLF(STS) ·	**8.0707**	9.7507	10.6799	8.4103
	FLF(C1) ·	1.4017	**1.3917**	1.4153	1.3962
	FLF(C2) ·	8.3638	**7.4050**	7.8857	7.7415
	FLF(C3) ·	17.3272	14.7533	**14.5387**	14.5567
	PM	**1.6890**	3.8954	4.9088	1.8033
	$PM_{VS} \cdot 10^{-3}$	**1.7267**	4.3624	5.5461	1.7733
	$PM_{SS} \cdot 10^{-3}$	4.5184	3.2019	**3.1100**	5.1608
XRP/USD	RLF(L) $\cdot 10^{-3}$	**5.7677**	7.8465	7.4876	6.1725
	RLF(STS) ·	**5.6807**	7.7065	7.3516	6.0565
	RLF(C1) ·	**0.0566**	0.1129	0.1093	0.0865
	RLF(C2) ·	**0.7561**	1.3285	1.2855	0.9463
	RLF(C3) ·	**0.3675**	0.5594	0.5405	0.4531
	FLF(STS) ·	**6.2024**	8.0598	7.7326	6.6928
	FLF(C1) ·	0.9725	**0.9530**	0.9623	0.9934
	FLF(C2) ·	6.4486	**5.4519**	5.6352	5.6517
	FLF(C3) ·	11.7213	**8.8927**	9.3912	10.2841
	PM	**1.8440**	9.2295	8.6622	5.2533
	$PM_{VS} \cdot 10^{-3}$	**1.2419**	6.9741	6.5271	3.7641
	$PM_{SS} \cdot 10^{-3}$	3.2520	**1.8737**	2.1079	3.9691

Note. As in Table 8.
Source: author's work.

Table 11. The results of the loss measures for VaR(5%): stocks

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	RLF(L) $\cdot 10^{-3}$	0.6325	0.6477	0.6669	**0.5874**
	RLF(STS) ·	0.4245	0.4377	0.4569	**0.3834**
	RLF(C1) ·	**0.0772**	0.0820	0.0801	0.0865
	RLF(C2) ·	**0.1573**	0.1647	0.1638	0.1588
	RLF(C3) ·	0.2003	0.2086	0.2118	**0.1939**
	FLF(STS) ·	**0.9444**	0.9628	0.9686	0.9768
	FLF(C1) ·	2.3212	2.3297	**2.3190**	2.3463
	FLF(C2) ·	4.8079	4.9064	**4.7376**	6.0414
	FLF(C3) ·	11.4991	11.6156	**11.3582**	12.8568
	PM	0.1342	0.1446	0.1493	**0.1323**
	$PM_{VS} \cdot 10^{-3}$	0.1100	0.1292	0.1448	**0.0681**
	$PM_{SS} \cdot 10^{-3}$	3.1390	3.1812	**3.0669**	3.2644
Apple	RLF(L) $\cdot 10^{-3}$	0.5681	0.5972	0.5112	**0.4457**
	RLF(STS) ·	0.3471	0.3632	0.3062	**0.2487**
	RLF(C1) ·	0.0882	0.0941	**0.0750**	0.0767
	RLF(C2) ·	0.1376	0.1463	0.1161	**0.1135**
	RLF(C3) ·	0.1932	0.2034	0.1708	**0.1518**
	FLF(STS) ·	0.7835	0.7898	**0.7666**	0.7736
	FLF(C1) ·	2.3089	**2.2974**	2.3450	2.3933
	FLF(C2) ·	3.9964	**3.9017**	4.2822	5.3016
	FLF(C3) ·	9.7129	**9.5570**	10.1199	11.3066
	PM	0.1131	0.1171	0.1084	**0.1050**
	$PM_{VS} \cdot 10^{-3}$	0.0924	0.1043	0.0730	**0.0428**
	$PM_{SS} \cdot 10^{-3}$	2.4387	**2.3648**	2.6329	2.6291
Google	RLF(L) $\cdot 10^{-3}$	0.5593	0.5958	0.5433	**0.4846**
	RLF(STS) ·	0.3583	0.3758	0.3473	**0.2926**
	RLF(C1) ·	0.0926	0.1015	**0.0882**	0.0931
	RLF(C2) ·	0.1725	0.1888	0.1650	**0.1646**
	RLF(C3) ·	0.1877	0.1981	0.1816	**0.1570**
	FLF(STS) ·	0.7687	0.7725	**0.7654**	0.8018
	FLF(C1) ·	2.3650	**2.3413**	2.3773	2.4475
	FLF(C2) ·	3.8830	**3.7322**	3.9813	5.5264
	FLF(C3) ·	8.9989	**8.7753**	9.1343	10.8414
	PM	0.1034	0.1043	0.1032	**0.1009**
	$PM_{VS} \cdot 10^{-3}$	0.0779	0.0860	0.0743	**0.0416**
	$PM_{SS} \cdot 10^{-3}$	2.4986	**2.3821**	2.5583	2.5763
Microsoft	RLF(L) $\cdot 10^{-3}$	0.4737	0.5225	0.4683	**0.4385**
	RLF(STS) ·	0.2767	0.2975	0.2753	**0.2385**
	RLF(C1) ·	0.0877	0.0990	**0.0868**	0.0917
	RLF(C2) ·	0.1443	0.1589	0.1434	**0.1392**
	RLF(C3) ·	0.1662	0.1805	0.1617	**0.1513**
	FLF(STS) ·	**0.6650**	0.6741	0.6666	0.6929
	FLF(C1) ·	2.3389	**2.3233**	2.3427	2.3870
	FLF(C2) ·	3.6320	**3.5223**	3.6758	4.7211
	FLF(C3) ·	8.6167	**8.4472**	8.6615	9.8578
	PM	0.0933	0.0966	0.0929	**0.0894**
	$PM_{VS} \cdot 10^{-3}$	0.0654	0.0760	0.0637	**0.0394**
	$PM_{SS} \cdot 10^{-3}$	2.2326	**2.1549**	2.2510	2.1770
NVIDIA	RLF(L) $\cdot 10^{-3}$	1.1063	1.1609	1.0660	**0.9396**
	RLF(STS) ·	0.9133	0.9609	0.8700	**0.7356**
	RLF(C1) ·	0.0822	0.0863	**0.0796**	0.0834
	RLF(C2) ·	0.2360	0.2495	0.2248	**0.2139**
	RLF(C3) ·	0.2849	0.2983	0.2749	**0.2517**
	FLF(STS) ·	1.6190	1.6527	1.5851	**1.5472**
	FLF(C1) ·	2.3288	**2.3164**	2.3338	2.3716
	FLF(C2) ·	6.5116	**6.3493**	6.6322	8.2795
	FLF(C3) ·	15.4668	**15.2363**	15.6374	17.4874
	PM	0.2165	0.2293	0.2033	**0.1877**
	$PM_{VS} \cdot 10^{-3}$	0.2181	0.2499	0.1877	**0.1259**
	$PM_{SS} \cdot 10^{-3}$	4.1113	**3.9930**	4.1836	4.1431

Note. As in Table 8.
Source: author’s work.

Table 12. The results of the loss measures for VaR(5%): currencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-	GARCH-GPD-P
EUR/USD	RLF(L) $\cdot 10^{-3}$	0.2222	0.2214	0.2236	**0.2160**
	RLF(STS) ·	0.0352	0.0354	0.0346	**0.0300**
	RLF(C1) ·	0.0707	0.0721	0.0712	**0.0703**
	RLF(C2) ·	0.0429	0.0429	0.0428	**0.0410**
	RLF(C3) ·	0.0560	0.0569	0.0555	**0.0498**
	FLF(STS) ·	0.2169	0.2158	**0.2157**	0.2354
	FLF(C1) ·	2.4041	2.3947	**2.3936**	2.4555
	FLF(C2) ·	1.6682	**1.6502**	1.6591	1.8337
	FLF(C3) ·	3.8794	**3.8549**	3.8669	3.9032
	PM	0.0299	0.0297	0.0295	**0.0280**
	$PM_{VS} \cdot 10^{-3}$	0.0069	0.0071	0.0064	**0.0041**
	$PM_{SS} \cdot 10^{-3}$	1.0597	**1.0470**	1.0522	1.0436
GBP/USD	RLF(L) $\cdot 10^{-3}$	0.3286	0.3450	0.3162	**0.3098**
	RLF(STS) ·	0.1176	0.1260	0.1152	**0.1038**
	RLF(C1) ·	0.0839	0.0983	0.0804	**0.0792**
	RLF(C2) ·	0.1152	0.1370	0.1121	**0.1097**
	RLF(C3) ·	0.0762	0.0821	0.0728	**0.0634**
	FLF(STS) ·	0.3030	0.3061	**0.3024**	0.3646
	FLF(C1) ·	2.4199	**2.4085**	2.4235	2.4602
	FLF(C2) ·	1.8040	**1.7435**	1.8210	1.9052
	FLF(C3) ·	4.0239	**3.9294**	4.0424	4.0484
	PM	0.0343	0.0351	0.0342	**0.0328**
	$PM_{VS} \cdot 10^{-3}$	0.0137	0.0183	0.0130	**0.0060**
	$PM_{SS} \cdot 10^{-3}$	1.1262	**1.0713**	1.1336	1.1398
USD/JPY	RLF(L) $\cdot 10^{-3}$	**0.2648**	0.2975	0.2818	0.2672
	RLF(STS) ·	0.0788	0.0875	0.0808	**0.0662**
	RLF(C1) ·	**0.0786**	0.0974	0.0860	0.0856
	RLF(C2) ·	0.0755	0.1035	0.0801	**0.0719**
	RLF(C3) ·	0.0743	0.0804	0.0781	**0.0699**
	FLF(STS) ·	0.2646	0.2682	**0.2623**	0.2677
	FLF(C1) ·	2.4877	2.4875	**2.4722**	2.4275
	FLF(C2) ·	1.8474	1.8174	**1.8035**	1.9426
	FLF(C3) ·	4.0593	**3.9858**	3.9910	4.0351
	PM	0.0338	0.0355	0.0334	**0.0308**
	$PM_{VS} \cdot 10^{-3}$	0.0116	0.0170	0.0124	**0.0085**
	$PM_{SS} \cdot 10^{-3}$	1.1205	**1.0833**	1.0872	1.1430

Note. As in Table 8.
Source: author's work.

Table 13. The results of the loss measures for VaR(5%)–: cryptocurrencies

Time series	Loss function	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
BTC/USD	RLF(L)$\cdot 10^{-3}$	3.5872	4.1160	3.8117	**3.4967**
	RLF(STS) ·	3.4682	3.9650	3.6807	**3.3617**
	RLF(C1) ·	0.0849	0.1307	0.0952	**0.0711**
	RLF(C2) ·	0.7562	0.9925	0.8195	**0.7209**
	RLF(C3) ·	0.3717	0.4469	0.4038	**0.3143**
	FLF(STS) ·	**4.2347**	4.6415	4.4130	4.6220
	FLF(C1) ·	1.7660	1.8090	**1.7594**	1.8124
	FLF(C2) ·	8.9334	**8.1458**	8.5988	8.9265
	FLF(C3) ·	17.1626	**15.5972**	16.5849	17.0877
	PM	0.5468	0.8304	0.6360	**0.5281**
	$\mathrm{PM_{VS}} \cdot 10^{-3}$	0.4400	0.8360	0.5659	**0.4015**
	$\mathrm{PM_{SS}} \cdot 10^{-3}$	4.7747	**4.0613**	4.5249	4.4202
ETH/USD	RLF(L) $\cdot 10^{-3}$	4.0074	4.4719	4.1703	**3.1127**
	RLF(STS) ·	3.9374	4.3889	4.0963	**3.0517**
	RLF(C1) ·	**0.0353**	0.0470	0.0399	0.0354
	RLF(C2) ·	0.4727	0.5584	0.4973	**0.3214**
	RLF(C3) ·	0.2754	0.3308	0.2982	**0.2444**
	FLF(STS) ·	4.4912	4.8781	4.6284	**4.0703**
	FLF(C1) ·	0.9677	**0.9407**	0.9620	1.0239
	FLF(C2) ·	6.0753	**5.2895**	5.8584	5.8557
	FLF(C3) ·	12.3425	**11.1961**	11.9597	11.4522
	PM	0.8659	1.1373	0.9703	**0.8122**
	$\mathrm{PM_{VS}} \cdot 10^{-3}$	0.4635	0.6946	0.5494	**0.2641**
	$\mathrm{PM_{SS}} \cdot 10^{-3}$	3.5066	**2.9737**	3.3584	3.9539
LTC/USD	RLF(L) $\cdot 10^{-3}$	5.3013	5.8459	6.4325	**3.9071**
	RLF(STS) ·	5.2053	5.7269	6.3185	**3.8301**
	RLF(C1) ·	0.0468	0.0613	0.0639	**0.0423**
	RLF(C2) ·	0.5601	0.6532	0.7493	**0.3916**
	RLF(C3) ·	0.4089	0.4843	0.5103	**0.3270**
	FLF(STS) ·	6.1904	6.6076	7.2043	**5.7100**
	FLF(C1) ·	1.4763	**1.4332**	1.4513	1.6000
	FLF(C2) ·	10.8787	**9.5686**	9.8804	10.2211
	FLF(C3) ·	21.4234	**19.5871**	19.7093	18.9232
	PM	0.7684	0.9737	1.0589	**0.7526**
	$\mathrm{PM_{VS}} \cdot 10^{-3}$	0.5474	0.8363	0.9315	**0.2353**
	$\mathrm{PM_{SS}} \cdot 10^{-3}$	6.5813	**5.6306**	5.7035	5.4774
XRP/USD	RLF(L) $\cdot 10^{-3}$	4.4567	5.5485	5.1969	**3.0706**
	RLF(STS) ·	4.4077	5.4825	5.1319	**3.0116**
	RLF(C1) ·	**0.0309**	0.0462	0.0422	0.0356
	RLF(C2) ·	0.4579	0.6333	0.5935	**0.3698**
	RLF(C3) ·	0.2601	0.3353	0.3136	**0.2392**
	FLF(STS) ·	5.0882	6.0244	5.7262	**4.6218**
	FLF(C1) ·	1.0286	**0.9887**	1.0041	1.0570
	FLF(C2) ·	8.2378	**6.4892**	7.1847	6.7012
	FLF(C3) ·	14.5282	**11.9674**	12.9762	13.1000
	PM	0.9431	1.5521	1.3568	**0.8611**
	$\mathrm{PM_{VS}} \cdot 10^{-3}$	0.4785	1.0102	0.8356	**0.2856**
	$\mathrm{PM_{SS}} \cdot 10^{-3}$	4.6401	**3.4052**	3.8779	4.8478

Note. As in Table 8.
Source: author's work.

Table 14. The results of the loss measures for VaR(10%): a model with the lowest loss measure

	Amazon	Apple	Google	Microsoft	NVIDIA	EURSUD	GBPUSD	USDJPY	BTCUSD	ETHUSD	LTCUSD	XRPUSD
RLF(L)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n
RLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n
RLF(C1)	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
RLF(C2)	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
RLF(C3)	GARCH-n	GARCH-GPD-P	GARCH-n/GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
FLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD	GARCH-n	GARCH-n	GARCH-n	GARCH-n	GARCH-n
FLF(C1)	GARCH-GPD-P	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-t	GARCH-t	GARCH-t
FLF(C2)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t
FLF(C3)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t
PM	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n
PM_{VS}	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-n	GARCH-n	GARCH-n
PM_{SS}	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t

Note. As in Table 8.
Source: author’s work.

Table 15. The results of the loss measures for VaR(5%): a model with the lowest loss measure

	Amazon	Apple	Google	Microsoft	NVIDIA	EURSUD	GBPUSD	USDJPY	BTCUSD	ETHUSD	LTCUSD	XRPUSD
RLF(L)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(STS)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(C1)	GARCH-n	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-GPD-P	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n	GARCH-GPD-P	GARCH-n
RLF(C2)	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
RLF(C3)	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
FLF(STS)	GARCH-n	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-GPD-P	GARCH-GPD	GARCH-GPD	GARCH-GPD	GARCH-n	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
FLF(C1)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-GPD	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t
FLF(C2)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t
FLF(C3)	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t
PM	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
PM_{VS}	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P	GARCH-GPD-P
PM_{SS}	GARCH-GPD	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t	GARCH-t

Source: author's work.

Tables 14 and 15 provide a summary of the models with the lowest loss measure for all times series used in the empirical analysis, for VaR at a 10% and 5% coverage level, respectively. At a 10% probability level, the GARCH-GPD-P and GARCH-n models resulted in 49 cases (out of 144) with the lowest values of the loss measures. At a 5% probability level, the GARCH-GPD-P model resulted in 78 cases with the lowest values of the loss measures.

Thirdly, we apply a predictive ability test for penalisation measure $PM(\varphi, VaR)$ proposed by Şener et al. (2012) to verify the obtained results statistically. Rejecting the null hypothesis means that a given model is worse in terms of VaR forecasting measured by the penalisation measure than any other competing model. Tables 16 and 17 present the results of the predictive ability test for VaR(10%) and VaR(5%). At a 10% coverage level, we do not reject the null hypothesis for the GARCH-GPD-P and the GARCH-n models (in almost all cases). This means that the differences in VaR forecasts from GARCH-n and GARCH-GPD-P across all four competing models are statistically significant. At a 5% coverage level, we can see that the GARCH-GPD-P and GARCH-t models are significantly more accurate than other models. These results are in line with the ones obtained for the loss functions.

Table 16. The *p*-values of the predictive ability test (Şener et al., 2012) for VaR(10%) based on the penalisation measure

Assets	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	0.6180	0.0000*	0.0000*	1.0000
Apple	0.0335*	0.0000*	0.0000*	1.0000
Google	1.0000	0.0000*	0.0000*	1.0000
Microsoft	1.0000	0.0000*	0.0000*	1.0000
NVIDIA	0.8460	0.0000*	0.0000*	1.0000
EUR/USD	0.0000*	0.0000*	0.0000*	1.0000
GBP/USD	1.0000	0.0000*	0.0000*	0.5023
USD/JPY	1.0000	0.0000*	0.0000*	0.6388
BTC/USD	1.0000	0.0000*	0.0000*	0.0000*
ETH/USD	1.0000	0.0000*	0.0000*	1.0000
LTC/USD	1.0000	0.0000*	0.0000*	1.0000
XRP/USD	1.0000	0.0000*	0.0000*	0.0000*

Note. * indicates that the null hypothesis is rejected at a 5% significance level.
Source: author's work.

Table 17. The *p*-values of the predictive ability test (Şener et al., 2012) for VaR(5%) based on the penalisation measure

Assets	GARCH-n	GARCH-t	GARCH-GPD	GARCH-GPD-P
Amazon	0.0000*	0.0000*	0.0000*	1.0000
Apple	0.0000*	0.3537	0.0000*	1.0000
Google	0.0000*	1.0000	0.0000*	0.8814
Microsoft	0.0000*	1.0000	0.0000*	1.0000
NVIDIA	0.0000*	1.0000	0.0001*	1.0000
EUR/USD	0.0000*	1.0000	0.0000*	1.0000
GBP/USD	0.0000*	1.0000	0.0001*	1.0000
USD/JPY	0.2369	0.0000*	0.0000*	1.0000
BTC/USD	0.0000*	0.0000*	0.0000*	1.0000
ETH/USD	0.0000*	0.0148*	0.0000*	1.0000
LTC/USD	0.0000*	0.1938	0.0000*	1.0000
XRP/USD	0.0000*	0.0000*	0.0000*	1.0000

Note. * indicates that the null hypothesis is rejected at a 5% significance level.
Source: author's work.

5.4. Forecasting an expected shortfall

In this subsection, we compare the proposed model (the GARCH-GPD-P) against the GARCH-GPD and two benchmarks, the GARCH-n and the GARCH-t, for the Expected Shortfall forecasting. The forecasting procedure is similar to the one for VaR in subsection 0.

Tables 18 19 present the results of the ES statistical properties for a 10% and 5% coverage level, respectively. At a 10% and 5% probability level only ES forecasts from the GARCH-GPD-P model result in the not-rejection of the null hypothesis. On the other hand, the GARCH-n model leads to the failing of the unconditional coverage property in 5 cases and the independence property in 3 cases, the GARCH-t model leads to the failing of the independence property in 3 cases and the GARCH-GPD model leads to the failing of the independence property in 2 cases, at a 10% probability level. At a 5% probability, the GARCH-n model leads to the failing of the unconditional coverage property in 6 cases and the independence property in 2 cases, the GARCH-t model leads to the failing of the unconditional property in 4 cases and the independence property in 3 cases and the GARCH-GPD model leads to the failing of the unconditional property and the independence property in 1 case. The results indicate that the Expected Shortfall forecasts obtained from the GARCH-GPD-P model are better than thos of the other competing models. This is somewhat confirmed by the mean of cumulative violations H_t that should be in theory equal to $\alpha/2$. The mean of cumulative violation process H_t for the GARCH-GPD-P is closer to the desired level than any other competing model.

Table 18. The results of backtesting for ES(10%) based on Du and Escanciano (2016) tests

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean H_t	*p*-value	Mean H_t	*p*-value	Mean H_t	*p*-value	Mean H_t
Amazon	DE_{UC}	0.7284	0.0510	0.5887	0.0515	0.4208	0.0523	0.1586	0.0460
	DE_{IND}	0.0010	-	0.0003	-	0.0012	-	0.5472	-
Apple	DE_{UC}	0.0126	0.0570	0.0004	0.0600	0.5367	0.0517	0.1700	0.0459
	DE_{IND}	0.4125	-	0.2759	-	0.3879	-	0.9371	-
Google	DE_{UC}	0.6399	0.0513	0.0670	0.0552	0.7052	0.0508	0.8382	0.0506
	DE_{IND}	0.0445	-	0.0206	-	0.0449	-	0.3718	-
Microsoft	DE_{UC}	0.5150	0.0518	0.0504	0.0555	0.4172	0.0523	0.2488	0.0468
	DE_{IND}	0.1564	-	0.0764	-	0.2140	-	0.3858	-
Nvidia	DE_{UC}	0.6284	0.0514	0.1995	0.0536	0.1572	0.0545	0.6640	0.0512
	DE_{IND}	0.0205	-	0.0207	-	0.0661	-	0.5348	-
EUR/USD	DE_{UC}	0.5672	0.0517	0.2451	0.0534	0.2319	0.0535	0.2739	0.0532
	DE_{IND}	0.7575	-	0.2791	-	0.8661	-	0.4593	-
GBP/USD	DE_{UC}	0.4309	0.0477	0.7011	0.0511	0.5965	0.0484	0.8642	0.0495
	DE_{IND}	0.5643	-	0.4843	-	0.2327	-	0.7765	-
USD/JPY	DE_{UC}	0.0245	0.0433	0.3083	0.0470	0.3649	0.0473	0.3156	0.0470
	DE_{IND}	0.3724	-	0.0529	-	0.1788	-	0.4759	-
BTC/USD	DE_{UC}	0.0390	0.0420	0.0527	0.0527	0.5175	0.0475	0.9848	0.0499
	DE_{IND}	0.5416	-	0.4537	-	0.5555	-	0.6605	-
ETH/USD	DE_{UC}	0.0447	0.0447	0.6197	0.0528	0.4724	0.0460	0.7224	0.0520
	DE_{IND}	0.3114	-	0.2075	-	0.9701	-	0.1606	-
LTC/USD	DE_{UC}	0.1379	0.0437	0.7778	0.0512	0.6524	0.0519	0.6753	0.0518
	DE_{IND}	0.4783	-	0.4317	-	0.4118	-	0.4350	-
XRP/USD	DE_{UC}	0.0277	0.0376	0.9871	0.0499	0.6980	0.0522	0.7553	0.0482
	DE_{IND}	0.8502	-	0.8329	-	0.8027	-	0.9801	-

Note. * indicates that the null hypothesis is rejected at a 5% significance level. For the independence test DE_{IND}, we calculate statistics up to 5 lags. DE_{UC}, DE_{IND} are the unconditional coverage and independence tests, respectively, proposed by Du and Escanciano (2016). H_t is the cumulative violation process.
Source: author's work.

Table 19. The results of backtesting for ES(5%) based on Du and Escanciano (2016) tests

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		p-value	Mean $H_t(5\%)$	*p*-value	Mean $H_t(5\%)$	*p*-value	Mean $H_t(5\%)$	*p*-value	Mean $H_t(5\%)$
Amazon	DE_{UC}	0.0157	0.0299	0.2135	0.0275	0.0464	0.0290	0.4256	0.0246
	DE_{IND}	0.0039	-	0.0276	-	0.0673	-	0.8085	-
Apple	DE_{UC}	0.0000	0.0347	0.0001	0.0329	0.2871	0.0272	0.5590	0.0262
	DE_{IND}	0.8345	-	0.7643	-	0.9919	-	0.2187	-
Google	DE_{UC}	0.0003	0.0323	0.0075	0.0304	0.5245	0.0263	0.7205	0.0257
	DE_{IND}	0.3295	-	0.5126	-	0.2664	-	0.6382	-
Microsoft	DE_{UC}	0.0052	0.0307	0.0176	0.0298	0.1282	0.0283	0.2330	0.0274
	DE_{IND}	0.3715	-	0.3548	-	0.6375	-	0.2524	-
Nvidia	DE_{UC}	0.0070	0.0305	0.1460	0.0280	0.3193	0.0270	0.5386	0.0263
	DE_{IND}	0.0005	-	0.0003	-	0.0227	-	0.7602	-
EUR/USD	DE_{UC}	0.3532	0.0269	0.7455	0.0256	0.8299	0.0254	0.7884	0.0245
	DE_{IND}	0.3936	-	0.5435	-	0.6237	-	0.2494	-
GBP/USD	DE_{UC}	0.0055	0.0306	0.0179	0.0297	0.8459	0.0254	0.9584	0.0249
	DE_{IND}	0.0719	-	0.0037	-	0.0669	-	0.5972	-
USD/JPY	DE_{UC}	0.4784	0.0264	0.7762	0.0244	0.8108	0.0245	0.9918	0.0250
	DE_{IND}	0.8998	-	0.2625	-	0.6726	-	0.1678	-
BTC/USD	DE_{UC}	0.0897	0.0292	0.1403	0.0287	0.1071	0.0290	0.8934	0.0247
	DE_{IND}	0.2789	-	0.0257	-	0.6941	-	0.1793	-
ETH/USD	DE_{UC}	0.4789	0.0273	0.3073	0.0284	0.2041	0.0208	0.6237	0.0234
	DE_{IND}	0.1311	-	0.1568	-	0.7552	-	0.1719	-
LTC/USD	DE_{UC}	0.7242	0.0260	0.6869	0.0261	0.5138	0.0270	0.9312	0.0252
	DE_{IND}	0.3733	-	0.5664	-	0.6328	-	0.3182	-
XRP/USD	DE_{UC}	0.4526	0.0225	0.5390	0.0230	0.2253	0.0210	0.3609	0.0220
	DE_{IND}	0.3556	-	0.1299	-	0.9194	-	0.3142	-

Note. As in Table 18.
Source: author's work.

6. Conclusions

The high and low prices and their range are believed to provide additional and useful information regarding the volatility of returns. Therefore, incorporating such prices in volatility models can lead to better estimates and forecasts of the conditional variance and covariance, but they may also be used to obtain more accurate estimates of risk measures. There is a growing body of literature showing that range-based models or models that use range-based estimators may outperform standard volatility models (see, see e.g. Asai, 2013; Brandt & Jones, 2006; Chou, 2005; Fałdziński et al., 2024; Fiszeder & Fałdziński, 2019; Fiszeder & Perczak, 2016; Fiszeder et al., 2019; Molnár, 2016; Xie, 2019). However, high and low prices are rarely used to describe the volatility of extreme observations. It seems natural that high and low prices provide additional insight into the dynamic behaviour of the returns that are at the tails of their distribution. In this paper, we propose an extension of the GARCH-GPD approach of McNeil and Frey (2000) by incorporating the range-based estimator to describe the magnitudes of threshold exceedances. We thus extend the Generalised Pareto Distribution by adding a meaningful covariate. The proposed model, the GARCH-GPD-P, is compared to the GARCH-GPD and two standard benchmarks i.e. the GARCH model with the normal and *t*-distributed errors.

We evaluate the competing models based on the Monte Carlo simulation and empirical time series. For the simulated time series, the GARCH-GPD-P is able to produce more accurate VaR and Expected Shortfall forecasts, especially at higher coverage levels (like 5%). At lower coverage levels, the differences in risk measures forecasting are not significant and it is difficult to determine which model is the best. As regards empirical time-series, there is even stronger evidence that the proposed GARCH-GPD-P model is able to perform more efficiently for high probabilities than the other competing models. For the Expected Shortfall forecasting, it seems to be of particular use as we obtained the most accurate estimates for the GARCH-GPD-P model.

This study can be extended in the future to better describe returns that are not deemed extreme observations but are forecasted by the GARCH-GPD-P model. One potential way to achieve this goal that is considered in the literature is to combine several VaR forecasting procedures (see Jeon & Taylor, 2013; McAleer et al., 2010, 2013).

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Appendix

Table A1. Summary of forecasting

Time series	First forecast date (1,000 obs. estimation)	First forecast date (500 obs. estimation)	Forecast end date	Number of forecasts (1,000 obs. estimation)	Number of forecasts (500 obs. estimation)
Amazon	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Apple	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Google	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
Microsoft	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
NVIDIA	Jan 2nd, 2010	Jan 2nd, 2008	May 31st, 2023	3,382	3,882
EURSUD	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
GBP/USD	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
USD/JPY	Nov 9th, 2009	Dec 5th, 2007	May 31st, 2023	3,512	4,012
BTC/USD	Sep 27th, 2017	May 15th, 2016	May 31st, 2023	2,073	2,573
ETH/USD	Sep 27th, 2020	May 16th, 2019	May 31st, 2023	977	1,477
LTC/USD	Sep 27th, 2018	May 15th, 2017	May 31st, 2023	1,708	2,208
XRP/USD	Sep 27th, 2020	May 16th, 2019	May 31st, 2023	977	1,477

Source: author’s work.

Table A2. The results of backtesting tests for VaR(10%): stocks

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
AMAZON	LR_U	4.0791	0.0434	0.5038	0.4778	0.3311	0.5650	0.1105	0.7395
	LR_I	2.4336	0.1188	9.5546	0.0020	7.0744	0.0078	13.224	0.0003
	LR_C	6.5127	0.0385	10.058	0.0065	7.4056	0.0247	13.335	0.0013
	J_{UC}	4.1365	0.0392	0.4398	0.5072	0.3868	0.5213	0.0784	0.7839
	J_{IND}	11.842	0.0199	25.228	0.0075	16.527	0.0134	17.415	0.0122
	J_{CC}	19.007	0.0019	26.697	0.0096	16.788	0.0176	17.650	0.0166
APPLE	LR_U	0.0018	0.9659	3.1891	0.0741	0.0959	0.7569	1.3078	0.2528
	LR_I	0.0000	0.9986	0.0987	0.7535	0.1254	0.7232	0.8083	0.3686
	LR_C	0.0018	0.9991	3.2877	0.1932	0.2213	0.8953	2.1161	0.3471
	J_{UC}	0.0092	0.9235	3.1811	0.0745	0.1301	0.7184	1.2320	0.2670
	J_{IND}	6.1819	0.0925	6.6702	0.0484	6.4365	0.0806	2.5328	0.3655
	J_{CC}	6.1249	0.2943	9.4953	0.0909	6.0720	0.1646	4.6959	0.4541
GOOGLE	LR_U	5.5084	0.0189	0.6183	0.4317	0.0257	0.8726	1.3650	0.2427
	LR_I	3.9694	0.0463	1.7509	0.1858	1.9726	0.1602	3.1286	0.0769
	LR_C	9.4778	0.0087	2.3692	0.3059	1.9983	0.3682	4.4937	0.1057
	J_{UC}	5.7103	0.0159	0.6895	0.4167	0.0469	0.8290	1.2840	0.2575
	J_{IND}	12.732	0.0201	4.7349	0.1478	19.336	0.0100	2.4504	0.3812
	J_{CC}	37.322	0.0057	5.3820	0.2064	18.469	0.0147	4.2231	0.3127
MICROSO FT	LR_U	5.2328	0.0222	0.8091	0.3684	0.1100	0.7402	0.1270	0.7216
	LR_I	0.9430	0.3315	1.2956	0.2550	1.3378	0.2474	1.7420	0.1869
	LR_C	6.1758	0.0456	2.1047	0.3491	1.4478	0.4849	1.8690	0.3928
	J_{UC}	5.4046	0.0199	0.8834	0.3520	0.1489	0.6999	0.0902	0.7711
	J_{IND}	7.2537	0.0632	5.1827	0.1177	7.4774	0.0575	0.9888	0.7242
	J_{CC}	21.074	0.0123	5.4394	0.1979	6.5939	0.1199	1.1588	0.8418
NVIDIA	LR_U	3.5074	0.0611	0.4528	0.5010	0.0257	0.8726	1.1061	0.2929
	LR_I	0.0068	0.9341	0.0442	0.8335	0.1928	0.6606	0.4240	0.5150
	LR_C	3.5142	0.1725	0.4970	0.7800	0.2185	0.8965	1.5300	0.4653
	J_{UC}	3.7415	0.0570	0.4496	0.4907	0.0274	0.8659	1.1375	0.2896
	J_{IND}	7.2300	0.0629	20.503	0.0081	15.013	0.0148	1.1509	0.6834
	J_{CC}	18.453	0.0164	16.590	0.0147	14.102	0.0260	2.7342	0.5162

Note. * indicates that the null hypothesis is rejected at a 5% significance level. LR_{UC} is the unconditional coverage test proposed by Kupiec (1995), LR_{IND}, LR_{CC} are the independence and conditional coverage tests, respectively, proposed by Christoffersen (1998). J_{UC}, J_{IND}, J_{CC} are the unconditional coverage, independence and conditional coverage tests, respectively, proposed by Candelon et al. (2011). For J_{IND} and J_{CC}, the number of moments is fixed to 5, *p*-values for J_{UC}, J_{IND}, J_{CC} are obtained by Dufour's (2006) Monte Carlo procedure based on 10,000 repetitions.
Source: author's work.

Table A3. The results of backtesting tests for VaR(10%) – currencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	P-value	Value	P-value	Value	P-value	Value	P-value
EUR/USD	LR_U	1.9391	0.1638	0.3823	0.5364	0.0134	0.9078	2.0916	0.0932
	LR_I	1.2127	0.2708	0.0097	0.9214	0.3539	0.5519	0.1695	0.6806
	LR_C	3.1518	0.2068	0.3920	0.8220	0.3673	0.8322	2.2611	0.3229
	J_{UC}	2.0339	0.1561	0.3746	0.5408	0.0135	0.9080	2.1980	0.1330
	J_{IND}	2.4774	0.3704	1.6541	0.5510	4.1887	0.1751	2.7031	0.3405
	J_{CC}	5.5107	0.1979	1.8269	0.6987	4.2697	0.2952	6.0168	0.1628
GBP/USD	LR_U	6.2695	0.0123	0.2628	0.6082	0.0003	0.9874	2.0172	0.1555
	LR_I	1.0918	0.2961	1.2412	0.2652	0.8109	0.3678	1.2425	0.2650
	LR_C	7.3613	0.0252	1.5040	0.4714	0.8112	0.6666	3.2597	0.1960
	J_{UC}	6.8452	0.0088	0.2674	0.6050	0.0002	0.9893	2.1179	0.1397
	J_{IND}	4.2050	0.1815	2.4302	0.3894	0.6305	0.8594	0.7408	0.8218
	J_{CC}	14.308	0.0240	3.0071	0.4712	0.6293	0.9513	2.9588	0.4771
USD/JPY	LR_U	6.3998	0.0114	0.1070	0.7436	0.0134	0.9078	2.9427	0.0863
	LR_I	3.3961	0.0654	5.0150	0.0251	3.7122	0.0540	1.6874	0.1939
	LR_C	9.7949	0.0075	5.1219	0.0772	3.7256	0.1552	4.6301	0.0988
	J_{UC}	6.6806	0.0085	0.0759	0.0784	0.0040	0.9417	2.9232	0.0833
	J_{IND}	1.8227	0.5077	7.2026	0.0599	6.0349	0.0906	5.5492	0.1085
	J_{CC}	9.3026	0.0613	7.3839	0.1065	6.0673	0.1701	9.2916	0.0618

Note. As in table A2.
Source: author’s work.

Table A4. The results of backtesting tests for VaR(10%): cryptocurrencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
BTC/USD	LR_{UC}	26.992	0.0000	0.5834	0.4450	0.232	0.6300	0.344	0.557
	LR_{IND}	6.3575	0.0117	4.7939	0.0286	2.776	0.0957	0.026	0.871
	LR_{CC}	33.350	0.0000	5.3773	0.0680	3.008	0.2222	0.371	0.830
	J_{UC}	33.518	0.0001	0.6637	0.4079	0.175	0.6737	0.285	0.581
	J_{IND}	5.2001	0.1149	12.865	0.0179	9.356	0.0331	3.955	0.196
	J_{CC}	64.762	0.0037	13.417	0.0275	9.576	0.0528	4.821	0.245
ETH/USD	LR_{UC}	16.633	0.0000	0.2473	0.6190	0.103	0.7474	2.876	0.089
	LR_{IND}	0.1029	0.7484	0.1566	0.6923	0.000	0.9885	0.465	0.495
	LR_{CC}	16.735	0.0002	0.4039	0.8171	0.104	0.9493	3.341	0.188
	J_{UC}	20.402	0.0001	0.1716	0.6716	0.055	0.8165	2.850	0.094
	J_{IND}	4.3022	0.1410	1.2979	0.5918	1.578	0.5162	1.881	0.489
	J_{CC}	34.508	0.0062	1.5168	0.7297	1.689	0.6895	6.078	0.163
LTC/USD	LR_{UC}	28.956	0.0000	0.3094	0.5780	0.420	0.5166	0.027	0.868
	LR_{IND}	2.5518	0.1102	1.1678	0.2799	3.192	0.0740	0.928	0.955
	LR_{CC}	31.508	0.0000	1.4772	0.4778	3.613	0.1642	0.955	0.620
	J_{UC}	38.443	0.0001	0.3174	0.5903	0.408	0.5206	0.007	0.912
	J_{IND}	1.0381	0.6958	1.8565	0.4593	4.404	0.1498	2.992	0.269
	J_{CC}	47.225	0.0053	2.3091	0.5715	4.580	0.2515	3.029	0.435
XRP/USD	LR_{UC}	32.036	0.0000	0.4531	0.5009	1.055	0.3044	1.172	0.278
	LR_{IND}	0.7016	0.4022	0.2634	0.6078	0.021	0.8846	0.994	0.318
	LR_{CC}	32.738	0.0000	0.7165	0.6989	1.076	0.5839	2.166	0.338
	J_{UC}	45.001	0.0001	0.3537	0.5651	0.928	0.3272	1.270	0.252
	J_{IND}	0.7966	0.7484	1.0683	0.6641	2.364	0.3541	2.802	0.292
	J_{CC}	61.381	0.0029	1.5788	0.7218	3.769	0.3306	3.806	0.330

Note. As in Table A2.
Source: author's work.

Table A5. The results of backtesting tests for VaR(5%) – stocks

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
AMAZON	LR_U	0.0224	0.8810	0.5991	0.4389	0.2143	0.6434	0.2143	0.6434
	LR_I	5.6770	0.0172	4.2360	0.0396	7.8459	0.0051	2.6128	0.1060
	LR_C	5.6994	0.0579	4.8351	0.0891	8.0602	0.0178	2.8272	0.2433
	J_{UC}	0.0515	0.8233	0.6948	0.3938	0.2847	0.5960	0.2847	0.5935
	J_{IND}	4.9644	0.1125	5.440	0.1166	7.4888	0.0474	4.7132	0.1280
	J_{CC}	4.9285	0.2174	4.9982	0.2178	6.7962	0.1182	4.2881	0.2814
APPLE	LR_U	1.3452	0.2461	4.9403	0.0262	0.1055	0.7454	0.2343	0.6283
	LR_I	0.1057	0.7452	0.0158	0.9001	0.0004	0.9845	0.5244	0.4690
	LR_C	1.4508	0.4841	4.9561	0.0839	0.1058	0.9485	0.7587	0.6843
	J_{UC}	1.4385	0.2279	4.7290	0.0291	0.0609	0.7908	0.1669	0.6873
	J_{IND}	2.3000	0.3695	0.3083	0.9429	2.3675	0.3566	1.6148	0.5066
	J_{CC}	3.4301	0.3773	5.2756	0.1901	2.7586	0.4780	1.7967	0.6730
GOOGLE	LR_U	0.1635	0.6860	0.5991	0.4389	0.6474	0.4211	0.3829	0.5361
	LR_I	1.1627	0.2809	1.3238	0.2499	1.6162	0.2036	0.8293	0.3625
	LR_C	1.3262	0.5153	1.9230	0.3823	2.2635	0.3225	1.2122	0.5455
	J_{UC}	0.1072	0.7456	0.6948	0.3875	0.5448	0.4677	0.4684	0.4959
	J_{IND}	21.430	0.0087	40.497	0.0031	17.582	0.0118	0.8869	0.7313
	J_{CC}	26.364	0.0098	27.592	0.0082	28.246	0.0086	1.1398	0.8248
MICROSOFT	LR_U	0.0521	0.8195	1.0188	0.3145	0.1481	0.7003	0.5232	0.4695
	LR_I	1.0671	0.3016	0.9807	0.3220	1.2124	0.2708	1.7593	0.1847
	LR_C	1.1192	0.5714	1.9925	0.3693	1.3606	0.5065	2.2825	0.3194
	J_{UC}	0.0925	0.7792	1.1114	0.2941	0.2094	0.6445	0.2847	0.6009
	J_{IND}	3.5941	0.2025	3.2414	0.2389	10.144	0.0247	4.0428	0.1580
	J_{CC}	3.4580	0.3708	3.7736	0.3289	8.6268	0.0666	3.9920	0.2986
NVIDIA	LR_U	0.4851	0.4861	1.5291	0.2163	0.0940	0.7592	0.5991	0.4389
	LR_I	0.2172	0.6411	0.0940	0.7592	0.0832	0.7729	0.2803	0.5965
	LR_C	0.7023	0.7039	1.6231	0.4442	0.1772	0.9152	0.8795	0.6442
	J_{UC}	0.4737	0.4807	1.4475	0.2207	0.0949	0.7589	0.5822	0.4320
	J_{IND}	6.2601	0.0726	10.686	0.0241	5.1656	0.1013	0.6773	0.8045
	J_{CC}	5.7040	0.1675	10.040	0.0507	4.8906	0.2194	1.0570	0.8488

Note. As in Table A2.
Source: author’s work.

Table A6. The results of backtesting tests for VaR(5%): currencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
EUR/USD	LR_U	0.9921	0.3192	1.1452	0.2846	0.7194	0.3963	1.1452	0.2846
	LR_I	0.0135	0.9074	0.0439	0.8341	0.1560	0.6929	0.9439	0.3313
	LR_C	1.0056	0.6048	1.1891	0.5518	0.8754	0.6455	2.0891	0.3518
	J_{UC}	1.0411	0.3045	1.2063	0.2737	0.7494	0.3954	0.0925	0.7655
	J_{IND}	6.8731	0.0639	1.2510	0.6170	4.2890	0.1583	1.0700	0.6629
	J_{CC}	10.662	0.0465	3.3908	0.3906	6.4944	0.1316	1.0976	0.8293
GBP/USD	LR_U	0.5862	0.4439	1.7761	0.1826	0.0022	0.9624	0.1662	0.6835
	LR_I	0.0004	0.9846	0.1071	0.7434	0.8959	0.3439	0.2623	0.6085
	LR_C	0.5865	0.7458	1.8832	0.3900	0.8981	0.6382	0.4286	0.8071
	J_{UC}	0.5658	0.0452	1.6718	0.1932	0.0022	0.9566	0.1631	0.6738
	J_{IND}	9.5343	0.0330	9.1258	0.0318	4.5468	0.1411	0.5752	0.8484
	J_{CC}	8.7007	0.0690	9.6805	0.0561	4.5092	0.2585	0.6444	0.9337
USD/JPY	LR_U	1.1452	0.2846	0.4570	0.4990	0.0008	0.9769	0.0008	0.9769
	LR_I	0.0177	0.8942	0.3870	0.5339	0.5052	0.4772	0.1308	0.7176
	LR_C	1.1629	0.5591	0.8439	0.6558	0.5060	0.7765	0.1317	0.9363
	J_{UC}	1.0411	0.2995	0.5396	0.4640	0.0102	0.9230	0.0102	0.9349
	J_{IND}	0.0303	0.9999	2.7671	0.3096	0.5864	0.8472	1.7285	0.4937
	J_{CC}	1.1197	0.8346	3.2947	0.4106	0.5890	0.9472	1.7212	0.6941

Note. As in Table A2.
Source: author’s work.

Table A7. The results of backtesting tests for VaR(10%): cryptocurrencies

Time series	Statistic	GARCH-n		GARCH-t		GARCH-GPD		GARCH-GPD-P	
		Value	*p*-value	Value	*p*-value	Value	*p*-value	Value	*p*-value
BTC/USD	LR_{UC}	26.992	0.0000	0.5834	0.4450	0.232	0.6300	0.344	0.557
	LR_{IND}	6.3575	0.0117	4.7939	0.0286	2.776	0.0957	0.026	0.871
	LR_{CC}	33.350	0.0000	5.3773	0.0680	3.008	0.2222	0.371	0.830
	J_{UC}	33.518	0.0001	0.6637	0.4079	0.175	0.6737	0.285	0.581
	J_{IND}	5.2001	0.1149	12.865	0.0179	9.356	0.0331	3.955	0.196
	J_{CC}	64.762	0.0037	13.417	0.0275	9.576	0.0528	4.821	0.245
ETH/USD	LR_{UC}	16.633	0.0000	0.2473	0.6190	0.103	0.7474	2.876	0.089
	LR_{IND}	0.1029	0.7484	0.1566	0.6923	0.000	0.9885	0.465	0.495
	LR_{CC}	16.735	0.0002	0.4039	0.8171	0.104	0.9493	3.341	0.188
	J_{UC}	20.402	0.0001	0.1716	0.6716	0.055	0.8165	2.850	0.094
	J_{IND}	4.3022	0.1410	1.2979	0.5918	1.578	0.5162	1.881	0.489
	J_{CC}	34.508	0.0062	1.5168	0.7297	1.689	0.6895	6.078	0.163
LTC/USD	LR_{UC}	28.956	0.0000	0.3094	0.5780	0.420	0.5166	0.027	0.868
	LR_{IND}	2.5518	0.1102	1.1678	0.2799	3.192	0.0740	0.928	0.955
	LR_{CC}	31.508	0.0000	1.4772	0.4778	3.613	0.1642	0.955	0.620
	J_{UC}	38.443	0.0001	0.3174	0.5903	0.408	0.5206	0.007	0.912
	J_{IND}	1.0381	0.6958	1.8565	0.4593	4.404	0.1498	2.992	0.269
	J_{CC}	47.225	0.0053	2.3091	0.5715	4.580	0.2515	3.029	0.435
XRP/USD	LR_{UC}	32.036	0.0000	0.4531	0.5009	1.055	0.3044	1.172	0.278
	LR_{IND}	0.7016	0.4022	0.2634	0.6078	0.021	0.8846	0.994	0.318
	LR_{CC}	32.738	0.0000	0.7165	0.6989	1.076	0.5839	2.166	0.338
	J_{UC}	45.001	0.0001	0.3537	0.5651	0.928	0.3272	1.270	0.252
	J_{IND}	0.7966	0.7484	1.0683	0.6641	2.364	0.3541	2.802	0.292
	J_{CC}	61.381	0.0029	1.5788	0.7218	3.769	0.3306	3.806	0.330

Note. As in Table A2.
Source: author's work.