Higher-order Markov chains for capital market decision-making

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Abstract. Analysing capital market returns is fundamental to decision-making by individual investors. Advanced methods require extensive knowledge and appropriate tools, whereas individual investors often make decisions intuitively or after a very simplified analysis. The aim of the study discussed in this paper is to present the idea of higher-order Markov chains and their models and to demonstrate that the combination of higher-order Markov chains with the technical analysis in its basic form provides support for investment decisions. This approach takes into account three aspects. The first one is the linguistic practice of observing rates of return through the construction of rate-of-return intervals (a large increase, a small decrease, no change, etc.), the second is related to investors' attitude towards risk through the aggregation of return intervals and the selection of investment strategies based on technical analysis, and the third concerns the investor's memory horizon through the construction of higher-order Markov chains.

Orlen's quotations at the Warsaw Stock Exchange (daily quotations – closing price) for the period from January 2000 to April 2025 serve as the numerical example in the study.

Keywords: Markov chain, decision-making, capital market analysis

JEL: C58, F47, G17, G41

1. Introduction

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The problem of decision-making under the conditions of uncertainty and risk has been analysed by many scholars. The scientific search for methods that could facilitate making investment decisions has been going on for over 100 years. It is linked to the scientific debate on preferences and probability (e.g. Keynes, 1921; Ramsey, 1928). Later, psychological aspects of decision-making were analysed (e.g. Allais, 1953; Kahneman & Tversky,1979). Many methods have been developed, taking into account various measurement conditions and the determination of multiple criteria (Trzaskalik, 2014). In many cases, the selection of decision-supporting methods has been automated (Cinelli et al., 2020). The search for quick answers under the conditions of uncertainty and risk often leads to the simplest mechanism, i.e. 'flipping a coin' for weather the rate of return will increase or decrease.

The belief that the process of decision- making based on observation has a simple 'rise and fall' mechanism is so obvious that no effort is made to verify whether 'the coin is not fake'. Nor is the independence of the throws of this 'coin' verified. The coin can be replaced by a multidimensional dice, i.e., by analysing of distributions in a finite *n*-dimensional state space, understanding states as small declines in rates of return, large declines in rates of return, and so on. The independence of the 'dice' throws should be replaced by the study of conditional distributions. Obviously, there are sophisticated econometric models for analysing the phenomenon of changes in the rate of return. Many of these methods incorporate behavioural determinants. There are several monographs, reprinted translations and current studies, concerning both models and decision analysis, within this popular field of research (Adamczyk-Kowalczuk, 2022; Borowski, 2014). Markov chains, including higher-order ones, have been widely used in the capital market analysis (Stawicki, 2004, 2016) and in many other analyses of economic processes, such as in the analysis of business cycle test results (Podgórska & Decewicz, 2001). Modern analyses use the Markov mechanism to make decisions by means of state models in a binary-time representation (Stasiak, 2025; Stasiak et al., 2025). The proposal of an extremely simple mechanism, such as the Markov chain, allows such a model to be linked to the well-known and widely used analyses in the field of technical analysis, particularly in the area of pattern analysis. These analyses are reduced to short observations, while retaining their analytical nature.

Technical analysis is the analysis of charts. The purpose of technical analysis (TA) is to determine the best times to buy or sell a given security, or when to hold a decision. Alongside fundamental analysis, it is a basic tool for stock market investors. The new proposals are based on previously developed principles (Murphy, 2017). Technical analysis is essentially based on the following three basic assumptions:

- the market discounts everything, i.e. the price of a company reflects everything that is happening on the market and in the environment (microeconomic and macroeconomic situation, economic conditions, political conditions and all other information relating to a given security);
- prices are subject to trends, i.e. share prices follow specific trends, either downward (bear market) or upward (bull market). A change in a trend is clearly signalled;
- history repeats itself, which means that using technical analysis, we examine the future based on the past, and individual formations that have occurred previously may provide information about the possible direction of change.

These charts usually refer to longer observations. The decisions are long-term. A similar analysis can be applied to subsequent daily observations. In this case, decisions concern a short period. Such a tool is only the aid to the decision-making process.

2. Higher-order Markov chains

Markov chains are a well-known tool used in economics (Ching & Ng, 2006; Decewicz, 2011; Kemeny & Snell, 1976; Podgórska et al., 2002; Stawicki, 2004; and others). A Markov process with a discrete time parameter and a discrete phase space is referred to as a Markov chain.

Let $\{Y_t\}$ be a stochastic process.

The Markov property, which is the basis for defining a Markov chain, has the following form:

$$Pr\{Y_{t+1}|Y_t, Y_{t-1}, \dots Y_1, Y_0\} = Pr\{Y_{t+1}|Y_t\}. \quad (1)$$

Let $S = \{s_1, s_2, \dots, s_r\}$ be a finite set of states in which process $\{Y_t\}$ is represented by observations $\{y_t\}$. We will define set S in an abbreviated form:

$$S = \{1, 2, \dots, r\}.$$

We will record our observations of process $\{Y_t\}$ as a sequence $\{y_t\}$. For example, the observation sequence presented in the next section will be the daily rate of return on securities listed on the stock exchange.

A Markov chain is defined by a sequence of stochastic matrices in the following form:

$$\mathbf{P}(\mathrm{t}) = \lfloor p_{ij}(t) \rfloor_{r \times r}, \quad (2)$$

i.e., matrices with positive elements and satisfying additional conditions expressed by:

$$\forall_t \forall_i \sum_j p_{ij}(t) = 1, \tag{3}$$

where $p_{ij}(t) = Pr\{Y_t = j | Y_{t-1} = i\}$ is a conditional probability.

By denoting the vector of unconditional distribution of random variable Y_t with $\mathbf{D}_t$, i.e.,

$$\mathbf{D}_t = [d_{1t}, \quad d_{2t}, \quad \cdots, \quad d_{rt}], \text{ where } d_{it} = Pr\{Y_t = i\}, \tag{4}$$

we determine the probability with which the process reaches the phase state in time *t*. The components of vector $\mathbf{D}_t$ satisfy the following conditions:

$$\forall_t \forall_i \quad d_{it} \geq 0, \tag{5}$$

and

$$\forall_t \quad \sum_i d_{it} = 1. \tag{6}$$

The dependence between unconditional distributions of random variables Y_t and Y_{t-1} is expressed by the formula resulting from the theorem on the total probability:

$$\mathbf{D}_t = \mathbf{D}_{t-1} \cdot \mathbf{P}(t). \tag{7}$$

Matrices $\mathbf{P}(\mathrm{t}) = \lfloor p_{ij}(t) \rfloor_{r \times r}$ reflect the mechanism of changes in the distribution of the analysed random variable Y_t over time.

A Markov chain $\{Y_t, \quad t \in N\}$ with a phase space $S = \{1, 2, ..., r\}$ is called *a homogeneous Markov chain* if the conditional probabilities $p_{ij}(t)$ of transition from state i to state j within a time unit, i.e., in the time period from $(t-1)$ to t, do not depend on the choice of the moment t, that is:

$$\forall_t \; p_{ij}(t) = p_{ij} . \tag{8}$$

In the case of a homogeneous Markov chain, the dependence (2.7) takes the following form:

$$\mathbf{D}_t = \mathbf{D}_{t-1} \cdot \mathbf{P} . \tag{9}$$

If the Markov property, which is the basis for defining a higher-order Markov chain, has the following form:

$$Pr\{Y_{t+1} | Y_t, Y_{t-1}, \dots, Y_1, Y_0\} = Pr\{Y_{t+1} | Y_t, Y_{t-1}, \dots, Y_{t-k}\}, \tag{10}$$

a higher-order Markov chain order k can be represented by matrix $\mathbf{Q}$, where each row represents a k-element variation with repetitions from an r-element set of process states, and each column represents one of the states of this process. This matrix has dimensions $r^k \times r$.

By denoting the vector of unconditional distribution of random variable Y_t with $\mathbf{D}_t$, i.e.,

$$\mathbf{D}_t = \begin{bmatrix} d_{1t}, & d_{2t}, & \cdots, & d_{rt} \end{bmatrix}\text{, where } d_{it} = Pr\{Y_t = i\},$$

and the vector of distribution of possible histories in the last k periods with $\mathbf{H}_t = \left[h_{1t}, h_{2t}, \dots, h_{r^k t}\right]$, i.e.,

$$h_{lt} = Pr\{[Y_{t-k}, Y_{t-k+1}, \dots, Y_{t-1}]\} \tag{11a}$$

or

$$h_{lt} = Pr\{i(k)_{t-k}, i(k-1)_{t-k+1}, \dots, i(1)_{t-1}\}, \tag{11b}$$

we determine the probability with which the process reaches phase state i at time t, in the following form:

$$\mathbf{D}_t = \mathbf{H}_t \cdot \mathbf{Q}. \tag{12}$$

This history is observed in the form of a vector of realised state sequences, i.e.,

$$\mathbf{H}_t = [0.0. \dots, 0,1,0, \dots, 0].$$

Matrix **Q** contains conditional distributions, where the condition is the history in the last k periods.

The observation of the process is based on microdata. The parameters of matrices **P** and **Q** are obtained using the maximum likelihood estimator formula:

$$\hat{p}_{(i_k,i_{k-1},\dots,i_1),j} = \frac{\sum_{t=k+1}^{T} v_{(i_k,i_{k-1},\dots,i_1),j}(t)}{\sum_{t=k+1}^{T} n_{(i_k,i_{k-1},\dots,i_1)}(t)}, \tag{13}$$

where

$$v_{(i_k,i_{k-1},\dots,i_1),j}(t) = \begin{cases} 1 \; if \; at \; time \; t \; after \; history \; (i_k, i_{k-1}, \dots, i_1), state \; j \; occurred, \\ 0 \; otherwise \end{cases}$$

and

$$n_{(i_k,i_{k-1},\dots,i_1)}(t) = \begin{cases} 1 \; if \; at \; time \; t, the \; observed \; history \; is \; (i_k, i_{k-1}, \dots, i_1), \\ 0 \; otherwise \end{cases}.$$

The quantity $\sum_{t=k+1}^{T} n_{(i_k,i_{k-1},\dots,i_1)}(t)$ from formula (2.13) determines the number of all the observed histories $(i_k, i_{k-1}, \dots, i_1)$, while the quantity $\sum_{t=k+1}^{T} v_{(i_k,i_{k-1},\dots,i_1),j}(t)$ determines the number of these histories followed by state *j*.

If the history is a single state *i*, the process becomes a classical Markov chain.

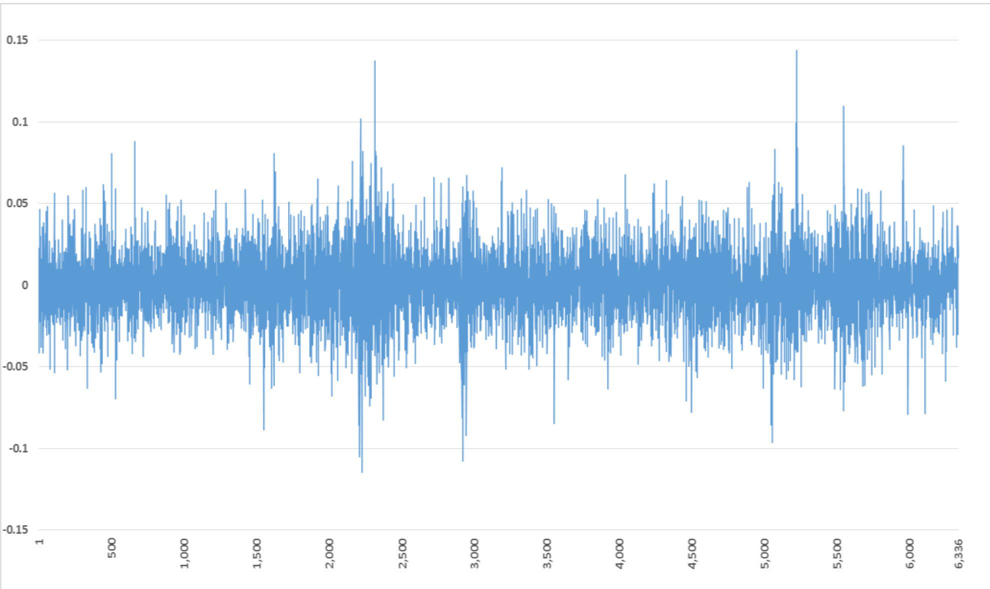
To test the hypothesis that a given row of matrix **Q** defining conditional transitions from history $(i_k, i_{k-1}, \dots, i_1)$ to individual states *j* is equal to the established probabilities, we use a chi-squared test with (r-1) degrees of freedom, using the statistic:

$$\chi^2 = \sum_{t=k+1}^{T} n_{(i_k,i_{k-1},\dots,i_1)}(t) \cdot \sum_{j=1}^{r} \frac{(\hat{p}_{(i_k,i_{k-1},\dots,i_1),j} - p_{(i_k,i_{k-1},\dots,i_1),j})^2}{p_{(i_k,i_{k-1},\dots,i_1),j}}. \tag{14}$$

3. Decision-making of individual investors: the example of Orlen company

We assume that investors on the stock market make decisions based on available observations of the rate of return on an asset. These observations cover a long period, but the decision is made on the basis of the last observation or several recent observations. The length of time covered by the memory is a specific attitude of the investor. Another specific feature of the investor is the way they view the observations. Investors may only be interested in the direction of changes in the rate of return or the ranges in which the rate of return is observed. The number of ranges and their sizes are determined by the decision-maker. In our study, the ranges will be determined according to specific rules.

Figure 1. Daily rate of return – closing price for Orlen (January 2000 – April 2025)



Source: authors' work.

Figure 1 shows the rate of return quotations that will be the basis for the analysis.

If the decision-maker only analyses stock market ups and downs, we consider a Markov chain with two states $s_1 = up, s_2 = down$. Therefore $\mathbf{D}_t = (u_t, d_t)$. For the entire studied period,

$$D = (0.5066, 0.4934).$$

It can be said that this vector represents the coin-tossing accurately. The Chi-square statistics is:

$$\chi^2 = 0.5347 \text{ where chi-square critical value is } \chi^2_{0.05} = 3.841.$$

The corresponding p-value is 0.3011.

The first-order Markov chain model shows that conditional distributions are also distributions $\overline{p} = (0.5\ , 0.5)$.

The matrix for this model has the following form:

$$\mathbf{P} = \begin{bmatrix} 0.4944 & 0.5056 \\ 0.5176 & 0.4824 \end{bmatrix},$$

and the corresponding chi-squared statistics are: for the first row (*u* is the condition of growth of the rate of return at the previous time) $\chi^2 = 0.384$, and for the second row (*d* is the condition of decrease of the rate of return at the previous time) $\chi^2 = 3.617$, where the chi-square critical value is $\chi^2_{0.05} = 3.841$. The corresponding p-values are 0.5354 and 0.0572.

The second-order Markov chain model is represented by matrix **Q**:

$$\mathbf{Q} = \begin{matrix} uu \\ ud \\ du \\ dd \end{matrix} \begin{bmatrix} 0.4580 & 0.5420 \\ 0.5007 & 0.4993 \\ 0.5266 & 0.4734 \\ 0.5348 & 0.4652 \end{bmatrix}.$$

Additionally, the form of the condition was marked with matrix **Q**. Chi-squared statistics for each row of matrix **Q** and p-value are presented in Table 1 below.

Table 1. Chi-square statistics and p-value for the second-order Markov chain

condition	chi-square statistic	p-value
uu	10.252	0.0014
ud	0.003	0.9584
du	4.150	0.0416
dd	6.688	0.0097

Source: authors' work.

Chi-square critical value is $\chi^2_{0.05} = 3.841$. Therefore, the conditions (uu, du, dd) significantly change the probabilities of transition to states u and d.

This is even more evident when looking at history going back three times. The matrix representing the third-order Markov chain model has the following form:

$$\mathbf{Q} = \begin{matrix} uuu \\ uud \\ udu \\ udd \\ duu \\ dud \\ ddu \\ ddd \end{matrix} \begin{bmatrix} 0.4537 & 0.5463 \\ 0.4823 & 0.5177 \\ 0.5251 & 0.4749 \\ 0.5239 & 0.4761 \\ 0.4511 & 0.5489 \\ 0.5253 & 0.4747 \\ 0.5289 & 0.4711 \\ 0.5510 & 0.4490 \end{bmatrix}$$

For each row of matrix **Q**, p-values for statistics chi-squared are as presented in Table 2.

Table 2. Chi-square statistics and p-value for third-order Markov chain

condition	Chi-square statistic	p-value
uuu	5.556	0.0184
uud	0.958	0.3277
udu	1.810	0.1785
udd	1.624	0.2026
duu	7.243	0.0071
dud	1.720	0.1897
ddu	2.371	0.1236
ddd	6.522	0.0107

Source: authors' work.

The chi-square critical value is $\chi^2_{0.05} = 3.841$. Therefore, the conditions (uuu, duu, ddd) significantly change the probabilities of transition to state u and state d.

Seeing the last three quotations as upwards, the decision-maker should take into account different probabilities of the occurrence of states u and d.

The second example, based on the same observations of the Orlen company's return quotation process, is constructed using Markov chain states as intervals in which the observed rate of return may be included. These states are defined as: s1– an increase, s2 – no change, and s3 – a decrease. The intervals for these states are determined by the decision-maker. Decision-makers differ from one another and establishing the same ranges for everyone is extremely difficult. The intervals below have been set so that each of them contains 1/3 of the observations from the entire period of the studied rate of return. This example refers to 'tossing a coin' or rather to 'throwing a three-sided dice'.

Table 3 below describes the states.

Table 3. Intervals and distribution for the entire observation period

	Return intervals for quotations	Number of observations	Unconditional probability
s_1	to -0.0076	2,099	0.3314
s_2	(-0.0076; 0.00818)	2,109	0.3328
s_3	from 0.00818	2,128	0.3358

Source: authors' work

A second-order Markov chain model is defined by matrix **Q**:

$$\mathbf{Q} = \begin{matrix} s1,s1 \\ s1,s2 \\ s1,s3 \\ s2,s1 \\ s2,s2 \\ s2,s3 \\ s3,s1 \\ s3,s2 \\ s3,s3 \end{matrix} \begin{bmatrix} 0.3246 & 0.2768 & 0.3986 \\ 0.3233 & 0.3262 & 0.3505 \\ 0.3253 & 0.2999 & 0.3748 \\ 0.3170 & 0.3511 & 0.3319 \\ 0.2925 & 0.3592 & 0.3483 \\ 0.3462 & 0.3805 & 0.2733 \\ 0.3433 & 0.3188 & 0.3379 \\ 0.3455 & 0.3581 & 0.2964 \\ 0.3647 & 0.3265 & 0.3088 \end{bmatrix}$$

Therefore, the estimated matrix does not give equal distributions identical to the distribution:

$$p = (\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}).$$

For each conditional distribution, the p-values for the chi-square test are presented in Table 4 below.

Table 4. A chi-square statistic and a p-value for the second-order Markov chain for the three states

condition	Chi-square statistic	p-value
s1,s1	15.5739	0.0004
s1,s2	0.8822	0.6433
s1,s3	6.5141	0.0385
s2,s1	1.1822	0.5537

s2,s2	5.6408	0.0596
s2,s3	12.5923	0.0018
s3,s1	0.7302	0.6941
s3,s2	4.5534	0.1026
s3,s3	3.3294	0.1892

Source: authors' work.

Observing the presented process two periods back, the distributions were significantly different from the uniform distribution only for two conditions.

The third example refers directly to the technical analysis used by decision-makers investing in the stock market. Technical analysis is a method of predicting the movement of stock prices by analysing charts of historical prices. Higher-order Markov chains fit into this understanding of price process analysis (return on investment).

In this example, six states of the Markov chain were assumed and described in linguistic terms. A state is understood as a range in which the rate of return will fall or rise. Table 5 presents the states in the studied example.

Table 5. Description of the Markov chain states

State	Linguistic description	Interval
s_1	A large drop in the rate of return	- 0.02 or less
s_2	A drop in the rate of return	(- 0.02 , - 0.01)
s_3	A small drop in the rate of return	(- 0.01 , 0.00)
s_4	A small increase in the rate of return	(0.00 , 0.01)
s_5	An increase in the rate of return	(0.01 , 0.02)
s_6	A large increase in the rate of return	0.02 or more

Source: authors' work.

Each decision-maker establishes an individual interval. This depends on the decision-maker's individual characteristics, their attitude towards risk (inclination or aversion), their knowledge and experience, and external circumstances influencing the course of economic processes. The proposed ranges are illustrative only and do not represent a specific decision-maker. The adoption of specific ranges generates the distribution of the rate of return over the entire period and the behaviour of the process within the examined time period. One such proposal is presented in Table 6.

Table 6. Probability distribution by the state of rate of return on Orlen quotations

state	probability
s1	0.141730
s2	0.143308
s3	0.224432
s4	0.195391
s5	0.143782
s6	0.151357

Source: authors' work.

Understanding the states is an individual characteristic of the investor. It is the investor who determines the ranges described by, for example, 'a small increase in rate of return' or 'a small decrease in rate of return'.

We present the transition matrix for a second-order Markov chain and the chi-square test for each conditional distribution in Table 7.

Table 7. Conditional distribution and the p-values for a second-order Markov chain

	s1	s2	s3	s4	s5	s6	p-value for a chi-square test
s1s1	0.2273	0.0584	0.1234	0.1883	0.1623	0.2403	0.0000
s1s2	0.1626	0.1301	0.1220	0.1789	0.1951	0.2114	0.0401
s1s3	0.1761	0.1056	0.1901	0.1901	0.1831	0.1549	0.4295
s1s4	0.1429	0.1429	0.1565	0.1701	0.1769	0.2109	0.1524
s1s5	0.1597	0.1345	0.2017	0.1849	0.1597	0.1597	0.9673
s1s6	0.1250	0.1369	0.1726	0.1726	0.1310	0.2619	0.0053
s2s1	0.1773	0.1560	0.2057	0.1489	0.1489	0.1631	0.6459
s2s2	0.1635	0.1346	0.1923	0.1635	0.2019	0.1442	0.5636
s2s3	0.1438	0.1918	0.1781	0.2192	0.1233	0.1438	0.4536
s2s4	0.1374	0.1209	0.2143	0.1923	0.1813	0.1538	0.7694
s2s5	0.1324	0.1691	0.1838	0.2279	0.1397	0.1471	0.7787
s2s6	0.1579	0.1053	0.1729	0.2331	0.1353	0.1955	0.3156
s3s1	0.1438	0.1438	0.1688	0.2250	0.1250	0.1938	0.3890
s3s2	0.1807	0.1145	0.1687	0.2229	0.1687	0.1446	0.2646
s3s3	0.1240	0.1240	0.1983	0.2273	0.1818	0.1446	0.3396
s3s4	0.0781	0.1641	0.2500	0.2188	0.1563	0.1328	0.0648

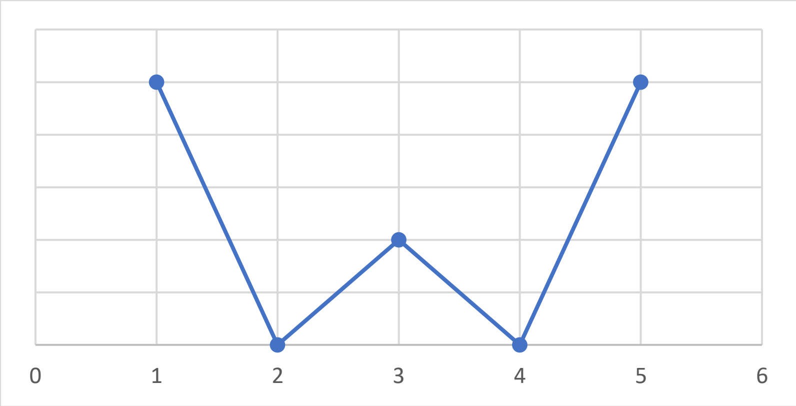
s3s5	0.0978	0.1250	0.2283	0.2663	0.1250	0.1576	0.1466
s3s6	0.1631	0.1560	0.2340	0.1915	0.1206	0.1348	0.9166
s4s1	0.1812	0.2101	0.1594	0.1667	0.1087	0.1739	0.0589
s4s2	0.1397	0.1285	0.1564	0.2626	0.1229	0.1899	0.0645
s4s3	0.1401	0.1440	0.2335	0.2179	0.1634	0.1012	0.3331
s4s4	0.1076	0.1659	0.2601	0.1973	0.1256	0.1435	0.4800
s4s5	0.0990	0.2178	0.2178	0.2178	0.0941	0.1535	0.0129
s4s6	0.1585	0.1402	0.2622	0.1768	0.1037	0.1585	0.6142
s5s1	0.1770	0.1858	0.1593	0.1150	0.1681	0.1947	0.0709
s5s2	0.1702	0.1206	0.1986	0.2270	0.1560	0.1277	0.6808
s5s3	0.1173	0.1676	0.2179	0.2346	0.1508	0.1117	0.4340
s5s4	0.1277	0.1702	0.2340	0.1809	0.1809	0.1064	0.3260
s5s5	0.1284	0.1743	0.2477	0.1651	0.1468	0.1376	0.8782
s5s6	0.1450	0.1679	0.2137	0.1985	0.1221	0.1527	0.9571
s6s1	0.1544	0.1342	0.1879	0.1879	0.1342	0.2013	0.5881
s6s2	0.1860	0.1318	0.1938	0.2171	0.1473	0.1240	0.6380
s6s3	0.1497	0.1551	0.2032	0.2674	0.1230	0.1016	0.1006
s6s4	0.1455	0.2061	0.1818	0.1576	0.2000	0.1091	0.0267
s6s5	0.1982	0.1081	0.1802	0.1892	0.1261	0.1982	0.2784
s6s6	0.2138	0.1887	0.2075	0.1321	0.1195	0.1384	0.0296

Source: authors' work.

By looking only at the last two observations, we can check the chances of an increase or a decrease in the price of the observed asset before making an investment decision. In the case under study, if the last two observations were significantly upward, a decline should be expected with a probability of over 0.6, or, more precisely, the sum of the transition probabilities from the state (s6,s6) to states s1, s2, s3 is 0.2138+0.1887+0.2075 = 0.61.

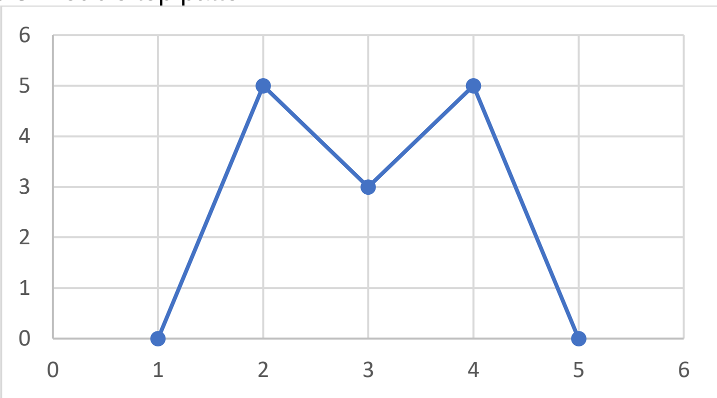
The analysis of somewhat complex patterns in technical analysis mentioned in the introduction requires a large number of observations. To examine the consequences of a double bottom or double top pattern in the short term, it is sufficient to know the last four observations, which become a condition in the fourth-order Markov chain (Figure 2 and 3).

Figure 2. Double bottom pattern



Source: authors' work.

Figure 3. Double top pattern



Source: authors' work

For the rate of return change process, this will be a sequence of successive states observed in history:

- a double bottom pattern, i.e. a large decline, a small increase, a small decline, a large increase;
- a double top pattern, i.e. a large increase, a small decline, a small increase, a large decline.

The fuzzy nature of such patterns can be assumed, including a similar behaviour of the rate of return in terms of declines and increases in the assumed sequence. The sequence of the last observed states, namely s1, s4, s3 and s6, is representative of such a sequence. For the double bottom pattern, the set of state sequences was assumed {s1,s4,s3,s6; s1,s5,s2,s6; s1,s4,s2,s6; s1,s5,s3,s6; s2,s4,s3,s5}. Similarly, for the double top pattern, the set of state sequences was assumed {s6,s2,s4,s1; s6,s3,s5,s2; s6,s3,s4,s1; s6,s2,s5,s1; s5,s3,s4 s2}.

Analysing the fourth-order Markov chain with states as in the last example and estimating conditional probabilities, we obtain the results presented in Table 8.

Table 8. Selected conditional probabilities for a fourth-order Markov chain

	s1	s2	s3	s4	s5	s6
double bottom pattern	0.2105	0.0000	0.2632	0.2105	0.1579	0.1579
double top pattern	0.0000	0.2500	0.1000	0.2500	0.1500	0.2500

Source: authors' work.

By comparing the obtained conditional distribution with the distribution presented in Table 6 and using the chi-square test, we get the following p-values: 0.4594 for the double bottom pattern, and 0.3141 for the double top pattern.

To support decision-making, any sequence of the observed states can be used to estimate the conditional distribution. For an observation period that is too short, estimating the conditional distribution may prove undesirable. However, if the investor's memory horizon is not too distant, the conditional distribution becomes a supportive forecast.

For example, if the observations at times t-4, t-3, t-2 and t-1 are as follows: s1, s4, s3, s6, then a vector of conditional probabilities takes the form of a double bottom pattern. The conditional distribution in this situation is presented in Table 9.

Table 9. Special case of conditional probability for a fourth-order Markov chain

	s1	s2	s3	s4	s5	s6
s1, s4, s3, s6	0.1667	0.0000	0.1667	0.5000	0.0000	0.1666

Source: authors' work.

This means that investors can expect growth rather than decline (the sum of the probabilities of transitions to states s4, s5, s6), and if there is a decline, it is most likely to be minor (the probability of transition to state s1 and s3). The small number of such observed sequences of states does not allow drawing far-reaching conclusions. Comparing this distribution with the pattern in Table 6 gives a p-value of 0.5619.

4. Conclusions

The proposed approach to investment decisions is based on a very simple Markov chain mechanism. To utilise this 'old-school' tool, we adopted a personalised approach to capital market decision-making. The numerical intervals of the rate-of-return were determined by incorporating a linguistic description of the rate's changes. This allows an individualised approach for each investor. The decision-maker's memory horizon for the observed rates of return has also been taken into account, and the assumption of constant transition probabilities has been adopted in the analysis. Modifying the transition probability matrix by incorporating characteristics for specific periods (e.g. macroeconomic observations, company developments or specific global situations influencing stock prices) will support investment decisions more effectively. The proposed simple approach to analysing rates of return can be used by individual investors in the situation where they do not see the need for a sophisticated analysis and want to make decisions quickly. This method may prove useful in an algorithmic trading system based on models with binary-time representation.

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